A Novel Concept of Embedding Orthogonal Basis Function Expansion in A Feedforward Neural Equaliser

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Abstract-**The proposed neural equaliser structure is based on an orthogonal basis function (OBF) expansion technique, motivated by genetic evolutionary concept, which utilizes a self-breeding approach to evolve new information so as to consolidate the final output.The equaliser structure developed using this novel approach has outperformed the conventional multilayer feedforward neural network (FNN) equaliser with a wide margin and its bit-error-rate performance is close to that of an optimal Bayesian equaliser. Also it learns faster with less training samples.Application of this proposed technique also reduces the structural complexity of a conventional FNN equaliser and has the potential to become a challenging candidate for real-time implementation issue.**

I. INTRODUCTION

Channel equalisation is a powerful technique for combating inter symbol interference and distortion in a communication channel. If equalisation is considered as a geometric classification problem rather than an inverse filter problem, however, the main objective becomes the separation of the received symbols in the output signal space. In this approach to equalization, it has been found that the optimal decision region boundaries in the signal space are highly nonlinear thus requiring the use of nonlinear classifiers, even with linear channels. Since neural networks are capable of realizing nonlinear mapping, neural equalizers are gaining widespread attention in order to increase receiver robustness. Efficient neural network based adaptive equalisations for digital communication channels have been suggested in recent past. Among the techniques based NN (such as multilayer perceptron (MLP) [1][2], radial basis function (RBF)[3], recurrent neural network (RNN)[4]and so on.) proposed to solve the linear and nonlinear channel equalization problem. Further, structure selection for an ANN equaliser has always been a point of concern because a less complex structure is much easier to implement in real-time using VLSI, DSP chips etc so also suitable for applications like mobile communication system, optical recording etc.[5],[6]. Development of faster training algorithms utilizing less number of samples is also a major research focus.

It has already been established that multilayer feedforward neural network (FNN) based equalisers have significant performance improvement over the conventional linear equalisers. This present work is based on a new concept motivated by self breeding phenomenon in genetic evolution. Orthogonal basis function (OBF) expansion blocks have been embedded in a FNN framework to evolve future generations. Here, the decision at a node of FNN termed as expert opinion of a generation that undergoes an orthogonal expansion in two dimensions. One of the outputs possessing the knowledge base for that generation participates in taking the final decision; while the other one is allowed to feed the information further ahead to generate the expert opinion for the next generation and the process continues. A collective judgment based on the expert opinions evolved from decisions of individual generations gives a more rational and heuristic solution at the equaliser output. The proposed equaliser structure is of decision feedback type and detected symbol is fed back. As the proposed OBFNN equaliser has been framed on a FNN backbone, so the existing Back-Propagation(BP)algorithm[7] has been suitably modified to take into account the error propagated through the OBF block for weight updation during training.

The organization of this paper is as follows. In Section II, the proposed OBFNN structure is described. The modifications incorporated in the existing BP algorithm is given in Section III. The performance analysis of the proposed neural equaliser through extensive simulations for linear and nonlinear channels is illustrated in Section IV. Finally, Section V summarizes the research work.

II. DESCRIPTION OF PROPOSED OBFNN EQUALISER STRUCTURE

The schematic diagram of the proposed neural equaliser structure is shown in Fig. 1. This structure resembles that of a

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multilayer FNN with the exception that each layer comprises of only one neuron and is based on orthogonal basis function expansion technique as shown in Fig. 1.

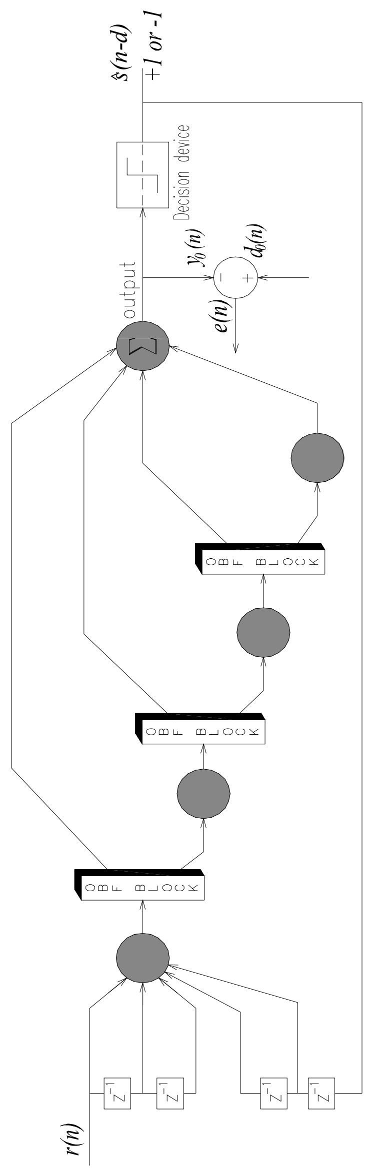

Fig.1 Structure of Proposed OBFNN equaliser with three OBF Blocks in cascade

The basic idea revolves around decomposing the signal '*y*' at a neuron output, into two orthogonal pairs as '*y cos y*'and '*y sin y*' such that the energy contained by transformed signals remains unaltered. Comparing this technique with self-breeding genetic approach, these two generated signals may be termed as *off springs* of the present generation '*y*' which take part in reproducing further to create a new generation. In this process out of these two off-springs only one is allowed to mutate and reproduce further in order to create a new generation, while the other one is constrained to remain as it is to preserve the knowledge of the corresponding generation which it forwards to the output node (expert) responsible for taking the final decision. The process of evolution continues for a number of generations depending upon the complexity of the problem under investigation. According to the back propagation algorithm the evaluation of local gradient at each node is of prime importance for updating synaptic weights connected to the corresponding unit. Calculation of local gradients at each node cannot be directly computed using BP algorithm, as OBF block is positioned in between the neurons of different layers. Here, an adhoc solution has been devised by considering the function(OBF) block which performs a non-linear mathematical operation to be equivalent to the sigmoidal nonlinearity of a neuron. The expanded view of OBF expansion illustrated in Fig. 2 is visualised as two parallel blocks, each comprising of a functional block along with a multiplier unit. This structural realization is utilized for proposed learning algorithm development.

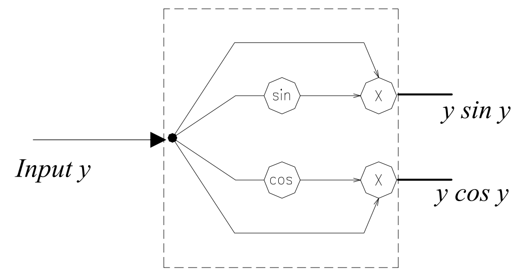

Fig. 2 Expanded view of OBF Block

III. PROPOSED LEARNING ALGORITHM

The Back-Propagation algorithm is suitably modified considering the stuctural modifications in the proposed equaliser. At time index n, the mx1 received signal vector $\boldsymbol{r}(n)=[r(n),\ r(n-1),\ \ldots,\ r(n-m+1)]$ and n_bx1 decision signal vector$[\hat{s}(n-d-1),\ \hat{s}(n-d-2),\ldots\ ,\hat{s}\ \ n-d-n_b)]$ are fed into the feedforward filter and feedback filter of the proposed OBFNN equaliser respectively. The various notations used in the development of the weight updation algorithm for the proposed neural structure are given as below:

j is the layers index, $1 \le j \le nn$

nx is the number of input to the equaliser.

V is the weight matrix connecting inputs to the neurons of first layers.

W is the weight matrix connecting the neurons of different layers.

The generalized algorithm is summarized as below:

- The synaptic weights and thresholds are initialised to small random values which are uniformly distributed.
- The signal at the input layer of the proposed equaliser can be represented by a $(m+n_b)$ x 1 vector as

$$\boldsymbol{x}(n) = [\ r(n), r(n-1), \ldots, r(n-m+1);\ \hat{s}\ (n-d-1)\ ,\quad ,\ \hat{s}\ (n-d-n_b)]^T \tag{1}$$

- Calculation of network output

The forward propagation of signal continues layer by layer till the final output $y_o(n)$ of the neural structure is calculated. The internal activity of the node at the first layer is given by

$$c_j(n) = \sum_{k=1}^{nx} x_k(n).v_k(n) + th_j(n), \tag{2}$$

For all the nodes in the middle layer, the internal activity of each node is calculated as

$$c_j(n) = y_i \cos y_i(n) \,.\, w_{ij}(n) + th_j(n), \tag{3}$$

where $1 < j < nn$ and $i = j - 1$

The local expert decisions at the nodes of all layers except the output are decided by the sigmoidal nonlinearity as given by

$$y_j(n) = F\{c_j(n)\} = \frac{1 - e^{-\phi . c_j(n)}}{1 + e^{-\phi . c_j(n)}}, \tag{4}$$

where $1 \le j < nn$

The node at the output layer combines the knowledge available from all generations (weighted sum of all the signals) to generate the final output.

$$y_j(n) = y_i(n)\, w_{ij}(n) + \sum_{k=1}^{nn-2} y_k \sin y_k(n).w_{k\,nn}(n),$$

where $j = nn$ and $i = j - 1$ (5)

- Computation of Error Terms:

As the final node is a summing unit, the error term at time index n is computed by comparing the output with the desired value.

$$\delta_{nn}(n) = e_j(n) = d_o(n) - y_j(n), \tag{6}$$

For the node in $(nn-1)^{\text{th}}$ layer, the error term is calculated as

$$\delta_j(n) = \delta_l(n)\, w_{jl}(n)\{1 - y_j(n)^2\}(\phi/2),$$

where $j = nn - 1$ and $l = j+1$ (7)

For the nodes in all the middle layers, the error terms are calculated as

$$\delta_j(n) = \{\delta_l(n)\, w_{jl}(n)\, (\cos y_j(n) - y_j \sin y_j(n))$$
$$+ \delta_{nn}(n) w_{jnn}(n)\, (\sin y_j(n) + y_j \cos y_j(n))\}\{1 - y_j(n)^2\}(\phi/2),$$

where $1 \le j \le nn - 2$ (8)

- Weight updation:

Updation of synaptic weights and thresholds is carried out using the generalized delta rule as follows.

$$w_{j\,nn}(n+1) = w_{j\,nn}(n) + \eta\,\delta_{nn}(n)\, y_j \sin y_j(n) +$$
$$\alpha \Delta w_{j\,nn}(n-1), \quad \text{where } 1 \le j \le nn-2 \tag{9}$$

$$w_{jl}(n+1) = w_{jl}(n) + \eta\,\delta_l(n)\, y_j \cos y_j(n) + \alpha\,\Delta w_{jl}(n-1),$$
$$1 \le j \le nn - 2 \tag{10}$$

$$w_{jl}(n+1) = w_{jl}(n) + \eta\,\delta_{nn}(n)\, y_j(n) + \alpha\,\Delta w_{jl}(n-1),$$
$$\text{where } j = nn - 1 \tag{11}$$

$$v_k(n+1) = v_k(n) + \eta\,\delta_l(n)\, x_k(n) + \alpha\,\Delta v_k(n-1),$$
$$\text{where } 1 \le k \le nx \tag{12}$$

$$th_j(n+1) = th_j(n) + \beta\,\delta_j(n), \quad 1 \le j < nn \tag{13}$$

where η and β are the *learning-rate parameters* of weights and thresholds respectively and α is the *momentum constant*. After training is completed the equaliser will self-adapt to the changes in the channel characteristics occurring during transmission (decision directed mode).

IV. SIMULATION STUDY

Adaptive channel equalization is a major issue in digital communications. An exhaustive computer simulation study has been undertaken for evaluating the performance of all the proposed neural equaliser structures based on FNN topologies for both linear and non-linear real communication channels models. The simulation model of an adaptive equaliser considered is illustrated in Fig. 2. A real-valued digital communication system with 2-PAM signal is considered to transmit a sequence of real-valued symbols which is denoted as $s(n)$ for nth time instance. The combined effect of the transmitter filter, the transmission medium and other components are included in the '*Channel*'. A finite impulse response (FIR) model is widely used to model a linear channel. The output of the channel is corrupted by noise, which is usually modeled as an additive white Gaussian noise (AWGN) process with a zero mean and variance ^{2}. And thus the received signal is passed into the equalizer which cancels the channel effects and recovers transmitted symbol $s(n)$ or $s(n-D)$ without any error from the knowledge of the received signal samples, where 'D' is the transmission delay associated with the physical channel. $d(n)$ denotes the desired signal and is defined by $d(n) = s(n-D)$. In this study, we do not consider any specific modulation (demodulation) strategy, and our specific focus here is on canceling the channel linear and nonlinear effects.

The Bit Error Rate (BER) performance for each SNR value under study are evaluated based on 10^7 more received symbols and averaged over 20 independent realizations, after the equaliser structure is trained using the proposed learning algorithms using 1000 samples. The proposed OBFNN equaliser structure yield superior result in terms of BER performance and faster learning in terms of less training samples when compared with a conventional FNN (CFNN).

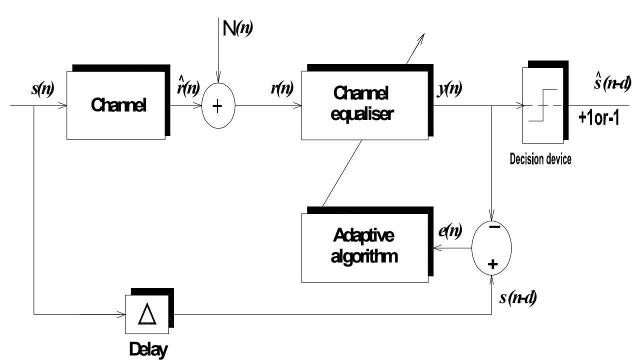

Fig. 3 Simulation model of an adaptive equaliser

An example studied here is a five-tap deep-null communication channel [8] with significant inter symbol distortion and characterised by the following transfer function.

$$H_1(z) = 0.9413 + 0.3841 z^{-1} + 0.5684 z^{-2} + 0.4201 z^{-3} + z^{-4} \quad (14)$$

It is also observed from the MSE convergence curves in Fig.4 that the proposed equaliser achieves a lower MSE in comparison to the conventional FNN equaliser trained with 2000 samples. Also the proposed neural structure learns faster with less number of training samples. The learning rate parameters and momentum constants in weight updation are chosen to be 0.005 and 0.03 respectively. Fig.5 illustrates the BER performance enhancement by the proposed OBFNN equaliser {a two layer (1,1) structure} when compared with a CFNN one {a two layer (5,1) structure} whose parameters are chosen as m=5, n_b=4 and d=4. The proposed OBFNN equaliser is able to provide better performance in terms of the minimum SNR to get a prefixed error probability level (around 18 dB against 20 dB to obtain BER=10^{-4}) and is close to the optimal Bayesian performance [9]in comparison to the conventional FNN equaliser.

In order to prove the robustness of the proposed FNN based structure, equalisation of a non-linear channel is again simulated. A typical non-minimum phase channel with nonlinear harmonics encountered in practical communication scenario is described by following transfer function and widely referred in technical literature[10] as a standard model.

$$H_2(z) = (0.3482 + 0.8704 z^{-1} + 0.3482 z^{-2}) + 0.2 (0.3482 + 0.8704 z^{-1} + 0.3482 z^{-2})^2 \quad (15)$$

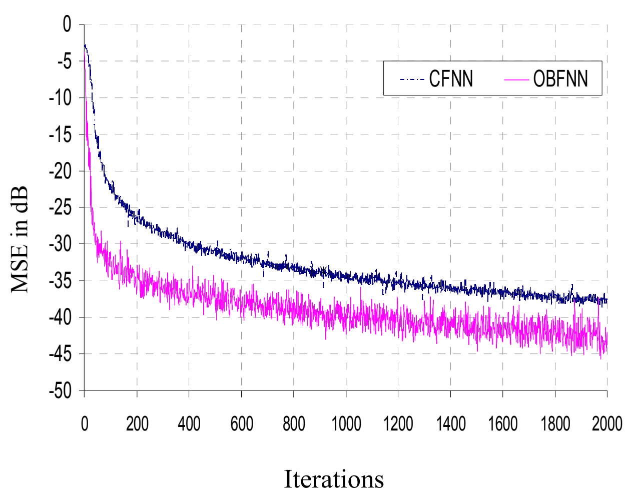

Fig. 4 Learning characteristic of OBFNN equaliser for channel $H_1(z)$

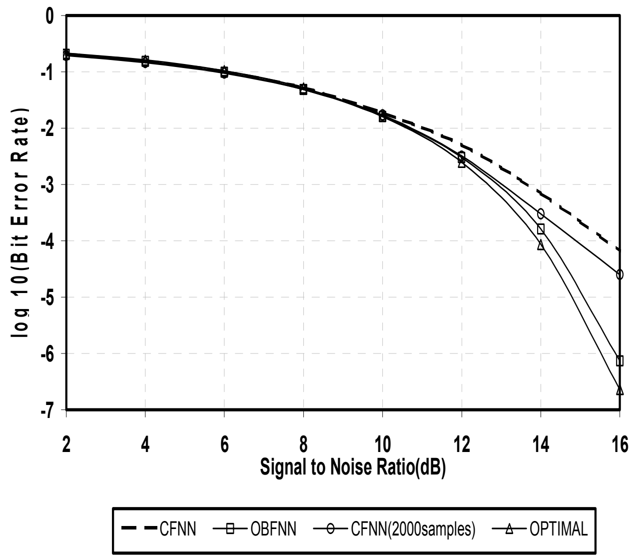

Fig. 5 BER performance comparison of OBFNN equaliser with conventional FNN for channel $H_1(z)$

The advantage gained in terms of performance of the proposed equaliser {a two layer (1,1) OBFNN structure} is clearly demonstrated by comparing their MSE convergence at SNR=20dB and and BER performance with a CFNN structure {a two layer (2,1) structure} with parameters chosen as m=3, n_b=2 and d=2. Simulation results obtained from the Fig. 6 show that the proposed OBFNN equaliser can achieve lower values of the MSE and faster of convergence speed than those of the FNN.The learning rate parameters and momentum constant in weight updation are chosen to be 0.004 and 0.08 respectively. It is observed in Fig. 7 that the proposed OBFNN equaliser yield a significant improvement in BER performance in comparison to their conventional counterpart and close to that of optimal Bayesian equaliser. However, under severe noise conditions (SNR < 8dB), the conventional FNN equaliser structure and all the proposed equaliser configurations yield similar performance. But the superiority in the performance of the proposed ones over the conventional ones is distinct as the signal to noise ratio improves (i.e. for more realistic SNR levels). For example, at a prefixed error probability level (BER) of 10^{-4} , the proposed OBFNN equalisers are able to provide SNR gain of about 1.8 dB over the FNN one.

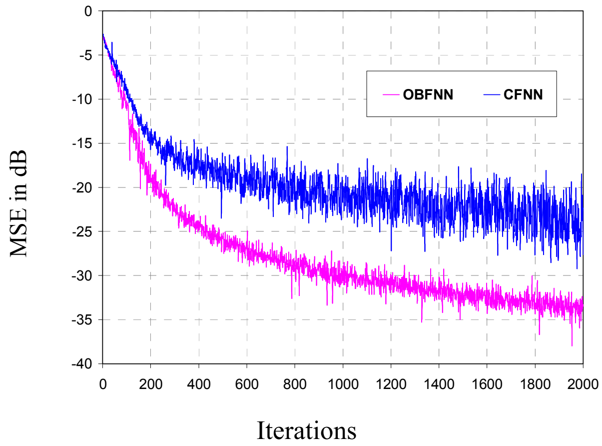

Fig.6 Learning characteristic of OBFNN equaliser for channel $H_2(z)$

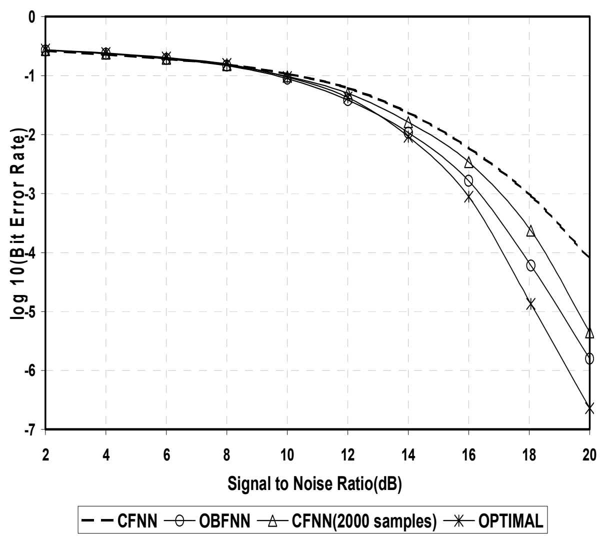

Fig. 7 BER performance comparison of OBFNN equaliser with conventional FNN for channel $H_2(z)$

V. Conclusion

A new FNN based equaliser termed as the Orthogonal basis function (OBFNN) equaliser based on self-breeding genetic framework is presented in this research work. The gain in performance is achieved due to the evolution concept, where the decision at a node (expert's opinion) instead of being directly conveyed to the next node undergoes a two dimensional orthogonal expansion. While one output preserves the information of the current generation to take part in the final decision, the other one is allowed to pass on the information to the next generation to generate a new expert opinion. Such configuration generally reinforces the information base of the final decision node to yield a better estimate of the output as compared with a conventional feedforward neural equaliser structure. The exhaustive simulation studies carried out prove that these novel technique improves the BER performance of FNN equalisers significantly and offer faster learning with less number of training samples as compared to conventional FNN equaliser. In addition to the reduced structural complexity of proposed one proves it efficacy for real-time implementation in DSP or FPGA processors

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