Parameter Estimation Techniques Applied to Power networks

B.Subudhi[1], P.K.Ray[1], A.M.Panda[3] and S.R.Mohanty[2]

Abstract--**This paper represents the estimation of frequency which is an important power system parameter by several variant of recursive techniques. Estimation of frequency is not new as found in the literature such as Discrete Fourier Transformation, Least Mean Square Technique etc. In this paper we have implemented Recursive least square (RLS) and Extended Least square (ELS) for estimation of frequency. These are well known algorithms for their simplicity in computations and good convergence properties. Such algorithms require three step calculations with estimation of amplitude and phase followed by frequency. They are simple and attractive with the implementation of covariance matrix. Again the choice of co-variance matrix is very crucial without which there might be delay in convergence for estimation of power system parameters. At the same time, feasibility of the above algorithms is tested with signal buried with noise. The above work can be extended for real time implementation, which will be immensely helpful for Modern Power System engineers.**

***Index Terms* -- Recursive Least Square(RLS) , Extended least square (ELS), Covariance matrix ,System Structure Matrix, Power system Parameters**

I. INTRODUCTION

IN a complex power system the fast and accurate estimation of supply frequency, voltage and it's variation in real-time is essential. Variations in system frequency from its normal value indicate the occurrence of a corrective action for its restoration to its original value. In this context a large number of numerical methods are available for frequency estimation from the digitized samples of the system voltage. Conventional methods assume that the power system voltage waveform is purely sinusoidal and therefore the time between two zero crossings is an indication of system frequency. However Discrete Fourier Transforms, Least error squares technique [3, 6], Kalman Filter [1,2], Adaptive notch filters [11-14] etc. are known signal processing techniques used for frequency measurements of power system signals. A large number of numerical techniques [7, 9] and its practical implementation are found in the literature but these set of approaches suffer from inaccuracies due to the presence of noise and harmonics and other system changing conditions such as change in fault inception angle, change in fault resistance etc. Keeping this in mind iterative techniques such as RLS and ELS [4, 5, 8] are presented in this paper for fast and accurate estimation of nominal and off-nominal power system frequency.

Although RLS and ELS methods are very popular frequency estimation techniques but unfortunately these have been overlooked by the researchers in this area. Fortunately, after applying these techniques to power system parameter estimation, it gives attractive result.

II. ESTIMATION ALGORITHMS

The algorithms used in this work are based upon the criterion of least square norms.

A. Recursive Least Square Algorithm

Let a signal buried with noise is represented by the following structure

$$y(t) = A_1 \sin(\omega_0 t + \phi_1) + \mu(t) \qquad (1)$$

So for the purpose of estimation the signal in the parametric form can be expressed as

$$y(t) = \begin{bmatrix} \sin\omega_0 t & \cos\omega_0 t \end{bmatrix} \begin{bmatrix} \alpha & \beta \end{bmatrix}^T + \mu(t) \qquad (2)$$

or in the standard form.

$$y(t) = \phi(t)\theta + \mu(t) \qquad (3)$$

Here y*(t)* is the noisy measurement .

$\phi(t)$is the system structure matrix.

---

[1] Department of Electrical Engineering, NIT Rourkela-769008, Orissa, India
[2] Department of Electrical Engineering MNNIT, Allahabad, India
[3] Department of Electrical Engineering, IGIT Sarang-759146, Orissa, India

θ(t) is the vector of unknown parameter.
The estimate for the required parameter

$$\hat{\theta}(t) = \hat{\theta}(t-1) + K(t)\varepsilon(t) \qquad (4)$$

Error in measurement is

$$\varepsilon(t) = y(t) - \phi(t)^T \hat{\theta}(t-1) \qquad (5)$$

The gain *K* is related with covariance of parameter vector

$$K(t) = P(t-1)\phi(t)[1+\phi(t)^T P(t-1)\phi(t)]^{-1} \qquad (6)$$

The updated covariance of parameter vector using matrix inversion lemma

$$P(t) = [I - K(t)\phi(t)^T]P(t-1) \qquad (7)$$

These equations are initialized by taking some initial values for the estimate at instants t, *θ(t)* and *P*. As the choice of initial covariance matrix is large it is taken $P=\alpha I$ where *α* is a large number and *I* is a square identity matrix.

α,β are the parameter to be estimated and are given by

$$\alpha = A_1 \cos\phi_1 \qquad (8)$$

$$\beta = A_1 \sin\phi_1 \qquad (9)$$

The actual required parameters are A_1 and ϕ_1, which can be given in terms of α and β as

$$A_1 = \sqrt{\alpha^2 + \beta^2} \qquad (10)$$

$$\phi_1 = \tan^{-1}\frac{\beta}{\alpha} \qquad (11)$$

Once the estimate of amplitude and phase is done, it is required to estimate the frequency. This can be evaluated from the noisy measurement *y(t)* as

$$f_0 = \frac{1}{2\pi t}[\sin^{-1}(\frac{y(t)-\mu(t)}{A_1}) - \phi_1] \qquad (12)$$

B. Extended Least Square Algorithm

Here the estimation of the unknown parameter *θ* is done by rearranging the observation matrix *Φ* and the steps for the algorithms are given below.

$$P(t+1) = P(t) + [\phi^T(t)\phi(t)]^{-1} \qquad (14)$$

$$\hat{\theta}(t+1) = \hat{\theta}(t) + P(t+1)\phi^T(t)[y(t+1) - \phi(t)\hat{\theta}(t)] \qquad (15)$$

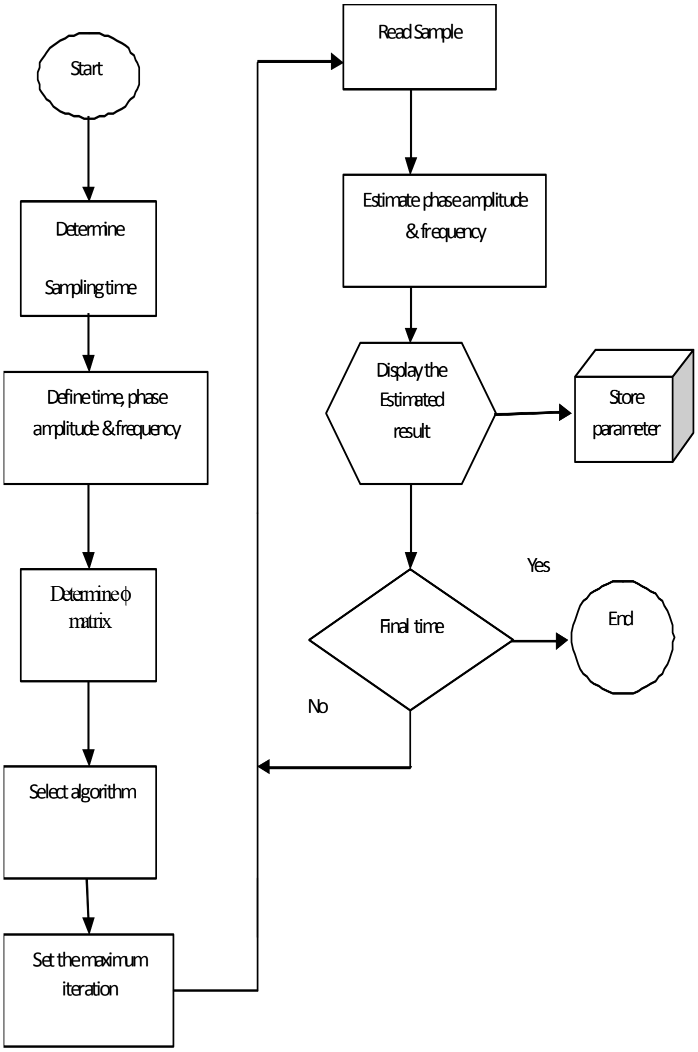

Fig.1 Estimation Strategy

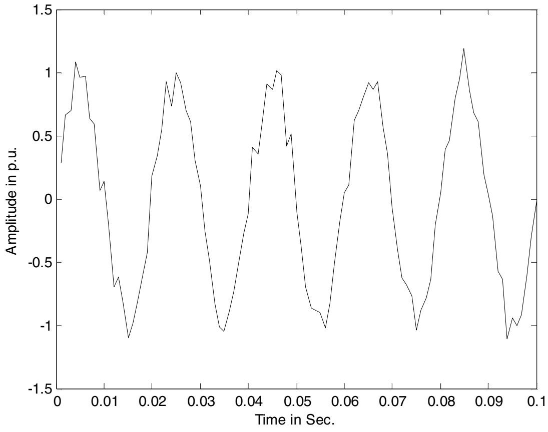

Fig.2 Sample Signal with noise

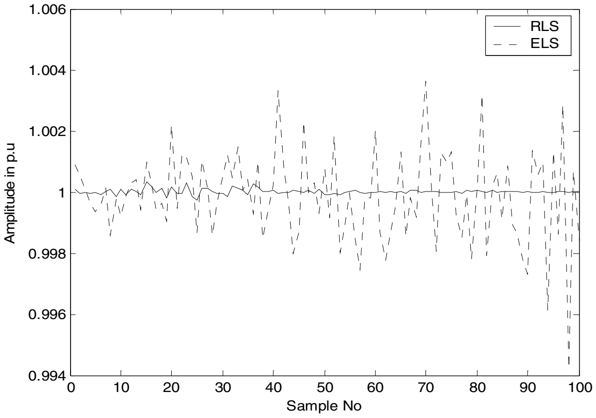

Fig.3. Estimation of Amplitude of Signal

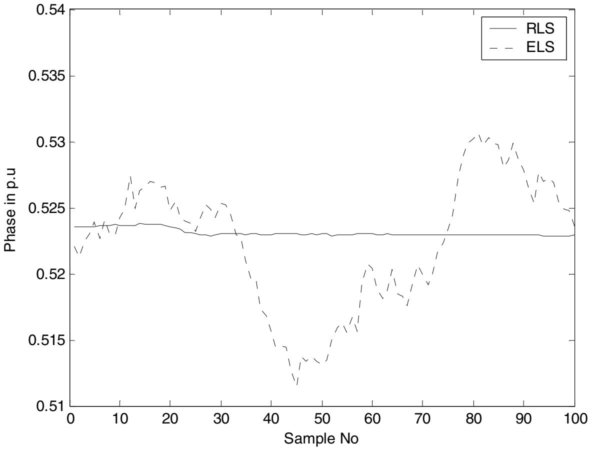

Fig.4. Estimation of Phase of Signal

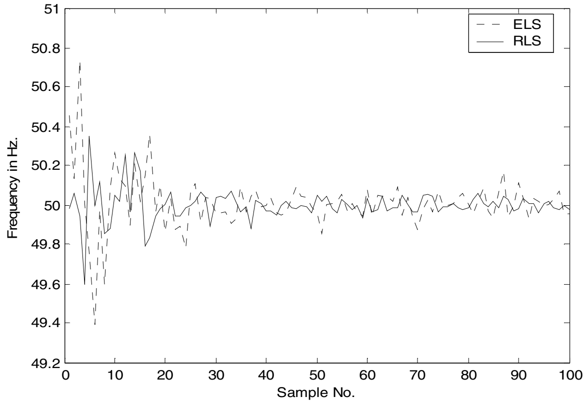

Fig.5. Estimation of Frequency of Signal

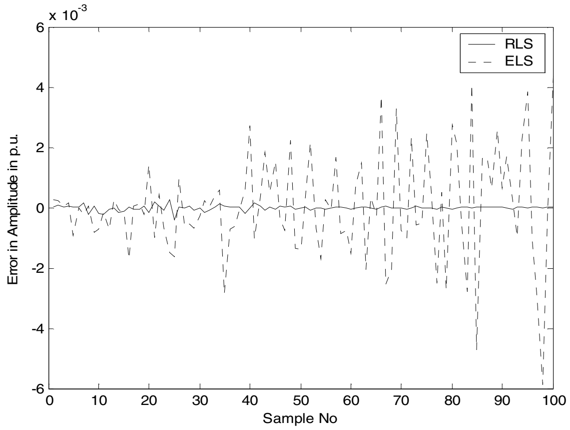

Fig.6.Estimation of Error in Amplitude of Signal

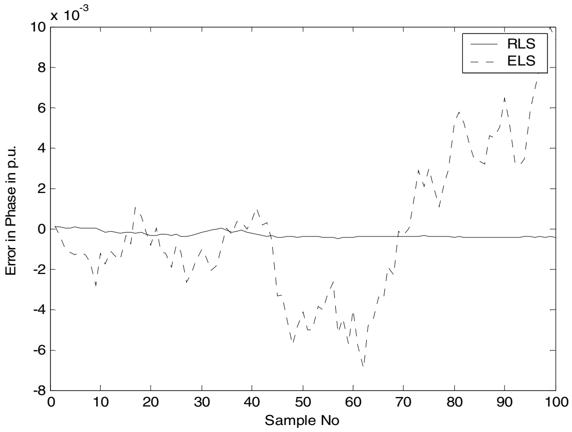

Fig.7. Estimation of Error in Phase of Signal

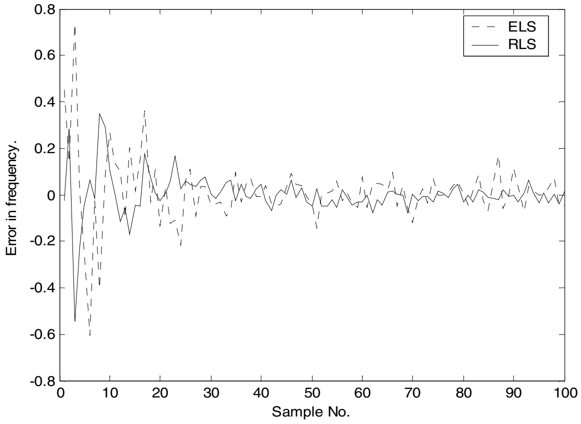

Fig.8. Estimation of Error in Frequency of Signal

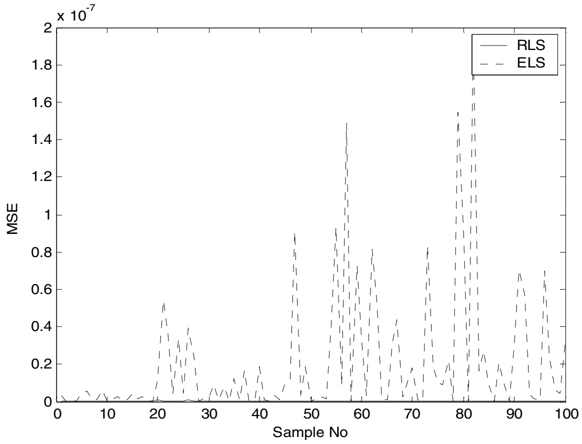

Fig.9. Estimation of Mean Squared Error of Amplitude of Signal

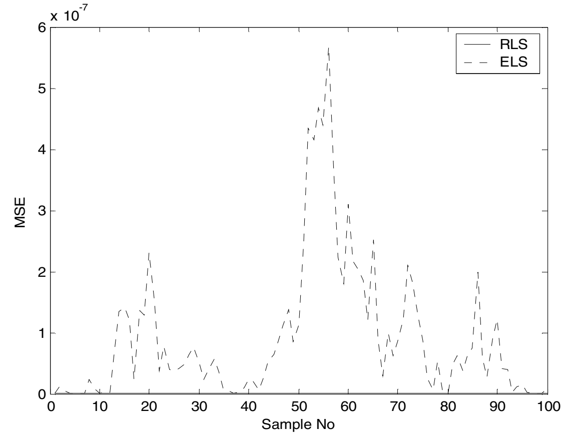

Fig.10. Estimation of Mean Squared Error of Phase of Signal

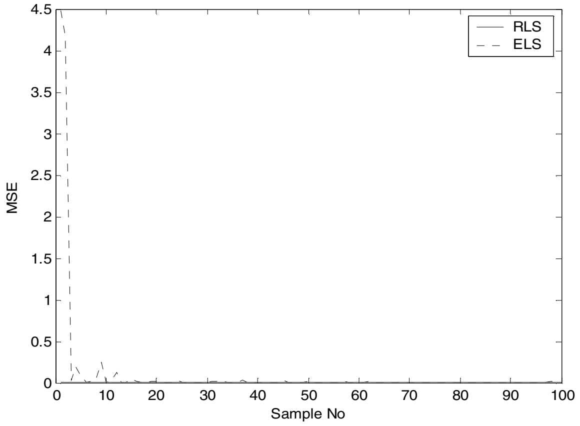

Fig.11. Estimation of Mean Squared Error of Frequency of Signal

III. DISCUSSION

A synthetic signal of 1 p.u amplitude, 50 Hz frequency and 0.5236 p.u phase angle is generated in MATLAB platform. Then the algorithms such as RLS and ELS are implemented for estimation of power system parameters with the sampling interval of 10microsecond. For different algorithms amplitude, phase and frequency are estimated. For power system signal, generated in MATLAB platform, signal to noise ratio is taken 200dB.

To improve the performance in the Least Square algorithm, many extensions and modifications are found in the literature [3,6]. The enhanced performances gained through these modifications are less deviations, faster convergence and recursive calculations. The initial covariance matrix is taken as αI, where I is the identity matrix and α=100. Here the system structure matrix as taken in both the algorithms is a (1×2) matrix. Estimation of frequency is done after the estimation of amplitude and phase of the signal.

Fig 1 represents the Estimation Strategy so that computation steps of estimation of power system parameter are clearly indicated in the flow chart. Fig 2 represents the generation of synthetic signal contaminated with noise with SNR 30 dB. dB Fig 3 represents the Comparison of estimation of amplitude of the signal with 1 p.u. amplitude when the signal is represented in the parametric form.

Fig 4 represents the Comparison of estimation of phase of the signal of 0.5236 p.u. which reflects the estimation is accurate with minimum estimation error in case of RLS Algorithm & there is little bit more oscillations in case of ELS Algorithm. After the estimation of amplitude and phase of the signal is over, estimation of frequency is done from the empirical relation given in equation (1) .From the profile of the comparison of frequency as shown in Fig.5, it is concluded that in case of ELS, the oscillation is more in first 10 samples. The estimation varies between (49.4-50.75) Hz with maximum value at about 4th samples. But in case of RLS the deviation of Frequency is very less.Fig.6 shows the comparison of Estimation of Error in Amplitude contaminated with noise having maximum estimation error of 6.0×10^{-3} p.u. which is at about 98th sample in the case of ELS but in case of RLS the maximum Estimation Error is about 0.1×10^{-3} p.u. Fig.7 represents the comparison of Estimation of Error in Phase with noise having maximum estimation error of 8×10^{-3} p.u. at about 95th sample for ELS but for the case of RLS, maximum Estimation Error is 0.1×10^{-3} p.u. Fig.8 shows the comparison of Estimation Error in Frequency with noise having maximum estimation error of 0.7Hz at about 4th sample in case of ELS but in case of RLS the maximum Estimation Error is 0.5Hz at about 4th sample. Fig.9 represents the comparison of Mean Squared Error in the Estimation of Amplitude of signal. It becomes zero for RLS and it is of the order of 10^{-7} for ELS. Fig.10

shows the comparison of Mean Squared Error in the Estimation of phase of signal. It becomes zero for RLS and of the order of 10^{-7} for ELS. Fig.11 shows the Mean Squared Error profile of the Estimation of Frequency by the two algorithms. It is seen that for both the cases the curves settle at zero. But the performance curve in case of RLS is better.

TABLE I
COMPARATIVE ASSESSMENT OF METHODS

Methods	Maximum Estimation Error in Amplitude (p.u)	Maximum Estimation Error in Phase (p.u)	Maximum Estimation Error in Frequency (Hz)	Computational time in Sec.
ELS	6.0×10^{-3}	8×10^{-3}	0.7	0.188
RLS	0.1×10^{-3}	0.1×10^{-3}	0.5	0.181

Table I. shows a comparative assessment among these two methods, from this table we conclude that both estimation error of all the parameters and computational time are reduced in case of RLS Algorithm. So this algorithm outperforms on ELS algorithm.

IV. CONCLUSION

This paper presents the estimation of frequency of a synthetic signal by various recursive estimation techniques. However choice of the covariance matrix is very crucial at the initial instant for RLS and ELS algorithm. Improper choice of covariance matrix leads to more computational time with more estimation error. But at the same time these recursive algorithms are very simple by representation of the parametric form of the signal. The computational time is less due to the simplicity of these algorithms and estimation error is also less. Validation of these algorithms can be done in MATLAB platform with various system changing conditions and all possible types of faults. Real time Implementation of the Algorithm can be realized by a DSP processor.

REFERENCES

[1] P.K.Dash, A.K.Pradhan and G.Panda, "Frequency estimation of distorted power system signals using extended complex Kalman Filter." *IEEE Trans.Power Del., vol. 14*, no. 3, pp. 761-766, Jul. 1999.

[2] P.K.Dash,A.K.Pradhan, G.Panda, R.K. Jena, S.K.Panda "On line tracking of varying harmonics using an integrated complex Kalman Filter and Fourier Linear Combiner"*IEEE Trans.on Power Del.* 2000 pp1575-1580.

[3] A.K.Pradhan, A.Routray and Abir Basak "Power System Frequency Estimation Using Least Mean Square Technique." *IEEE Trans. Power Del.,vol 20*,no. 3,.july2005.

[4] Maamar Bettayeb, Uvais Qidwai "Recursive estimation of power system harmonics" *Electric Power System Research 47* (1998) 143-152.

[5] E.A. Abu Al-Feilat, I. El-Amin, M. Bettayeb "Power system harmonic estimation a comparative study", *Electric Power System Research 29* (1994) 91-97.

[6] M.S. Sachdev, M.M. Giray "A Least Error Square technique for determining power system frequency" *IEEE Trans. Power Apparatus Syst. 104*(2) (1985) 437-443.

[7] D.C. Rife, R.R. Boorstyn, "Single tone parameter estimation from discrete time observations" IEEE Trans. Infor. Theory 20(5) (1974) 591-598.

[8] T.S. Sidhu and M.S. Sachdev, "An Iterative Technique for Fast and Accurate Measurement of Power System Frequency", IEEE Trans. PWRD, Vol.13, No.1, pp.109-115, 1998.

[9] M.S. Sachdev, M.M. Giray, "Off-Nominal Frequency Measurements in Electric Power Systems" IEEE Trans.on Power Delivery, Vol.4, No.3, pp.1573-1578, 1989.

[10] L.L. Lai, C.T. Tse, W.L. Chan, A.T.P. So, "Real-Time Frequency and Harmonic Evaluation using Artificial Neural Networks" IEEE Trans.on Power Delivery, Vol.14, No.1, January 1999.

[11] J.M. Mendel, "*Lessons in Digital Estimation Theory*" Prentice-Hall, Englewood Cliffs, NJ, 1987.

[12] T. Soderstrom and P. Stocia "*System Identification*" Prentice-Hall, Englewood Cliffs, NJ, 1989

[13] L. Ljung and T. Soderstrom "*Theory and Practice of Recursive Identification*" MIT Press, Cambridge, MA, 1983.

[14] Ljung, L.. System Identification: Theory for the User, Prentice- Hall ,Englewood Cliffs, NJ.1987