

# DIGITAL PROTECTIVE RELAYING USING AN ADAPTIVE NEURAL NETWORK

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## Summary

The paper presents a new approach to the estimation of voltage and current phasors of a faulted power system by using an adaptive neural network represented by adalines. The neural estimator uses a nonlinear weight adjustment algorithm for the effective rejection of dc offset and noise. The fault apparent resistance, reactance and fault location of a typical transmission line are calculated using the estimated phasors. The results are quite comparable to the Kalman filter approach which requires more computation than the present approach.

**Keywords :** Neural estimation, adaptive neuron fault impedance calculation, fault location.

## 1. Introduction

In the last decade, the development of new digital protection concepts of power systems was very intensive. The development of new generation of numerical protection devices was mostly encouraged by the demand for greater adaptability, uniformity of hardware, integrations of functions and improved flexibility. In addition, digital protection systems allow communication with station monitoring and control systems.

Recently several papers have applied various techniques for the estimation of the Fourier coefficients of voltage and current for digital impedance protection[1]. Thorp and Phadke presented a symmetrical component distance relay based on phasor measurements by using discrete Fourier transforms (DFT)[2]. The computational cost of this DFT based algorithm is very low[3], but its performance can be adversely affected by decaying DC components or low SNR. Sachdev and Baribeau used batch-processing least square estimates of the Fourier coefficients[4]. Grigis applied Kalman filtering to the same[5] problem assuming a stochastic model for the post-fault parameter changes. This algorithm has a long transient response time and high

computational requirement. Recently Kamwa and Grodin presented two recursive algorithms (the RLS and LMS) for tracking voltage phasors in transmission and distribution systems. These two algorithms produce fast tracking of voltage and current phasors but are subject to high computational requirements and filtering of signals corrupted with noise prior to computing.

This paper presents a new approach to the estimation of voltage and current signals in a power system for both impedance and differential protection using a Fourier linear combiner. The linear combiner is realised using a 2-layer neural network. The input layer consists of  $2n+2$  elements, where  $n$  is the number of harmonics to be modeled. The network learns the signal components using the Widrow-Hoff delta rule[6]. The output layer has one element, whose output is the signal sample. This neural model estimates the Fourier coefficients of the signal, in contrast with the other neural estimation schemes presented in the literature in which indirect learning approach is addressed.

## 2. Basic Model and Neural Estimation

For a basic signal model, we assume the measured signals (voltages and currents) can be described by its harmonic components and exponentially decaying transient. If we denote the signal  $y(t)$  we have, with  $n$  harmonic components

$$y(t) = d_0 e^{-t/\tau} + \sum_{k=1}^n C_k \sin(\omega_0 k t + \phi_k) + v(t) \quad (1)$$

where  $v(t)$  is the measurement noise. Using, for example, upto 3rd harmonics and two terms of the Taylor series expansion of  $e^{-t/\tau}$ , the signal  $y(t)$  becomes

$$y(t) = d_0 + d_1 t + \sum_{k=1}^3 (a_k \sin \omega_0 k t + b_k \cos \omega_0 k t) + v(t) \quad (2)$$

Where,

$$\begin{aligned} d_1 &= d_0 / \tau \\ C_k &= \sqrt{a_k^2 + b_k^2} \\ \tan \phi_k &= \frac{b_k}{a_k} \end{aligned} \quad (3)$$

Now if the fundamental frequency  $\omega_0$  and the measurement time  $t$ , are known then equation (3) is linear in the unknown parameters,  $a_k$ ,  $b_k$  and  $d_k$ .

Equation (3) is rewritten as a regression in the following way

$$y(t) = \phi^T(t) \theta + v(t) \quad (4)$$

Where the regression vector

$$\phi^T(t) = [1 \ t \ \sin \omega_0 t \ \cos \omega_0 t \ \sin 2\omega_0 t \ \cos 2\omega_0 t \ \sin 3\omega_0 t \ \cos 3\omega_0 t]$$

(5)

and the unknown parameters collected in the vector  $\theta$

$$\theta^T(t) = [d_0 \ d_1 \ a_1 \ b_1 \ a_2 \ b_2 \ a_3 \ b_3] \quad (6)$$

One of the most widely used adaptive algorithms for the signal estimation is least mean square algorithm (LMS) developed by Widrow and Hoff. The algorithm is simple and computationally efficient and is used to generate the estimates of the constant or time varying parameters of the signal described by equations (1) and (3). This algorithm, however is slow in convergence and does not yield an accurate estimate under noisy observations.

The parameters of the signal model given by equation (6) are identified by using an adaptive neural network consisting of linear adaptive neurons called adaline. The regression input vector for this network shown in Fig.1 is given by

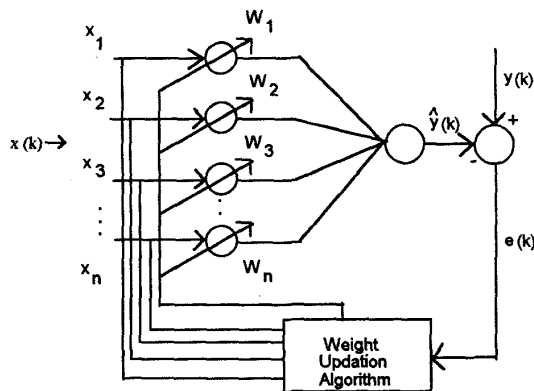


Figure 1: Block Diagram of an Adaline

$$x^T(k) = [k\Delta t \ \sin \omega_0 k\Delta t \ \cos \omega_0 k\Delta t \ \dots \ \sin 3\omega_0 k\Delta t \ \cos 3\omega_0 k\Delta t] \quad (7)$$

$$\text{where, } \Delta t = \text{sampling time} = \frac{2\pi}{\omega_0 N_s}$$

$N_s$  = sample rate

The Widrow-Hoff adaptation algorithm is used to adapt the adaline's weight vector. The delta rule which minimizes the mean square error between the signal sample  $y(k)$  and the output of the net  $\hat{y}(k)$  over all  $k$ , can be written as

$$W(k+1) = W(k) + \frac{\alpha e(k) x(k)}{\lambda + x^T(k) x(k)} \quad (8)$$

where  $k$  = time index or sample number,

$$W(k) = [W_1(k) \ W_2(k) \ \dots \ W_N(k)]^T$$

is the weight vector at time  $k$ ,

$e(k) = \hat{y}(k) - y(k)$  is the error at time  $k$ ,

$\alpha$  = learning parameter

and  $\lambda$  is a constant chosen to make

$$\lambda + x^T(k) x(k) \neq 0. \quad (9)$$

The error dynamics for the adaline for the weights are adjusted by the delta rule can be obtained as follows:

The neural estimator (equations 7-9) satisfies the following properties:

$$(i) \ \|W(k) - W_0\| \leq \|W(k-1) - W_0\| \leq \|W(0) - W_0\|, \quad k > 1 \quad (10)$$

$$(ii) \ \lim_{k \rightarrow \infty} \frac{e^2(k)}{\lambda + x^T(k) x(k)} = 0 \quad (11)$$

where  $W_0$  is the weight vector that corresponds to perfect learning of the signal parameters and

$$y(k) = x^T(k) \cdot W_0 \quad (12)$$

The error signal for the adaline whose weights are adjusted by the delta rule can be obtained as follows:

$$\tilde{W}(k) = W(k) - W_0$$

and

$$e(k) = y(k) - \hat{y}(k) = x^T(k) W_0 - x^T(k) W(k) = x^T(k) \tilde{W}(k) \quad (13)$$

Subtracting  $W_0$  from both sides equation (8) we get

$$\tilde{W}(k+1) = \tilde{W}(k) - \frac{\alpha x^T(k) x(k)}{\lambda + x^T(k) x(k)} \cdot \tilde{W}(k) = (1 - \alpha') \tilde{W}(k) \quad (14)$$

where,

$$\alpha' = \frac{\alpha x^T(k) x(k)}{\lambda + x^T(k) x(k)} \approx \alpha \quad (15)$$

From equations (13) and (14) we get

$$\|\tilde{W}(k+1)\|^2 - \|\tilde{W}(k)\|^2 = \left[ -2\alpha + \frac{\alpha^2 x^T(k) x(k)}{\lambda + x^T(k) x(k)} \right] \frac{e^2}{\lambda + x^T(k) x(k)} \quad (16)$$

If  $\alpha$  lies between 0 and 2, and  $\lambda > 0$  and is a small quantity it can be verified that for

$$\left[ -2 + \frac{\alpha x^T(k) x(k)}{\lambda + x^T(k) x(k)} \right] < 0 \quad (17)$$

$\|\tilde{W}(k)\|^2$  is a bounded non increasing function and  $\tilde{W}(k)$  will reduce to a null vector after a few iterations.

The error signal  $e(k+1)$  at  $(k+1)$  the iteration is given by

$$e(k+1) = -x^T(k+1) \tilde{W}(k+1) = -x^T(k+1) (1 - \alpha) \tilde{W}(k) \quad (18)$$

On comparison of equations (13) and (18) are finds that  $|e(k+1)| < |e(k)|$

for  $0 < \alpha < 2$

Usually with a value of  $\alpha = 0.8$  or  $1.3$ ,

$$(1 - \alpha)x(k+1) < x(k) \quad (20)$$

and the inequality in (19) is satisfied.

Thus the tracking error signal will asymptotically reduce to zero after a few iterations. After the weight error vector reduces to zero asymptotically after a few iterations. The final weight vector  $W_0$  will yield the parameters regressor vector of the faulted current or voltage signal

$$\theta^T = W_0 \quad (21)$$

To provide faster convergence and noise suppression, nonlinearity is introduced to the learning rule in the following manner :

$$W(k+1) = W(k) + \frac{\alpha e(k)X(k)}{\lambda + x^T(k)X(k)} \quad (22)$$

where  $X(k)$  is a nonlinear function and the simplest one is chosen as

$$X(k) = \text{SGN}\{x(k)\} \quad (23)$$

where,

$$\text{SGN}(x_i(k)) = \begin{cases} +1 & \text{if } x_i(k) > 0 \\ -1 & \text{if } x_i(k) < 0 \\ 0 & \text{if } x_i(k) = 0 \end{cases} \quad (24)$$

The learning rate is made adaptive for fast convergence and noise rejection by choosing

$$\alpha = \frac{\alpha_0}{(1 + k/\beta)} \quad (25)$$

where  $\alpha_0$  = initial learning rate

$\beta$  = decaying rate constant

The choice of  $\alpha_0$  and  $\beta$  depends on the signal to noise ratio and the nature of the signal whose parameters are to be estimated. For noisy signal a value of  $\alpha_0 > 1$  and  $20 < \beta < 100$  will be adequate. For time varying signals, the value  $\beta$  is  $\geq 100$ .

### 3. Fault Location and Impedance Estimation

When data is available at one location, the apparent impedance is defined as the ratio of a selected voltage to a selected current based on the fault type and the faulted phases. Upon the detection of disturbance, the phasor quantities of the voltages and currents are obtained using the neural estimator described in the earlier section. The change in the magnitude of the current phasors is used to classify the fault type and faulted phases. Upon the classification of the fault type, a voltage pair is selected to compute apparent impedance. Using the fault boundary conditions and the sequence network parameters, the following equation holds good for a single-line-to-ground fault on a 3-phase power system at a distance D(in km) from the sending end source (Fig.2).

$$V_a = (I_a + kI_0)Z_1 + 3I_0R_f \quad (26)$$

where

$$k = \frac{(Z_0 - Z_1)}{Z_1} \quad (27)$$

Thus the apparent impedance is defined as

$$Z_{app} = V_{select} / I_{select} = R_{app} + jX_{app} \quad (28)$$

where  $R_{app}$  = apparent resistance at the relaying point

$X_{app}$  = apparent reactance at the relaying point.

Here  $V_{select} = V_a$

$$\text{and } I_{select} = I_a + kI_0 = I_d + jI_q \quad (29)$$

Thus

$$Z_{app} = Z_1 + \frac{(3I_0R_f)}{(I_a + kI_0)} \quad (30)$$

To compensate for the unknown fault resistance, the current fed into the fault must be considered. For a single-line-to ground fault, the compensating current is proportional to the change in the zero sequence current in the faulted line section. Thus the equation for the apparent impedance is

$$Z_{app} = DZ_1 + \frac{3I_{comp}R_f}{(I_a + kI_0)} \quad (31)$$

A little manipulation will yield

$$D = (R_{app} \cdot A - B \cdot X_{app}) / (r_1 \cdot A - x_1 \cdot B) \quad (32)$$

where

$r_1, x_1$  are the per unit resistance and reactance of the line

$$A = (-I_{cd}I_q + I_{cq}I_d) / I_m^2 \quad (33)$$

$$B = (I_{cd}I_d + I_{cq}I_q) / I_m^2$$

and

$$I_m^2 = I_d^2 + I_q^2 \quad (34)$$

$$I_{comp}^2 = I_{cd}^2 + I_{cq}^2$$

with  $I_{comp} = 3I_0$

In a similar way for a line-to-line fault

$$V_{select} = V_a - V_b, I_{select} = I_a - I_b \quad (35)$$

and  $I_{comp} = \Delta I_a - \Delta I_b$

where  $\Delta$  represents the change.

### 4 Numerical Experimentation

For digital protection, the fault current and voltage data are obtained from the power system shown in Fig.1 using EMTDC.

Operating Parameters :

Base voltage = 210 kV

Base current = 2300 Amp.

$Z_1 = 0.5655 + j.9270 \Omega/\text{km}$

$Z_2 = 0.1202 + j.0840 \Omega/\text{km}$

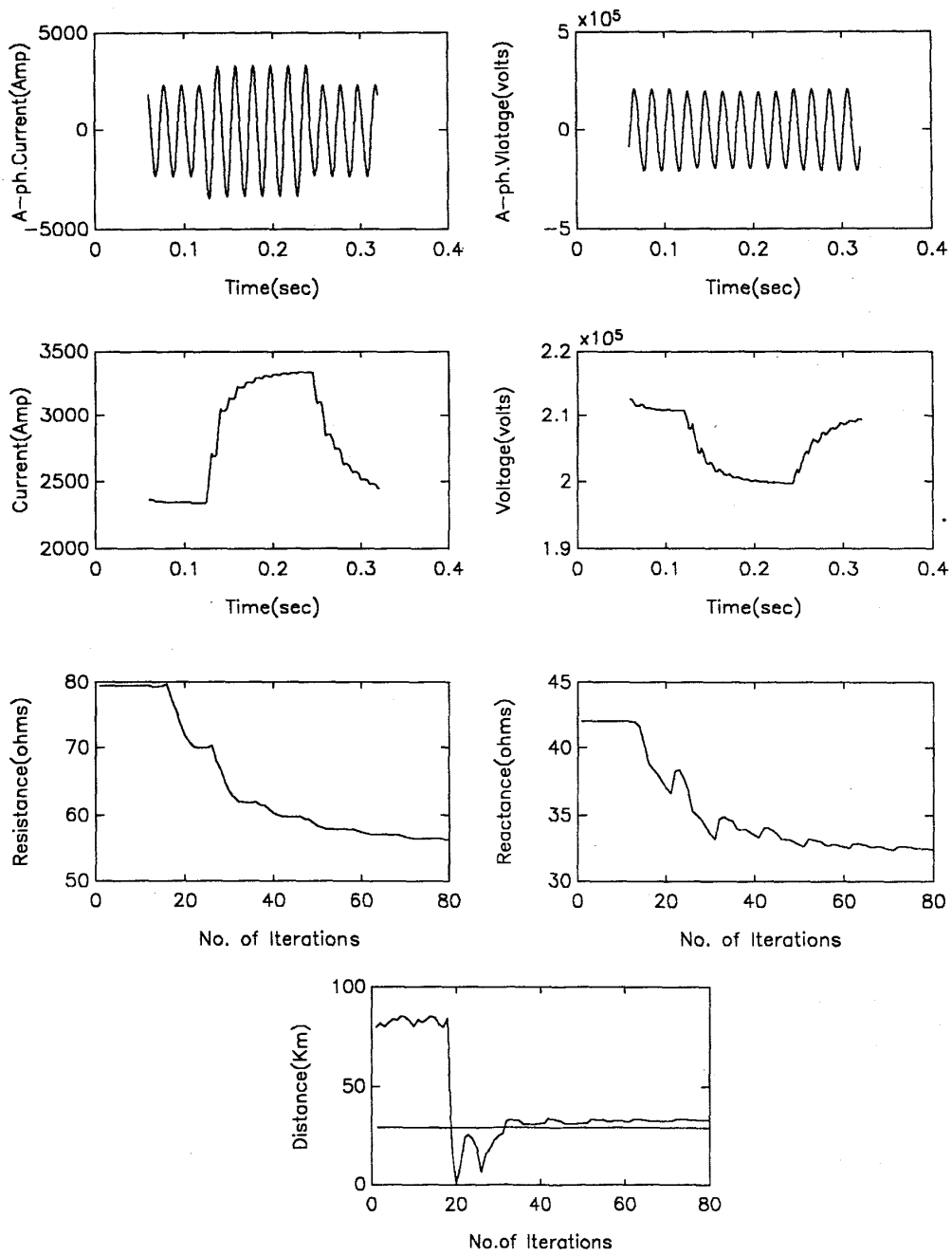
$Z_0 = 0.1119 + j.0804 \Omega/\text{km}$

Source impedance  $Z_s = 1.367 + j8.58 \Omega$

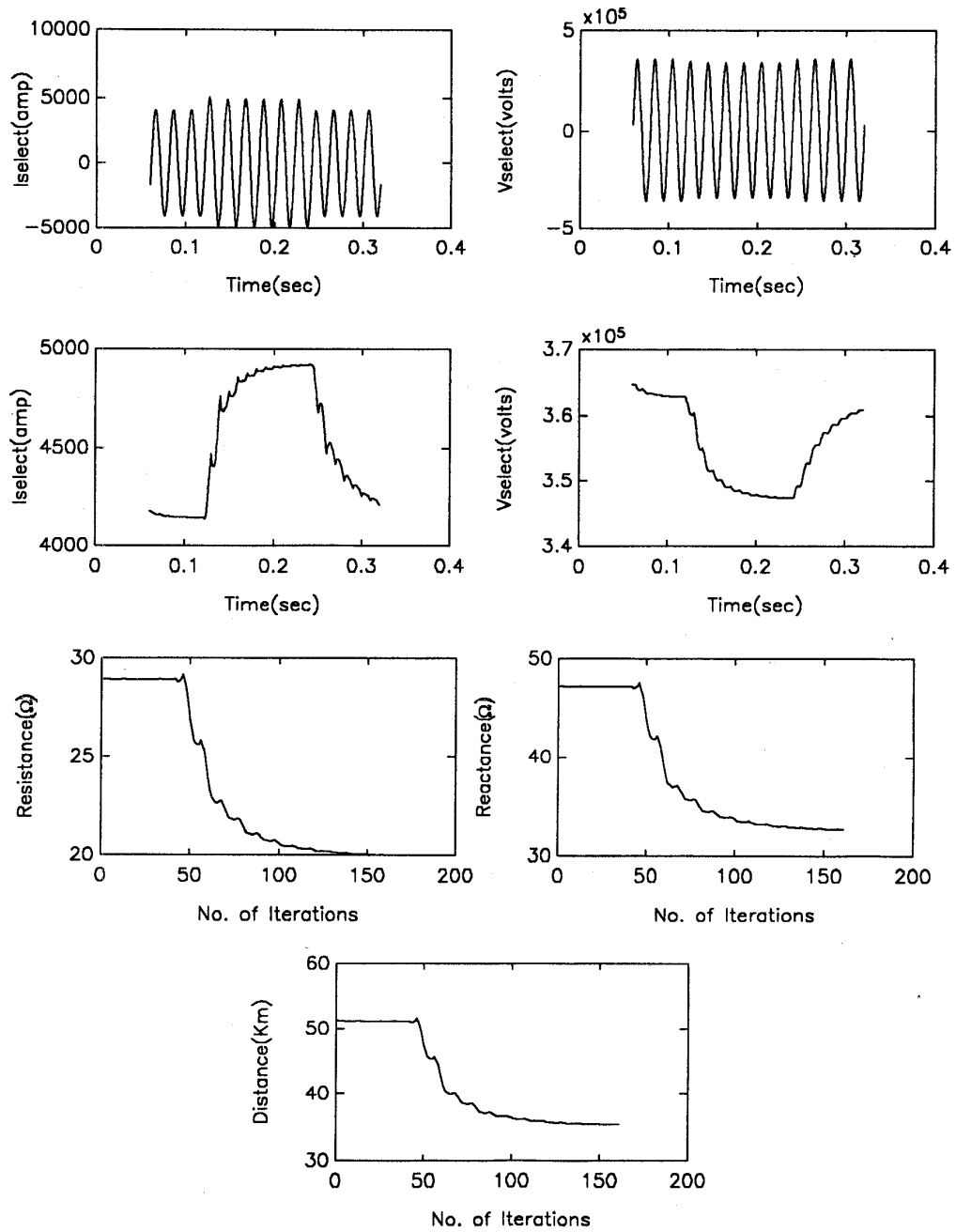
Load impedance =  $50 + j 15.7 \Omega$

Fault resistance  $R_f = 1.0 \Omega$

Fig.2 shows the voltage, current, peak voltage and current,  $R_{app}$ ,  $X_{app}$ , and the distance D obtained at the relaying using the adaline for a single-line to ground fault at a distance of 30 km from the relaying point. From



**Figure 2:** Calculation of resistance, reactance & distance of Single-line to Ground fault, using Adalines.



**Figure 3:** Calculation of resistance, reactance & distance of L-L fault using Adaline.

the figure it is observed that the adaptive neural estimator tracks the voltage, current, apparent resistance, reactance, and the distance accurately. The error of nearly 5% is due to the computation of unknown fault resistance accurately.

Fig.3 depicts the above mentioned quantities of the power system for a line-to-line fault at 30 km from the relaying point. The tracked parameters are found to be accurate and are comparable to the values generated using a Kalman filtering.

## 5. Conclusions :

The paper presents a neural estimator approach for digital protective relaying of power transmission lines. The neural estimator uses an adaline and nonlinear weight adjustment algorithm using SGN function. A fault distance calculation routine is used to estimate the distance of the fault from the relaying point using the estimated phasor quantities. The adaline provides an accurate and robust technique for fault distance calculation.

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## 7. References :

- [1] Phadke A.G., Hlebika T., Ibrahim M., "Fundamental basis for distance relaying with systematical components", IEEE Trans. on PAS, Vol.PAS-96, 1977, pp.635-646.
- [2] Thorp, J.S., Phadke, A.G., Horowitz, S.H., and Beehler, J.E., "Limits to Impedance Relaying", IEEE Trans., 1979, PAS-98, pp.246-260.
- [3] Jr. A.W. Brooks, "Distance relaying using least-squares estimates of voltage, current and impedance", Proceedings of IEEE PICA Conference, No.77 CH 1131-2-PWR, May 1977, pp.394-402.
- [4] Sachdev, M.S. and Baribeau, M.A., "A new algorithm for digital impedance relays", IEEE Trans. 1979, PAS-98, pp.2232-2240.
- [5] Girgis, A.A., "A New Kalman filtering based digital distance relay", IEEE Trans., 1982, PAS-101, pp.3471-3479.
- [6] Kezunovic, M., "Digital protective relaying algorithms and systems-An overview", Electr. Power Syst. Res., 1981, 4, pp.167-180.
- [7] Wainear P.L., Elangovan, and A.C.Liew, "Fault Impedance estimation algorithm for digital rules", IEEE Trans. on PD, vol.9, no.3, 1994, pp.1375-1384.
- [8] Widrow, B. and M.A.Lehr, "30 years of adaptive neural networks", Proc. IEEE, Vol.78, 1990, pp.1415-1442.