Packed Bed Pressure Drop and Incipient Fluidization Condition in a Conical Bed of Spherical Particles : A Mathematical Model

S. K. AGARWAL AND G. K. ROY
Chemical Engineering Department, R.E. College. Rourkela 769 008

Based on the balance between the forces acting on the solid particles initially at rest subsequently suspended in air in a conical bed, a model has been proposed. Equations have been derived from the model for the prediction of packed bed pressure drop and the incipient fluidizing velocity (onset velocity) in terms of superficial velocity at t/ie bottom of the conical bed. Experimental investigation's were carried out in a conical bed with varying apex angles using air as the fluidizing medium and spherical glass beads, urea and mustard; seeds as the bed materials. Experimental data agreed fairly well with the proposed model, thereby establishing the practical utility of the derived equations.

PACKED and fluidized beds find extensive use in many transfer operations and solid-catalysed reactions. The use of conical beds in place of the conventional cylindrical ones has preferential application in situations where continuously varying particle size is encountered as in solid fuel combustion and gassification, roasting of ores and biological waste treatment. The advantages of a conical fluidized bed are similar to those of a tapered bed with the possibility of using mixed-size particles avoidance of hot spots near the distributor (in the case of highly exothermic reactions), grid-less operation at high velocity and relatively smooth operation for deep gas-solid beds.[1]

Static and dynamic characteristics of conical beds are quite different from those of columnar beds : they have not been exhaustively invesigated. Available information regarding conical beds includes the prediction of packed bed pressure drop,[2,3] minimum fluidization velocity[4,5] and the prediction of fluidization quality in terms of fluctuation ratio.[6-9] Most of the earlier work is empirical in nature and thus has limited applicability. For the prediction of the two important features of the conical bed — the packed bed pressure drop and the incipient fluidization velocity — the earlier investigators have used Ergun's equation[10] and Baskakov's modification" for the cone geometry and proposed empirical correlations.

In this communication a mechanistic model has been proposed for the onset of the fluidization condition in a conical bed. Equations have been derived for the prediction of pressure drop for packed beds and at incipient fluidization velocity.

Development of the Model

In the course of experiments it has been observed that, at a particular velocity, the pressure drop reaches a maximum and the particles in the bed are lifted slightly upward by the fluid. This is followed by the particles at the bottom of the bed beginning to fluidize. Once the particles are unlocked there is a sharp decline in the pressure drop. Evidently, fluidization is initiated when the force exerted by the fluidizing medium flowing through the bed is equal to the total effective weight of the particles in the bed. It is assumed that the laterial velocity of the fluid is relatively small and can be neglected.

The pressure drop through a packed bed over a differential height of dh is given by Ergun[10] as follows :

$$-dp = (AU + BU^2)\, dh \ldots\ldots\ldots\ldots \tag{1}$$

where,

$$A = 150. \frac{(1-\varepsilon_o)^2}{\varepsilon_o^3} . \frac{\mu}{(\varphi_s \quad d_p)^2} \tag{1.a}$$

$$B = 1.75. \frac{(1-\varepsilon_o)}{\varepsilon_o^3} \frac{\rho_f}{\varphi_s \quad D_p} \tag{1.b}$$

The overall pressure drop across the entire bed height, H, is obtained by integrating equation (1), i.e..

$$-(\Delta p) = \int_{P_{h_o}}^{P_{H+h_o}} -(dp) = \int_{ho}^{H+ho} (Au + BU^2)\, dh \qquad (2)$$

Now, for a conical bed with an apex angle of α, we have

$$\frac{U}{U_o} = \frac{D_o^2}{D^2} \quad \text{and} \quad \frac{D}{D_o} = \frac{h}{h_o}$$

$$U = U_o \frac{h_o^2}{h^2}$$

So, equation (2) becomes

$$-(\Delta p) = \int_{ho}^{H+ho} \left(A.U_o \frac{h_o^2}{h^2} + B.U_o^2 \frac{h_o^4}{h^4}\right) dh$$

$$= A.Uo.\frac{H\,ho}{(H+h_o)} + \frac{B.Uo^2.ho}{3}\left[\frac{(H+ho)^3 - ho^3}{(H+ho)^3}\right] \qquad (3)$$

$$\text{Where, } ho = \frac{Do}{2\tan \alpha/2}$$

The area of cross-section of a conical bed increases continuously from the bottom to the top. So the force exerted by the fluidizing medium on the solid particles is not directly propotional to the pressure drop. The force in a differential bed height of dh is equal to the product of the pressure drop through it, — (dp), and the cross-sectional area of the bed, $\frac{\pi}{4} D^2$

$$\text{i.e. } df = \frac{\pi}{4} D^2 (-dP) = \frac{\pi}{4} D^2 (AU + BU^2)\, dh \qquad (4)$$

$$\text{or, } F = \int_{ho}^{H+ho} \left[\left(A.U_o \frac{h_o^2}{h^2} . \frac{\pi}{4} D_o^2 . \frac{h^2}{h_o^2}\right) + \left(B.U_o^2 \frac{h_o^4}{h^4} . \frac{\pi}{4} Do^2 \frac{h^2}{h_o^2}\right)\right] dh$$

$$= \frac{\pi}{4}\left[A.Uo.Do^2H + \frac{B.U_o^2\, D_o^2\, ho.H}{(H+ho)}\right]$$

$$\text{Now, } \frac{H+ho}{ho} = \frac{D_1}{Do} = \frac{(Do + 2\,H \tan \alpha/2)}{4}$$

$$\therefore F = \frac{B\,Do^3H}{4\,(Do + 2H \tan \alpha/2)} . U_o^2 + \frac{\pi\,A\,Do^2\,H}{4} . Uo$$

$$= A'\,U_o^2 + B'\,Uo \qquad (5)$$

Also, the over-all effective weight of the particles in the bed is given by

$$G = \int_{ho}^{H+ho} g\,(1-\varepsilon_o)(\rho_s - \rho_f) . \frac{\pi}{4} D^2 . dh \qquad (6)$$

On integration for a conical bed, we get

$$G = \frac{K}{3}\left[(H+ho)^3 - ho^3\right] = C' \text{ (say)} \qquad (7)$$

$$\text{Where } K = g\,(1-\varepsilon_o)(\rho_s - \rho_f)\frac{\pi\,Do^2}{4\,ho^2}$$

Now, according to the proposed model the particles in the bottom start fluidizing when F=G. Therefore, the minimum fluidizing velocity, Umf, can be found by equating equations (5) and (7), which gives

$$A'\,Umf^2 + B'\,Umf = C'$$

$$\text{or, } A'\,Umf^2 + B'\,Umf - C' = 0$$

$$\text{or, } Umf = \frac{-B' + \sqrt{B'^2 + 4\,A'C'}}{2\,A'} \qquad (8)$$

Putting in equation (3), Uo = Umf

$$-(\Delta P) = -(\Delta P)_{mf} = A.\frac{Hho}{H+ho}\,Umf + \frac{Bho}{3}\left[\frac{(H+ho)^3 - ho^3}{(H+ho)^3}\right] Umf \qquad (9)$$

Experimental

The details of the experimental set-up are given elsewhere.[3] The structure of the conical bed is shown in *Figure 1.* A known weight of the bed material was charged in the column from the top. The surface of the bed material was levelled by first fluidizing and then allowing it to settle slowly for a number of times. The corresponding static bed height was noted. Compressed dry air was admitted to the column from a constant pressure tank maintained at 0.20 MN/m^2. The bed pressure drop was recorded against the gradual increase of air flow rate till the fluidization condition was attained. From these data the pressure drop-velocity curve was plotted and the maximum pressure drop and minimum fluidizing velocity where noted from the curve.

The range of experimental conditions is given in *Table I.*

TABLE I

RANGE OF EXPERIMENTAL CONDITIONS

		Properties of Solid Particles			
α Degree	material	d_p m	W_s Kg	ρ_s Kg/m^3	ε_o
14.64	glass beads	0.0010	0.35	2300	0.37
	"	0.0015		"	"
	"	0.0020	"	"	
	"	"	0.25	"	
	"	"	0.45	"	
	"	"	0.55	"	
	urea	0.0015	0.35	1335	
	mustard seeds	"	"	1150	
12.52	glass beads	0.0020	"	2300	
18.63	"	"	"	"	

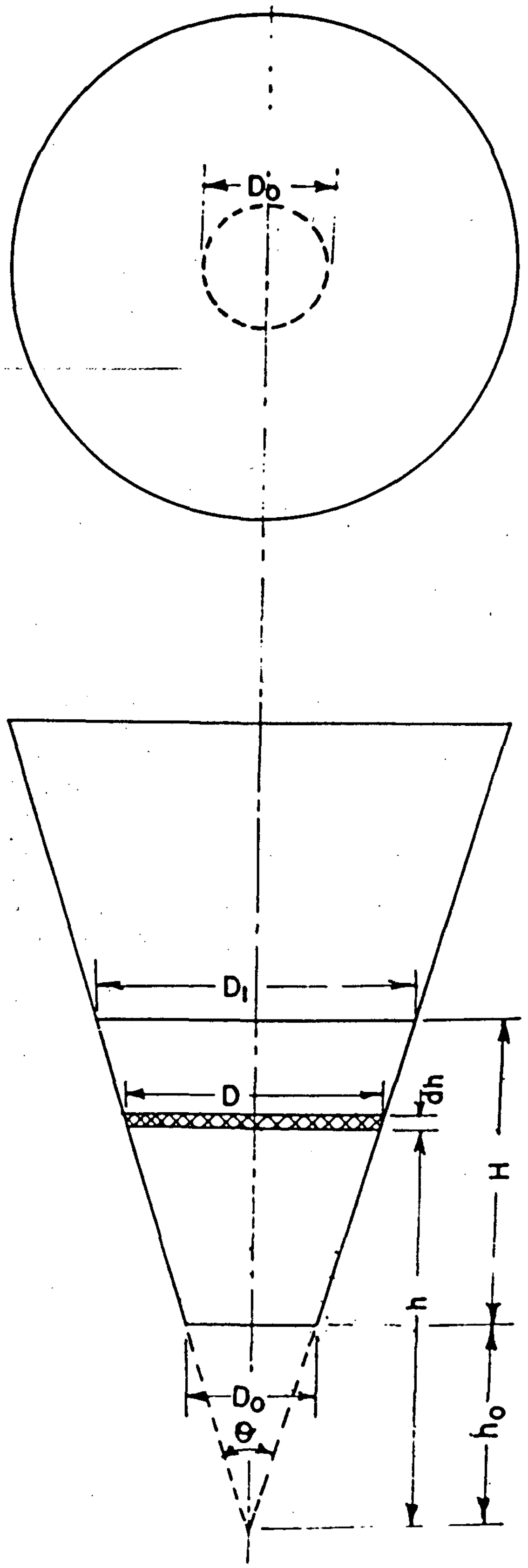

Fig. 1 Structure of the conical bed

Results and Discussion

The packed bed pressure drops calculated with the help of equation[3] have been compared with their respective experimentally measured values in *Figure 2*. The mean and standard deviations for forty-seven readings have been found to be 12.59 per cent and 21.94 per cent respectively. Experimental values of incipient fluidization velocity have been compared with the values calculated by equation[8] in *Figure 3*. *Figure 2* and *3* indicate satisfactory agreement between the calculated and experimental values, establishing thereby that the proposed model is valid and the derived equation is of practical applicability.

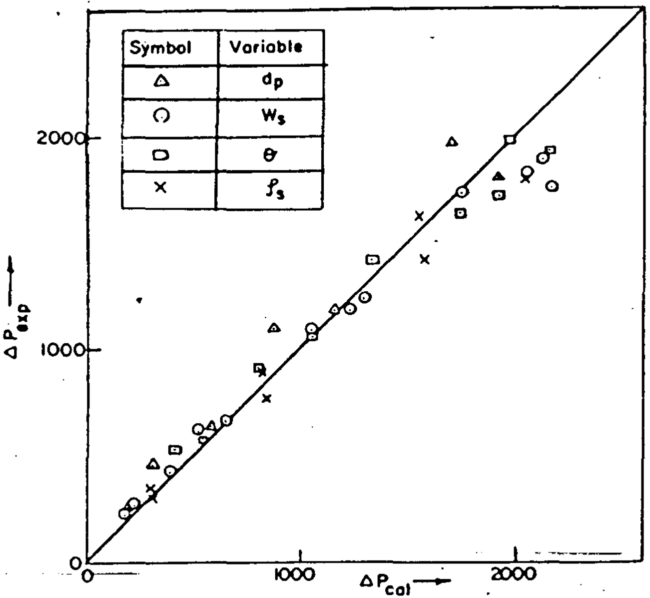

Fig. 2 Comparison between the experimental and calculated pressure drops in packed bed condition

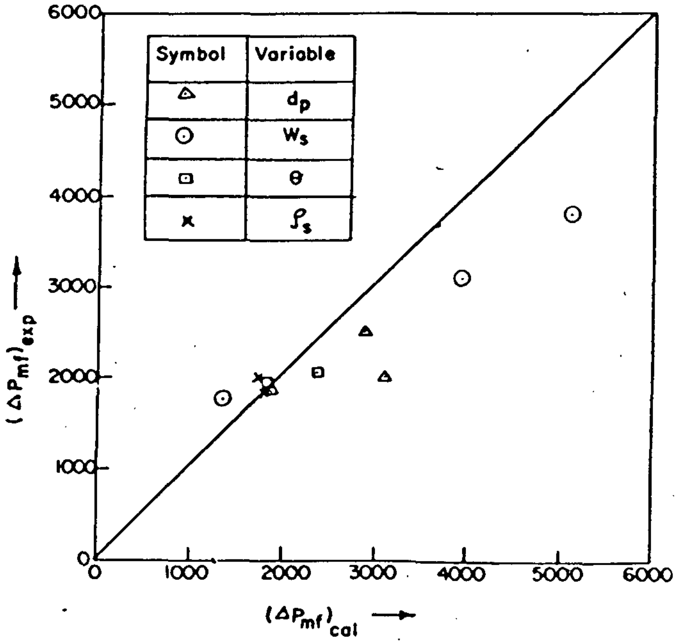

Fig. 3 comparison of experimental and calculated values of pressure drop at minimum fluidization condition

Pressure drops for the incipient fluidization condition have also been calculated with the help of equation (9) and compared with the experimental values. However, there has been a somewhat appreciable difference between the calculated and the experimental values for some of the cases. This can be attributed to the fact that the same value of the porosity for the packed bed and the onset of fluidization conditions has been used in the calculation of equation (9) which, in actual practice, could be different. It may be noted that the equation is very sensitive to porosity variation.[12]

Nevertheless,. the overall agreement between the experimental data and the calculated values, based on the equations derived from the proposed model establishes its validity and practical utility for gas-solid fluidized conical beds.

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NOMENCLATURE

d_p	particle Size, m
D	inside diameter of the cone at a distance h from the appex, m
D_o	inside diameter of the cone at the inlet, m
F	force exerted by the fluidizing medium on the particles in the bed, kgf
g	acceleration due to gravity, m/sec^2
G	effective weight of material in the bed, kgf
H	static bed height, m
ΔP	pressure drop across the static bed, Kgf/m^2
ΔP_{mf}	pressure drop across the bed corresponding to onset of fluidization condition, kgf/m^2
U	linear velocity of fluid at a distance h from the apex, m/sec. (based on empty cross-section)
U_o	linear velocity of fluid at the inlet, m/sec
U_{mf}	linear velocity of fluid at onset of fluidization condition, m/sec
W_s	amount of bed material, Kg

Greek Letters

ε_o	porosity of the packed bed, dimensionless
α	apex angle of the cone, degree
φ_s	sphericity, dimensionless
μ	viscosity of the fluid, Kg/m. sec.
ρ_f	density of the fluid, Kg/m^3
ρ_s	density of the solid partical, Kg/m^3

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