Folded Dualcube: A New Interconnection Topology For Parallel Systems

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Abstract-With the advent of VLSI technology, the demand for higher processing has increased to a large extent. Study of parallel computer interconnection topology has been made along with the various interconnection networks emphasizing the cube based topologies in particular. This paper proposes a new cube based topology called the Folded dualcube with better features such as reduced diameter, cost and improved broadcast time in comparison to its parent topologies: viz: Folded hypercube and Dualcube. Two separate routing algorithms one-to-one and one-to-all broadcast have been proposed for the new network.

Keywords: folded hypercube; metacube; dualcube

I. INTRODUCTION

Cube based networks have received much attention over the past few years since they offer a rich interconnection structure with large bandwidth, logarithmic diameter and high degree of fault tolerance. Many interconnection networks such as trees and multidimensional meshes can be embedded in the cube. Extensive research has resulted in several variations such as Hyercube [3], Folded hypercube,[4] Dualcube [5] and Metacube [6].

Metacube network [6] has two level cube structures. An MC(k,m) network can connect 2^{k+m2^k} nodes with (k+m) links per node. MC(1,m) is the Dualcube [5]. As shown in Fig.1, m (m=2) links are used within clusters to construct an m cube and the single link is used to connect a node in a cluster of the other class. There is no link between clusters of same class. If two nodes are in same cluster or in two clusters of different classes then, distance between them is equal to the hamming distance. Otherwise it is hamming distance + 2 [5]. The Folded hypercube [4] of dimension n called as FHC (n) is constructed from standard n-cube by connecting each node to the unique node that is farthest from it. Thus, FHC(n) is a regular network of degree (n+1). The Hypercube of degree 3 is converted to FHC (3) network as shown in Fig.2

The current work proposes a new interconnection topology called "Folded Dualcube" (FDC) which inherits some of the useful properties of the Dualcube [5] and Folded hypercube [4]. The Dualcube is augmented with some extra links. The aim is to improve the diameter and broadcast time.

The next section presents the topological properties of the new network. Section 3 of the paper proposes two routing algorithms. Results are presented in Section 4. Section 5 concludes the paper.

II. PROPOSED TOPOLOGY

A. Folded Dualcube FDC

The Folded dualcube is constructed from Dualcube G_r (V, E) as shown in Fig.1, by connecting each node to a node farthest from it similar to FHC as shown in Fig. 2. r=m+1. Some of the complementary links are shown. The Folded dualcube is a graph F_r (V,E') as shown in Fig. 3, with the same set of vertices as in G_r and with the edge set E' that is a super set of E.

$E' = |E| + (\text{Total no of nodes}) / 2 = r.2^{2r-2} + 2^{2r-1}/2 = (r+1)\,2^{2r-2}$

Now G_r is a spanning sub graph of F_r and e(u, v) $\in$ E' iff $\|a(u) \oplus a(v)\|=1$ or 2r-1 where $\oplus$ is the XOR operator giving hamming distance.

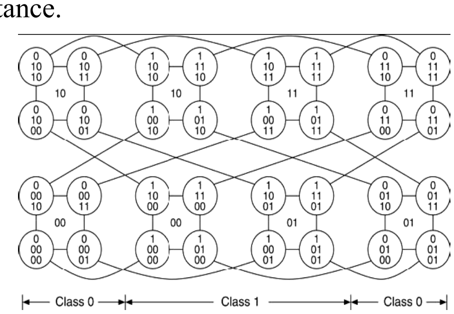

Figure 1. Dualcube G_3

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B. Topological Properties of FDC:

Proposition 1: The node connectivity of F_r is (r+1).

Proof: Every node with (2r-1) bit address a(u) in F_r is connected to r nodes at hamming distance 1 and one node at hamming distance (2r-1). So degree of F_r is $d_F(u)$ = r+1 and F_r is regular of degree (r+1) .

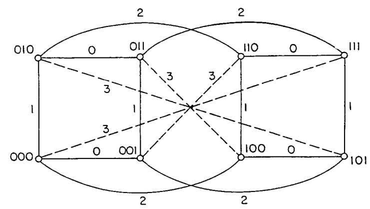

Figure2. Folded Hypercube FDC(3)

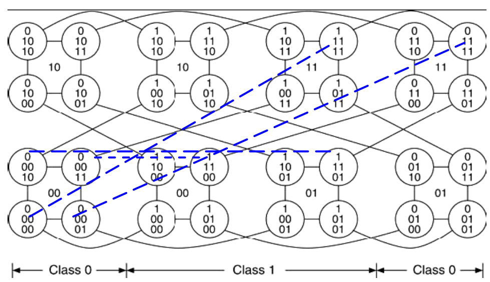

Figure 3. Folded dualcube FDC(3)

Proposition 2: The number of node disjoint paths between any two nodes of F_r is (r+1).

Proof: Since every node has (r+1) neighbors so it is necessary to remove at least (r+1) nodes to disconnect F_r.

Proposition 3: Diameter of F_r is 2r-2.

Proof: As discussed in [5], suppose s & t ∈ V in F_r differ in k bit positions. Then in Folded dualcube, if s and t are in same cluster, then d'(s t) = min.(k, (2r-1)-k+1). If s and t are of different class, then d'(s t) = min.(k, (2r-1)-k+1). If s and t are in different clusters of same class, then

d'(s t) = min.(k+2, (2r-1)-k+1)

When k=2r-2, maximum distance in DC is 2r, but maximum distance in FDC, is (2r-2).

Proposition 4: Bisection Width of F_r is $2^{2r-1}/2$.

Proof: For a Dualcube the bisection width is $2^{2r-1}/4$. In Folded dualcube number of augmented edges = $2^{2r-1}/2= 2^{2r-2}$.

So bisection width is $2^{2r-1}/4 + 2^{2r-1}/4 =2^{2r-1}/2$.

All the topological properties of Folded dualcube are summarized in Table1.

Table 1: Comparison of Topological properties

Network	Degree	Diameter	Cost	Bisection Width	No. of edges
HC	**n**	**n**	**n^2**	**$2^n/2$**	**$n.2^{n-1}$**
FHC	**n+1**	$\lceil n/2 \rceil$	$\lceil n/2 \rceil$*** (n+1)**	**$2^n/4$**	**$(n+1)\ 2^{n-1}$**
DC	**(n+1)/2**	**n+1**	**$(n+1)^2/2$**	**$2^n/4$**	**$(n+1)\ 2^{n-2}$**
FDC	**(n+3)/2**	**n-1**	**$(n^2+2n-3)/2$**	**$2^n/2$**	**$(n+3)\ 2^{n-2}$**

III. ROUTING IN FDC

The routing in a network depends upon shortest path, the Hamming distance. In F_r the hamming distance is 1 or (2r-1). Two distinct algorithms for one-to-one and one-to-all communication are proposed below.

A. One-to-one Routing:

This algorithm performs the routing between any pair of nodes namely u, v∈ V of Fr.

Algorithm One-to-one (a(u),a(v),r)

begin

a(w)=(a(u) ⊕ *a(v);*

If ||a(w)|| < 2r-2

Route the message sent from u via a path composed of links with labels corresponding to bit position which are 1's in a(w)

Else

send the message to u' via the complementary link, route the message via a path composed of links with labels corresponding to bit positions that are 0's in a(w).

end;

So length of shortest path in Folded dualcube is atmost (2r-2), the diameter of the network.

B. Broadcasting One-to-all :

The broadcasting process should satisfy the following desirable properties.

1. A node should not send (receive) the message to (from) more than one of its neighbors.
2. A node receives the message exactly once for the whole duration of the broadcasting.

Let s be source node in class0. Then (r+1) neighbors of s are s^i, $0<i<=r+1$.

$s=(0,s_{2r-2},\ldots\ldots,s_1)$. Then $s^i=(0,s_{2r-2},\ldots,s_{i+1},s_i',s_{i-1},s_1)$ where $0<i<=r-1$.

$s^r=(1,s_{2r-2},\ldots,s_1)$ and $s^{r+1}=(s'_{2r-2},\ldots,s_1')$.

Algorithm:

Source s will send a message to its neighbor s^r through cross edge. Then s and s^r will broadcast simultaneously in their clusters using binomial trees. Then a spanning broadcast tree (SBT) [3] can be constructed for F_r.

In SBT each node is connected by cube edge or a cross edge ,if the hamming distance is less than 2r-2. Next if the next node belongs to different cluster of same class then a complementary edge is used.

So the height of the spanning broadcast tree is atmost 2r-2. Hence broadcasting is done in (log p-1) time, where $p=2^{2^{r-1}}$.

IV. RESULTS AND DISCUSSIONS

Different parameters of the FDC are evaluated and compared against those of the parent networks. The Figures 5, 6 and 7 depict the variation of degree, diameter and cost with the network dimension respectively.

Due to augmentation of complementary links the degree of FDC is observed to be slightly greater than Dualcube but quite less than HC and FHC. Diameter of FDC is found to be smaller than that of Dualcube.

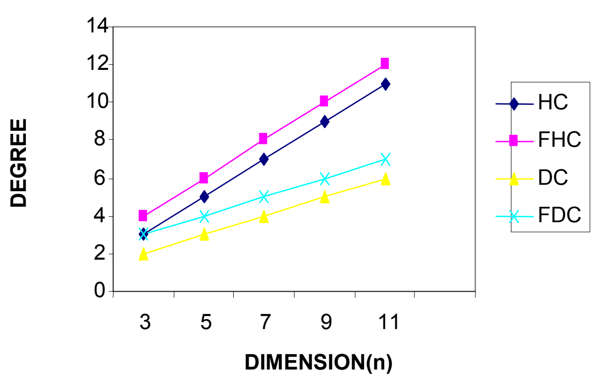

Figure 5. Comparison of Degree

The new network is found to have least cost. In Figure 8, broadcast time of FDC is compared with Dualcube, HC and FHC. FDC exhibits quite a good improvement in broadcast time over its parent networks with millions of nodes.

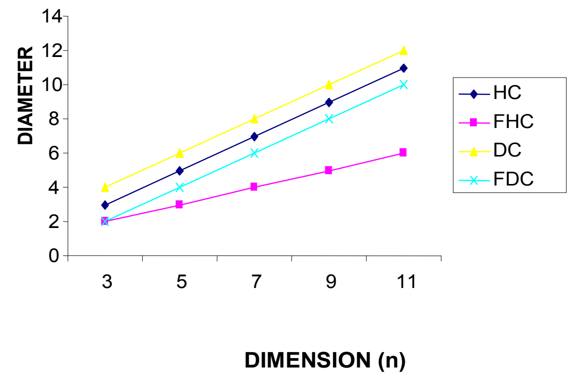

Figure 6. Comparison of Diameter

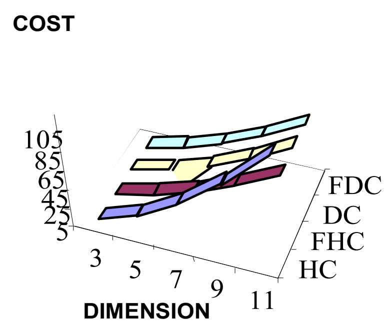

Figure 7. Cost Versus Dimension

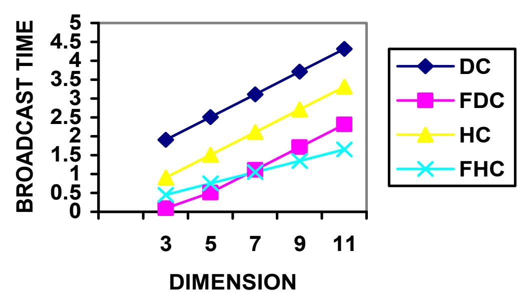

Figure 8. Broadcast Time Versus Dimension

V. CONCLUSION

This paper proposed a new interconnection topology called Folded dualcube. Its basic properties are compared with the parent networks. Two routing algorithms are proposed for this new topology with lesser time complexities. The cost of the proposed topology is found to be less. This proposed topology will help to speed up the overall operation of large scale parallel systems.

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