

ART based cell formation using combined operation sequence and time

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Abstract – The cell formation (CF) problem mainly deals with clustering of parts into part families and the machines into machine cells. The parts are grouped into part families based on similarities in their manufacturing and design attributes and the machines are allocated into machine cells to produce the identified part families. The zero-one part-machine incidence matrix is commonly used as input to any clustering algorithm. The output is generated in the form of block diagonal structure. Production data such as operation time, sequence of operations, batch size etc. that have significant bearing on smooth flow of materials are not considered in such methods. In this paper, an attempt has been made to develop an algorithm based on Adaptive Resonance Theory (ART) neural network to address this issue by considering combination of operation sequence and operation time of the parts to enhance the quality of the solution obtained for the CF problem. A new performance measure is proposed to assess the goodness of the solution quality obtained through proposed algorithm. The performance of the proposed algorithm is tested with example problems and the results are compared with the existing methods found in the literature. The results presented clearly shows that the performance of the proposed algorithm is comparable with other methods for small size problems and better for large size problems.

Keywords: Cell formation; ART1; Grouping efficiency

I. INTRODUCTION

The primary concern in group technology (GT) is to identify the machine cells and part families such that movement of parts from one GT cell to another cell can be minimized. The part-machine incidence matrix (PMIM) is presented as input. The part families and machine cells are identified from the diagonal blocks of the output matrix. If any value exists in the off diagonal blocks, it indicates the inter-cell movements of the respective parts that are called as exceptional elements. There have been several methods to solve this cell formation (CF) problem viz., array manipulation, hierarchical clustering, non-hierarchical clustering, mathematical programming, graph theory, heuristics, etc. These methods are found to produce good solutions for well structured matrices where part families and machine cells exist naturally. However, they fail to produce so, for ill structured matrices in the block diagonalization and will end up with many exceptional elements.

In this work, an attempt is made to use the operation sequence and the operation time of the parts which are

obtained through the route sheets to group the parts into part families and machines into machine cells with an idea to maximize the proposed performance measure. The proposed algorithm employs the principle of ART1

network found in the literature [1, 2, 3]. The proposed algorithm first converts the given non-binary data into a zero-one binary matrix known as part machine precedence matrix (PMPM) and feed the ART1 network with PMPM as the input matrix. In section II, the step by step procedure for CF with combined data i.e. operation sequence and time is presented.

II. THE PROPOSED ALGORITHM FOR CELL FORMATION WITH COMBINED DATA

The input to the algorithm is the operational time and sequence based part-machine incidence matrix (PMIM) of size $N \times M$.

Step 1: Using the given PMIM with the sequence data, for every part, a machine-machine precedence matrix (MMPM) of size $m \times m$ is constructed. Each row of a MMPM represents a machine and the ones in the row indicate all other machines which are required subsequently for the part considered. However, for the row corresponding to the first machine to be visited by the part, the '1's will appear in all the machines required by the part; thus it has the maximum number of '1's in the MMPM for the particular part. For the rows corresponding to the machine which are not required by the part, all the elements are assigned with zero.

Step 2: Using the n number of MMPMs, a single Part-Machine precedence matrix of size $N \times (M \times M)$ is constructed. Each row of the PMIM corresponds to a part and the element of the row is obtained by placing all the rows of the MMPM in a linear sequence.

Step 3. Multiply all the ones present in the PMPM by the respective workload data from the work load matrix which represents operational time of the parts. The new matrix is a combination of ordinal and ratio level data which is named as matrix of combined data (MCD).

Step 4. The MCD will be the input to the modified ART1 which is given in steps 5 to 15.

Step 5: Set nodes in the input layer equal to N (number of parts) and nodes in output layer equal to $(M \times M)$. Set vigilance threshold (ρ).

Step 6: Initialize top-down connection weights. Top-down weights wt_{ji} ($0 = 0$ for $i = 1, 2, \dots, N$ and $j = 1, 2, \dots, (M \times M)$).

Step 7: Let $q=1$. The first input vector X_1 (first row of the MCD) is presented to the input layer and assigned to the first cluster. Then, first node in the output layer is activated.

Step 8: The top-down connection weights for the present active node are set equal to the input vector.

Step 9: Let $q = q+1$. Apply new input vector X_q . (input vectors are the rows of the PMPM).

Step10: Compute Euclidean distance between X_q and the exemplar stored in the top-down weights (wt_{ji}) for all active nodes i as given in the Eq. (1). This distance function is used to calculate similarity between the stored pattern and the present input pattern. If the similarity value is less than or equal to ρ (vigilance threshold), the present input is categorized under the same cluster as that of stored pattern.

$$e_i = \sqrt{\sum_{j=1}^M (x_{qj} - wt_{ji})^2} \quad (1)$$

Step11: Perform vigilance test: Find out minimum euclidean distance.

Step12: If $\min e_i \leq \rho$ (threshold value), select output node for which euclidean distance is minimum. If tie occurs, select the output node with lowest index number. Suppose output node k is selected then allocate the vector X_q to the node k (cell) and activate node k . Make increment to the number of parts in the active node k by one. If e_i 's for all active nodes are greater than ρ , then go to step 13.

Step13: Start a new cell by activating a new output node.

Step14: Update top-down weights of active node k .

The decision for belongingness of an input vector to a node (cluster) is determined using similarity between previously stored exemplar with present input pattern. In other words, top-down weights play the role of storing exemplars (for active nodes) for comparison purpose. Therefore, top-down weights must contain relevant information of all the input vectors already classified under an active node (cluster) in aggregate nature. The top down weights are updated each time when a new input vector is presented and clustered to an active node. When a vector is selected (to be allocated to an output node), its top-down weights are updated using more information of the previously stored exemplar and a relatively less information of the input vector (pattern) as shown in Eq. (2).

$$wt_{jk} = \left(\frac{n}{m} \cdot wt_{jk}\right) + \left(\frac{1}{m} \cdot x_{qj}\right) \quad (2)$$

Step15: Go to step 5 and repeat till all the rows are assigned in the output nodes (cells).

Step16: Check for singleton part family. If a singleton is found in any part family, perform the following operations

to merge the part family with one part into any other part families.

Determine average of processing time in each part family.

Calculate the Euclidean distance between the part families.

Merge the part family containing single part with another part family in such a way that euclidean distance between them is minimum than other part families.

Step17: Allocate machines to the part families using following supplementary procedure.

The number of operations of a part in a particular machine is computed. If the part has maximum number of operations in machine i then the machine i is allocated to that part family where the part exists.

In case tie occurs the machine is allocated with the part family where minimum intercell moves are possible.

In case tie occurs again the machine is allocated with the small identification number of part family.

III. MEASURE OF PERFORMANCE

There are some popular measures like grouping efficiency and group efficacy [4] for measuring the goodness of the block diagonal structure of the output matrix in CF problems. However, all these measures treat all the operations equally and are suitable only for the binary matrix. These measures cannot be adopted for generalized cell formation problem where operational sequence and time of the parts are considered.

Therefore, group technology efficiency (GTE) given by Harhalakis et al. [5] as given in Eq. (3) and modified grouping efficiency by Mahapatra and Pandian [6, 7] as given in Eq. (4) can be conveniently used to measure the performance considering sequence of parts and operational times of the parts respectively.

$$GTE = \frac{I_p - I_r}{I_p} \quad (3)$$

$$MGE = \frac{\sum_{k=1}^c T_{ptk}}{N_e + \sum_{k=1}^c T_{ptk} + \sum_{k=1}^c (N_{vi} \times \frac{\sum T_{ik}}{N_p})} \quad (4)$$

But in case of modified grouping efficiency, the operation sequence is not addressed to make it a generalised one. GTE is used to measure the performance of the output matrix with the information of sequence of parts. There is no measure found in the literature to include both the information combined. Hence, a new measure of grouping efficiency termed as Ratio Ordinal

Combined Efficiency (ROCE) has been proposed in this work to find out the goodness of the grouping in the cell formation problem that deals with both operation sequence and time in the input matrix with due consideration of equal weightages to both the data. Ratio-Ordinal Combined Efficiency is calculated using the Eq. (5). Ratio Ordinal Combined Efficiency (ROCE) is defined as the weighted average of modified grouping efficiency (MGE) and group technology efficiency (GTE).

$$ROCE = q(MGE) + (1 - q)(GTE) \quad (5)$$

where,

T_{ptk}	Total processing time inside cell k.
T_{ik}	Total time taken by i^{th} machine in cell k
N_p	number of parts having operation in i^{th} machine
N_{vi}	Number of voids in i^{th} machine.
N_e	Total number of exceptional elements
I_p	Maximum number of inter-cell travel possible in the system.
I_r	Number of inter-cell travel actually required by the system.
n	Number of operations ($w=1,2,3,\dots,n$)

The problem of size 12×10 from literature [10] is considered for illustrating the proposed performance measure (ROCE). Initially, the input matrix is fed into the algorithm. After the output being generated, the total number of exceptional elements of the output matrix is counted ($N_e = 4$). The total processing time inside each cell has been calculated and found to be (T_{ptk}) => ($T_{pt1}=11.4$), ($T_{pt2}=10.05$), ($T_{pt3}=4.01$). The number of voids in each column is a count of zeros present in the respective columns inside the cells (N_{vi}) => ($N_{v1}=1$, $N_{v4}=1$, $N_{v5}=1$, $N_{v8}=2$, $N_{v9}=1$, $N_{v10}=1$). The average of the operational time of parts in each column is calculated which is multiplied by the respective N_{vi} values to get the void factor. The maximum number of inter-cell travel possible in the system (I_p) is found to be 25 and the number of inter-cell travel actually required (I_r) by the system is 4. The maximum number of operations (n) is given as 5. The values of GTE, MGE and ROCE are calculated using Eqs. 3, 4 and 5 respectively. Hence the value of MGE is 0.7645, GTE = 0.84 and ROCE = 0.8023. These values are calculated for the output of the existing methods and presented in Table I. It is observed that the results of the proposed performance measure outperforms the existing methods ACCORD [8] and analytical iterative approach [9]

IV. RESULTS AND DISCUSSIONS

The proposed algorithm has been coded in C++ and executed in a Pentium III, 700 MHz system. All the problems illustrated and solved in the literature are tested with the proposed algorithm. For all trial data sets, the

input matrix is generated with uniformly distributed random numbers in the range of 0.5 to 5 for operational time and 1 to 7 for operational sequence. The problem sizes considered in this work range from 5×4 to 90×35 .

The effect of vigilance threshold is inversely proportion with the number of cells. The vigilance threshold is varied from 1 to 9 for each problem till desired solution is obtained. The solutions obtained through the proposed algorithm are compared with the solution reported in the literature [8, 10]. In Table II, the comparison of the results of the proposed method with that of CASE [10] for all the example problems is given. In addition, a new weighted average performance measure ROCE is proposed that measures the performance of the algorithm proposed and tested with fifteen trial data sets. The results are found to be consistent for all the data sets tested which are shown in Table III. It is found that the proposed algorithm for machine cell formation with sequence data gives satisfactory results which are either superior or same as the existing methods found in literature.

V. CONCLUSIONS

In this work, ART1 based algorithm has been developed to solve the cell formation problem taking into account the production sequence data and operation time of the parts combined. In the proposed algorithm the non binary data is converted into a binary data and fed into a modified ART1 network. The performance of the proposed algorithm is compared with that of the other popular algorithms found in the literature for all the example problems cited in the literature and it is found that the proposed method outperforms the existing algorithms for larger size problems in terms the exceptional elements and the newly proposed performance measure ROCE in this work. The methodology of converting the non binary data into a suitable binary data and subsequently by feeding to the ART1 networks to solve the CF problem can be suitably modified or extended to solve the CF problem with other production data like batch sizes, machine capacity, etc, for different objective criteria.

REFERENCES

- [1] Moon, Y. (1990) Interactive activation and competition model for machine-part family formation, Proceedings of the International Conference on Neural Networks, Washington DC 2:667-670.
- [2] Carpenter, G.A. and Grossberg, S. (1987) A massively parallel architecture for a self-organizing neural pattern recognition machine, Comput Vision, Graph Image Process 37:54-115.

- [3] Dagli, C. and Huggahalli. R. (1995) Machine-part family formation with the adaptive resonance theory paradigm, *International Journal on Production Research* 33(4):893–913
- [4] Kumar, C.S. and Chandrasekharan, M.P. (1990) Grouping efficacy: a quantitative criterion for goodness of block diagonal forms of binary matrices in group technology, *International Journal of Production Research*, 28 (2), pp 233-243.
- [5] Harhalakis, G., Nagi, R. and Proth, J.M. (1990) An efficient heuristic in manufacturing cell formation for group technology applications, *International Journal of Production Research*, 28 (1), pp.185-198.
- [6] Mahapatra, S.S. and SudhakaraPandian, R. (2006) Adaptive Resonance Theory applied to generalized cell formation”, *International Journal for Manufacturing Science and Technology*, vol. 8, pp. 13-22.
- [7] Mahapatra, S.S. and SudhakaraPandian, R. (2007) Genetic cell formation using ratio level data in cellular manufacturing systems *International Journal of Advanced Manufacturing Technology*, DOI 10.1007/s00170-007-1029-5.
- [8] Nair, J.G and Narendran, T.T. (1999) ACCORD: A bicriterion algorithm for cell formation using ordinal and ratio-level data, *International Journal of Production Research* 37:539–556.
- [9] George, A.P, Rajendran, C. and Ghosh, S. (2003). An analytical-iterative clustering algorithm for cell formation in cellular manufacturing systems with ordinal-level and ratio-level data. *International Journal of Advanced Manufacturing Technology* 22:125–133.
- [10] Nair, J.G. and Narendran, T.T. (1998) CASE: A clustering algorithm for cell formation with sequence data, *International Journal on Production Research*, 36(1),157-179.

TABLE I
COMPARISON OF PROPOSED METHOD OVER EXISTING METHODS FOR THE PROBLEM SIZE 12 x 10

Factors considered	ACCORD [4]	Analytical iterative approach [10]	Proposed method
Exceptional elements	5	5	4
Grouping efficiency	0.881	0.881	0.897
Grouping efficacy	1.026	1.026	1.026
Modified grouping efficiency (%)	69.24	69.24	76.45
Group technology efficiency (%)	80.00	80.00	84.00
Ratio ordinal combined efficiency (%)	74.62	74.62	80.23

TABLE II
COMPARISON OF RESULTS OF THE PROPOSED ALGORITHM WITH CASE WITH EXAMPLE PROBLEMS

Problem Size	No. of Cells	CASE (1998)			Proposed Algorithm		
		Exceptional Elements	Intercell Moves	Group Technology Efficiency	Exceptional Elements	Intercell Moves	Group Technology Efficiency
7 x 7	2	2	4	69.25	2	4	69.25
	3	3	6	53.85	3	6	53.85
20 x 8	3	10	17	58.54	10	17	58.54
20 x 20	4	NA	NA	NA	12	15	74.58
	5	15	19	67.8	16	18	69.49
40 x 25	5	NA	NA	NA	26	22	72.04
	8	35	31	66.67	35	31	66.67

TABLE III
PERFORMANCE OF THE PROPOSED ALGORITHM ON TEST DATA SETS

S.No	Problem Size	Exceptional Elements	Intercell Moves	GTE	MGE	ROCE
1	5 x 4	0	0	100.00	83.48	91.74
2	5 x 5	1	1	85.71	81.15	83.43
3	7 x 5	6	5	64.30	72.01	68.16
4	8 x 6	2	2	84.61	70.15	77.38
5	19 x 12	8	9	83.93	65.08	74.51
6	20 x 12	11	10	78.00	59.56	68.78
7	20 x 20	3	3	94.00	84.25	89.13
8	30 x 15	21	17	76.71	60.02	68.37
9	37 x 20	25	25	71.59	60.99	66.29
10	50 x 25	49	46	69.13	58.39	63.76
11	55 x 20	15	19	81.20	66.03	73.62
12	60 x 28	39	38	70.50	57.20	63.85
13	65 x 30	58	52	76.68	59.59	68.14
14	80 x 32	53	59	74.57	62.28	68.43
15	90 x 35	54	56	77.69	62.26	69.98