

Some Generalized Information and Divergence Generating Functions: Properties, Estimation, Validation and Application

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Abstract

We propose Renyi information generating function and discuss its properties. The relation between the Renyi information generating function and Shannon entropy of order $q > 0$ is established and several bounds are obtained. Furthermore, we introduce Renyi divergence information generating function and discuss its effect under monotone transformations. We present non-parametric estimators of the Renyi information generating function and Golomb information generating function. For three coherent systems, we calculate the values of the Renyi information generating function and other well-established uncertainty measures and similar behaviour of the Renyi information generating function is observed. Finally, two chaotic maps are considered and then used to establish a validation of the proposed information generating function.

Keywords: Information generating function, Rényi entropy, Rényi divergence, Monotone transformation, Estimation, Coherent system, Chaotic maps.

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- Entropy measures the amount of uncertainty of a random variable.
- The **Shannon entropy** (Shannon (1948)) of a non-negative absolutely continuous random variable X with PDF $f(\cdot)$ is defined as

$$H(X) = - \int_0^{\infty} f(x) \log f(x) dx. \quad (1)$$

- The Shannon entropy compresses information or data in communication theory. It is used in other fields, such as biology, behavioural science, statistical mechanics and neuroscience.



- The **Rényi entropy** of X and **Rényi divergence** between X and Y are, respectively, given by (see Rényi (1961))

$$H_\alpha(X) = \frac{1}{\alpha} \log \int_0^\infty f^\alpha(x) dx \text{ and } RD^\alpha(X, Y) = \frac{1}{\alpha} \log \int_0^\infty f^\alpha(x) g^{1-\alpha}(x) dx, \quad (2)$$

where $\delta(\alpha) = \frac{1}{1-\alpha}$, $\delta^*(\alpha) = \frac{1}{\alpha-1}$, $0 < \alpha < \infty$, $\alpha \neq 1$.

- Golomb (1966) introduced a generating function of information measure, known as the **Golomb's IGF** of X , for $\gamma > 0$, is defined as

$$G_\gamma(X) = \int_0^\infty f^\gamma(x) dx. \quad (3)$$

- Guiasu and Reischer (1985) proposed **relative IGF**. For random variables X and Y , the relative IGF, for $\theta > 0$, is given by

$$RI_\theta(X, Y) = \int_0^\infty f^\theta(x) g^{1-\theta}(x) dx. \quad (4)$$



Literature Review & Motivation

- Golomb (1966) introduced an information generating function for discrete and continuous random variables. The author studied some properties and discussed how the Shannon entropy in (1) generates from the generating function in (3).
- Guiasu and Reischer (1985) proposed relative IGF using the Golomb's IGF for discrete and continuous random variables.
- Kharazmi and Balakrishnan (2021) discussed various properties of information generating function for continuous random variable. They also proposed Jensen information generating function and connection with Jensen-Shannon, Jensen-entropy and Jensen-Taneja divergent measures.
- Recently, Saha and Kayal (2024) introduced general weighted IGF, general weighted relative IGF, residual general weighted IGF and residual general weighted relative IGF. They discussed various theoretical properties of these IGFs. These IGFs are studied for escort, generalised escort and mixture distributions. Finally, they have proposed non-parametric estimator of residual general weighted IGF.



Definition 1.

Let X be a continuous random variable with PDF $f(\cdot)$. Then, for $0 < \alpha < \infty$, $\alpha \neq 1$, $\beta > 0$, the RIGF of X is

$$R_{\beta}^{\alpha}(X) = \delta(\alpha) \left(\int_0^{\infty} f^{\alpha}(x) dx \right)^{\beta-1} = \delta(\alpha) \left[E(f^{\alpha-1}(X)) \right]^{\beta-1}, \quad (5)$$

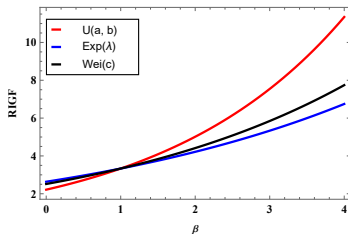
where $\delta(\alpha) = \frac{1}{1-\alpha}$.

- The RIGF is convex with respect to β for $\alpha < 1$ and concave for $\alpha > 1$.
- Some important observations of the proposed RIGF are as follows:
 - $R_{\beta}^{\alpha}(X)|_{\beta=1} = \delta(\alpha)$; $\left. \frac{\partial R_{\beta}^{\alpha}(X)}{\partial \beta} \right|_{\beta=1} = H_{\alpha}(X)$, where $H_{\alpha}(X)$ is as in (2);
 - $R_{\beta}^{\alpha}(X)|_{\beta=2, \alpha=2} = -IE(X)$, where $IE(X) = \int_0^{\infty} f^2(x) dx$ is the informational energy.

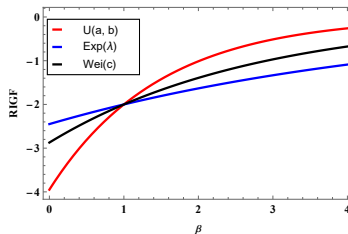


Table 1: The RIGF and Rényi entropy for uniform, exponential, and Weibull distributions. For convenience, we denote $\omega_1 = \frac{\alpha(c-1)+1}{c}$.

PDF	RIGF	Rényi entropy
$f(x) = \frac{1}{b-a}, x \in (a, b)$	$\delta(\alpha)(b-a)^{(1-\alpha)(\beta-1)}$	$\log(b-a)$
$f(x) = \lambda e^{-\lambda x}, x \geq 0, \lambda > 0$	$\frac{\delta(\alpha)\lambda^{(\alpha-1)(\beta-1)}}{\alpha^{\beta-1}}$	$\delta(\alpha) \log\left(\frac{\lambda^{\alpha-1}}{\alpha}\right)$
$f(x) = cx^{c-1}e^{-x^c}, x \geq 0, c > 1$	$\delta(\alpha)\left(\frac{c^{\alpha-1}}{\alpha\omega_1}\Gamma(\omega_1)\right)^{\beta-1}$	$\delta(\alpha) \log\left(\frac{c^{\alpha-1}}{\alpha\omega_1}\Gamma(\omega_1)\right)$



(a)



(b)

Figure 1: Plots of the RIGFs of uniform distribution ($U(a, b)$) for $x \in [0.1, 4]$, exponential distribution ($\text{Exp}(\lambda)$) for $\lambda = 1.5$, and Weibull ($\text{Wei}(c)$) distribution for $c = 1.4$, (a) for $\alpha = 0.7$ and (b) for $\alpha = 1.5$ in Table 1.



Proposition 1.

Suppose the random variable X has PDF $f(\cdot)$. Then, for $a > 0$ and $b \geq 0$, the RIGF of $Y = aX + b$ is

$$R_{\beta}^{\alpha}(Y) = a^{(1-\alpha)(\beta-1)} R_{\beta}^{\alpha}(X), \quad \alpha > 0, \quad \alpha \neq 1, \quad \beta > 0. \quad (6)$$

- The Shannon entropy of order q of a continuous random variable X is defined as (Kharazmi and Balakrishnan (2021))

$$\xi_q(X) = \int_0^{\infty} f(x)(-\log f(x))^q dx. \quad (7)$$

Proposition 2.

Let $f(\cdot)$ be the PDF of a random variable X . Then, for $\beta \geq 0$ and $0 < \alpha < \infty$, $\alpha \neq 1$, the RIGF of X can be represented as

$$R_{\beta}^{\alpha}(X) = \delta(\alpha) \left(\sum_{q=0}^{\infty} \frac{(1-\alpha)^q}{q!} \xi_q(X) \right)^{\beta-1}, \quad (8)$$

where $\xi_q(X)$ is as given in (7).

Proposition 3.

Suppose X is a continuous random variable with PDF $f(\cdot)$. Then,

(A) for $0 < \alpha < 1$, we have

$$R_{\beta}^{\alpha}(X) \begin{cases} \leq \delta(\alpha) G_{\alpha\beta-\alpha-\beta+2}(X), & \text{if } 0 < \beta < 1 \text{ and } \beta \geq 2, \\ \geq \frac{1}{2} R_{2\beta-1}^{\frac{\alpha+1}{2}}(X), & \text{if } \beta \geq 1, \\ \leq \frac{1}{2} R_{2\beta-1}^{\frac{\alpha+1}{2}}(X), & \text{if } 0 < \beta < 1; \end{cases} \quad (9)$$

(B) for $\alpha > 1$, we have

$$R_{\beta}^{\alpha}(X) \begin{cases} \leq \delta(\alpha) G_{\alpha\beta-\alpha-\beta+2}(X), & \text{if } 1 < \beta < 2, \\ \leq \frac{1}{2} R_{2\beta-1}^{\frac{\alpha+1}{2}}(X), & \text{if } \beta \geq 1, \\ \geq \frac{1}{2} R_{2\beta-1}^{\frac{\alpha+1}{2}}(X), & \text{if } 0 < \beta < 1, \end{cases} \quad (10)$$

where $G_{\alpha\beta-\alpha-\beta+2}(X) = \int_0^{\infty} f^{\alpha\beta-\alpha-\beta+2}(x) dx$ is the IGF of X .



- Suppose X and Y are two random variables with corresponding PDFs $f(\cdot)$ and $g(\cdot)$ and CDFs $F(\cdot)$ and $G(\cdot)$, respectively. Then, X is less dispersed than Y , denoted by $X \leq_{disp} Y$ (Shaked and Shanthikumar (2007)), if $g(G^{-1}(x)) \leq f(F^{-1}(x))$, for all $x \in (0, 1)$.
- The quantile representation of the RIGF of X is given by

$$R_{\beta}^{\alpha}(X) = \delta(\alpha) \left(\int_0^1 f^{\alpha-1}(F^{-1}(u)) du \right)^{\beta-1}.$$

Proposition 4.

Consider two random variables X and Y such that $X \leq_{disp} Y$. Then, we have

(A) $R_{\beta}^{\alpha}(X) \leq R_{\beta}^{\alpha}(Y)$; for $\{\alpha < 1; \beta \geq 1\}$ or $\{\alpha > 1; \beta \geq 1\}$,

(B) $R_{\beta}^{\alpha}(X) \geq R_{\beta}^{\alpha}(Y)$, for $\{\alpha < 1; \beta < 1\}$ or $\{\alpha > 1; \beta < 1\}$.



Definition 2.

Suppose X and Y are two continuous random variables and their PDFs are $f(\cdot)$ and $g(\cdot)$, respectively. Then, the Rényi divergence information generating function (RDIGF) is given by

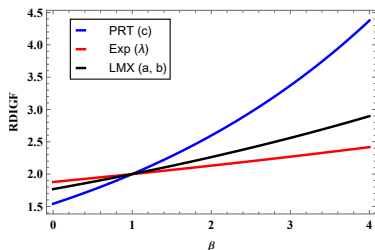
$$RD_{\beta}^{\alpha}(X, Y) = \delta^{*}(\alpha) \left(\int_0^{\infty} \left(\frac{f(x)}{g(x)} \right)^{\alpha} g(x) dx \right)^{\beta-1} = \delta^{*}(\alpha) \left(E_g \left[\frac{f(X)}{g(X)} \right]^{\alpha} \right)^{\beta-1}. \quad (11)$$

- The integral in (11) exists for $0 < \alpha < \infty$ and $\beta > 0$.
- The following observations from (11) can be readily made:
 - $RD_{\beta}^{\alpha}(X, Y)|_{\beta=1} = \delta^{*}(\alpha)$; $\frac{\partial}{\partial \beta} RD_{\beta}^{\alpha}(X, Y)|_{\beta=1} = RD^{\alpha}(X, Y)$;
 - $RD_{\beta}^{\alpha}(X, Y) = \alpha \delta^{*}(\alpha) RD_{\beta}^{1-\alpha}(Y, X)$,where $RD^{\alpha}(X, Y)$ is the Rényi divergence between X and Y given in (2).

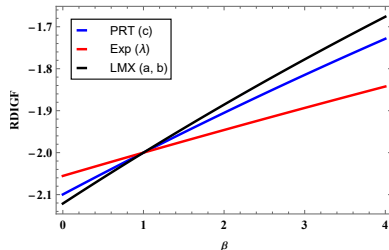


Table 2: The RDIGF and Rényi divergence for Pareto type-I, exponential, and Lomax distributions.

PDFs	RDIGF	Rényi divergence
$f(x) = c_1 x^{-(c_1+1)}, g(x) = c_2 x^{-(c_2+1)}, x > 1, c_1, c_2 > 0$	$\delta^*(\alpha) \left(\frac{c_1^\alpha c_2^{1-\alpha}}{\alpha c_1 + (1-\alpha)c_2} \right)^{\beta-1}$	$\delta^*(\alpha) \log \left(\frac{c_1^\alpha c_2^{1-\alpha}}{\alpha c_1 + (1-\alpha)c_2} \right)$
$f(x) = \lambda_1 e^{-\lambda_1 x}, g(x) = \lambda_2 e^{-\lambda_2 x}, x > 0, \lambda_1, \lambda_2 > 0$	$\delta^*(\alpha) \left(\frac{\lambda_1^\alpha \lambda_2^{1-\alpha}}{(\alpha-1)\lambda_2 - \alpha\lambda_1} \right)^{\beta-1}$	$\delta^*(\alpha) \log \left(\frac{\lambda_1^\alpha \lambda_2^{1-\alpha}}{(\alpha-1)\lambda_2 - \alpha\lambda_1} \right)$
$f(x) = \frac{b_1}{a} \left(1 + \frac{x}{a}\right)^{-(b_1+1)}, g(x) = \frac{b_2}{a} \left(1 + \frac{x}{a}\right)^{-(b_2+1)}, x > 0, a, b_1, b_2 > 0$	$\delta^*(\alpha) \left(\frac{b_1^\alpha b_2^{1-\alpha}}{\alpha(b_1-b_2)+b_2} \right)^{\beta-1}$	$\delta^*(\alpha) \log \left(\frac{b_1^\alpha b_2^{1-\alpha}}{\alpha(b_1-b_2)+b_2} \right)$



(a)



(b)

Figure 2: Plots of the RDIGFs of Pareto type-I (PRT) with $c_1 = 0.8$, $c_2 = 1.5$, exponential (Exp) with $\lambda_1 = 0.8$, $\lambda_2 = 0.5$, and Lomax (LMX) distributions with $a = 0.5$, $b_1 = 0.8$, and $b_2 = 0.4$ when (a) $\alpha = 0.5$ and (b) $\alpha = 1.5$.



Proposition 5.

Let X be a continuous random variable and Y be an uniform random variable, i.e. $Y \sim U(0,1)$. Then, the RDIGF of X reduces to the RIGF of X .

Proposition 6.

Suppose $f(\cdot)$ and $g(\cdot)$ are the PDFs of X and Y , respectively, and $\psi(\cdot)$ is strictly monotonic, differential, and invertible function. Then,

$$RD_{\beta}^{\alpha}(\psi(X), \psi(Y)) = \begin{cases} RD_{\beta}^{\alpha}(X, Y), & \text{if } \psi \text{ is strictly increasing,} \\ -RD_{\beta}^{\alpha}(X, Y), & \text{if } \psi \text{ is strictly decreasing.} \end{cases}$$



Non-parametric Estimator of RIGF

- Denote by $\hat{f}(\cdot)$ the kernel estimator of the PDF $f(\cdot)$, given by

$$\hat{f}(x) = \frac{1}{n\beta_n} \sum_{i=1}^n J\left(\frac{x - X_i}{\beta_n}\right), \quad (12)$$

where $J(\cdot)$ (≥ 0) is known as kernel (see Rosenblatt (1956) and Parzen (1962)) and $\{\beta_n\}$ is a sequence of real numbers, known as bandwidths, satisfying $\beta_n \rightarrow 0$ and $n\beta_n \rightarrow 0$ for $n \rightarrow 0$.

- The non-parametric kernel estimator of the RIGF in (5) is defined as

$$\hat{R}_\beta^\alpha(X) = \delta(\alpha) \left(\int_0^\infty \hat{f}^\alpha(x) dx \right)^{\beta-1}, \quad 0 < \alpha < \infty, \alpha \neq 1, \beta > 0. \quad (13)$$

- A non-parametric kernel estimator of the IGF given in (3) is obtained as

$$\hat{G}_\alpha(X) = \int_0^\infty \hat{f}^\alpha(x) dx, \quad \alpha > 0. \quad (14)$$



Simulation Study

- We carry out a Monte Carlo simulation study to examine the performance of the non-parametric estimators of the RIGF and IGF given in (13) and (14), respectively.
- To generate data from Weibull distribution with shape parameter $k > 0$ and scale parameter $\lambda > 0$ for different sample sizes.
- The SD, AB, and MSE of the kernel based non-parametric estimators of the RIGF in (13) and IGF in (14) are then obtained based on 500 replications.
- The Gaussian kernel is given by

$$k(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}, \quad -\infty < z < \infty. \quad (15)$$



Table 3: Comparison between the non-parametric estimators of the IGF in (14) and RIGF in (13) in terms of the AB, MSE, and SD for different choices of α , β , k , λ , and n .

α	n	$\beta = 1.1, \lambda = 1.5, k = 2$				β	n	$\alpha = 0.3, \lambda = 1, k = 2$			
		IGF		RIGF				IGF		RIGF	
		SD	AB (MSE)	SD	AB (MSE)			SD	AB (MSE)	SD	AB (MSE)
0.3	150	0.10189	0.06812 (0.01502)	0.00659	0.00448 (0.00006)	0.4	150	0.07575	0.04745 (0.00799)	0.02484	0.01615 (0.00088)
	300	0.07850	0.05581 (0.00928)	0.00505	0.00364 (0.00004)		300	0.05915	0.04524 (0.00555)	0.01949	0.01510 (0.00061)
	500	0.06867	0.04703 (0.00693)	0.00440	0.00305 (0.00003)		500	0.05283	0.03557 (0.00406)	0.01723	0.01182 (0.00044)
0.8	150	0.01249	0.00861 (0.00023)	0.00521	0.00360 (0.00004)	0.6	150	0.07737	0.04824 (0.00831)	0.01927	0.01231 (0.00052)
	300	0.00865	0.00506 (0.00010)	0.00360	0.00211 (0.00002)		300	0.06291	0.04286 (0.00579)	0.01544	0.01078 (0.00035)
	500	0.00678	0.00344 (0.00006)	0.00281	0.00143 (0.00001)		500	0.05248	0.04012 (0.00436)	0.01289	0.00999 (0.00027)
1.2	150	0.00916	0.02136 (0.00054)	0.00558	0.01288 (0.00020)	0.9	150	0.07915	0.04806 (0.00857)	0.00587	0.00365 (0.00005)
	300	0.00655	0.01697 (0.00033)	0.00397	0.01020 (0.00012)		300	0.06017	0.04170 (0.00536)	0.00443	0.00312 (0.00003)
	500	0.00495	0.01447 (0.00023)	0.00299	0.00868 (0.00008)		500	0.05430	0.03995 (0.00454)	0.00400	0.00298 (0.00002)
1.5	150	0.01601	0.02631 (0.00095)	0.00502	0.00818 (0.00009)	1.2	150	0.07676	0.04366 (0.00780)	0.01357	0.00788 (0.00025)
	300	0.01083	0.02100 (0.00056)	0.00338	0.00649 (0.00005)		300	0.06111	0.03892 (0.00525)	0.01074	0.00696 (0.00016)
	500	0.00921	0.01779 (0.00040)	0.00286	0.00548 (0.00004)		500	0.05310	0.03829 (0.00429)	0.00935	0.00681 (0.00013)
2.0	150	0.02045	0.02765 (0.00118)	0.00477	0.00636 (0.00006)	1.6	150	0.07740	0.04885 (0.00838)	0.05220	0.03325 (0.00383)
	300	0.01539	0.02244 (0.00074)	0.00353	0.00511 (0.00004)		300	0.06528	0.03730 (0.00565)	0.04387	0.02533 (0.00257)
	500	0.01145	0.01959 (0.00051)	0.00262	0.00442 (0.00003)		500	0.05503	0.03502 (0.00425)	0.03704	0.02371 (0.00193)

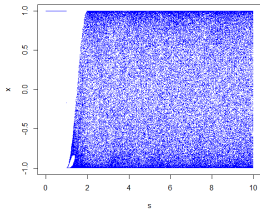
Validation Using Chaotic Maps

(A) Chebyshev Map

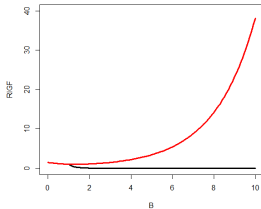
The Chebyshev map is defined as

$$x_{r+1} = \cos(s^2 \arccos(x_r)), \quad r = 1, 2, \dots, n-1, \quad (16)$$

where $x_r \in [-1, 1]$ and $s > 0$. We have considered the initial value $x_1 = 0.1$ and sample size $n = 10000$. For $0 < s \leq 1$, we get $x_r \in [0, 1]$ and $x_r \in [-1, 1]$ for $s > 1$.



(a)



(b)

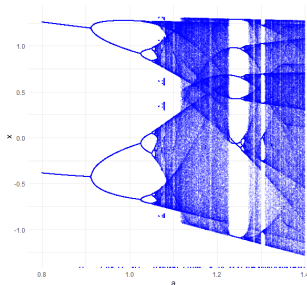
Figure 3: (a) The bifurcation diagram of the Chebyshev map in (16) and (b) the plots of the RIGF for the Chebyshev map when $s = 0.8$ (black) and $s = 2.0$ (red) with respect to β (B).

(B) Hénon map

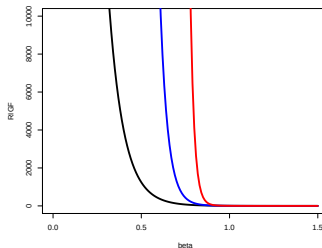
Michel Hénon (Hénon (1976)) first introduced the map as a simplified version of the Poincaré section of the Lorenz model. The Hénon map is defined as

$$\begin{aligned}x_{i+1} &= y_i + 1 - ax_i^2; \\y_{i+1} &= bx_i,\end{aligned}\tag{17}$$

where $i = 1, 2, \dots, n-1$, and $a, b > 0$.



(a)



(b)

Figure 4: (a) Bifurcation diagram of the Hénon map in (17) and (b) the plots of the RIGF of Hénon map for $a = 1.4$ (red line), $a = 1.2$ (blue line), and $a = 1.0$ (black line) for all $b = 0.3$.

Application in Reliability Engineering

- We consider a coherent system with n components and lifetime of the coherent system is denoted by T .
- The random lifetimes of n components of the coherent system are identically distributed (i.d.) with a common CDF and PDF $F(\cdot)$ and $f(\cdot)$, respectively. The CDF and PDF of T are defined as

$$F_T(x) = q(F(x)) \quad \text{and} \quad f_T(x) = q'(F(x))f(x), \quad (18)$$

respectively, where $q : [0, 1] \rightarrow [0, 1]$ is the distortion function (Navarro et al. (2013)) and $q' \equiv \frac{dq}{dx}$ with $q(0) = 0$ and $q(1) = 1$.

- The RIGF of T can be expressed as

$$R_{\beta}^{\alpha}(T) = \delta(\alpha) \left(\int_0^{\infty} \psi_{\alpha}(F_T(x)) dx \right)^{\beta-1} = \delta(\alpha) \left(\int_0^1 \frac{\psi_{\alpha}(q(u))}{f(F^{-1}(u))} du \right)^{\beta-1}, \quad (19)$$

where $\psi_{\alpha}(q(u)) = f_T^{\alpha}(F_T^{-1}(q(u)))$, for $0 \leq u \leq 1$.



- The IGF of T is

$$I(T) = \int_0^\infty f_T^\alpha(x) dx = \int_0^1 \frac{\psi_\alpha(q(u))}{f(F^{-1}(u))} du, \quad \alpha > 0. \quad (20)$$

- The Rényi entropy of T is defined as

$$H_\alpha(T) = \delta(\alpha) \log \int_0^\infty f_T^\alpha(x) dx = \delta(\alpha) \log \int_0^1 \frac{\psi_\alpha(q(u))}{f(F^{-1}(u))} du, \quad \alpha > 0 (\neq 1). \quad (21)$$

- The varentropy of T is

$$\begin{aligned} VE(T) &= \int_0^\infty f_T(x) \left(\log f_T(x) \right)^2 dx - \left\{ \int_0^\infty f_T(x) \log f_T(x) dx \right\}^2 \\ &= \int_0^\infty \frac{\psi_1(q(u))}{f(F^{-1}(u))} \left(\log \frac{\psi_1(q(u))}{f(F^{-1}(u))} \right)^2 dx - \left\{ \int_0^\infty \frac{\psi_1(q(u))}{f(F^{-1}(u))} \log \frac{\psi_1(q(u))}{f(F^{-1}(u))} dx \right\}^2. \end{aligned} \quad (22)$$



- We consider the power distribution with CDF $F(x) = \sqrt{x}$, $x > 0$, as a baseline distribution (component lifetime).
- The numerical values of the RIGF, IGF, Rényi entropy, and varentropy for the series, 2-out-of-3, and parallel systems with $\alpha = 1.2$ and $\beta = 0.5$ are reported.

Table 4: The values of the RIGF, IGF, Rényi entropy, and varentropy for the series, 2-out-of-3, and parallel systems.

System	RIGF	IGF	Rényi entropy	Varentropy
Series ($X_{1:3}$)	-4.144032	1.455774	0.7510748	2.940702
2-out-of-3 ($X_{2:3}$)	-4.852534	1.061702	0.1197473	0.194906
Parallel ($X_{3:3}$)	-4.958784	1.016692	0.03310925	0.1111111



Conclusions

- We have proposed two information generating function, namely Rényi information generating function and Rényi divergent information generating function.
- Several properties of these information generating functions have been discussed.
- A non-parametric estimator of RIGF have been proposed.
- For the validation of an information measure have been used the Chebyshev and Hénon maps.
- An application of RIGF related to the reliability engineering have been reported.



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Thank you!

