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Prediction of fatigue crack growth and residual life using an exponential model: Part I (constant amplitude loading)

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A B S T R A C T

In the present investigation an attempt has been made to introduce a life prediction methodology by adopting an 'Exponential Model' that can be used without integration of fatigue crack growth rate curve. The predicted results are compared with experimental crack growth data obtained for 7020-T7 and 2024-T3 aluminum alloy specimens under constant amplitude loading. It is observed that the results obtained from this model are in good agreement with experimental data and cover both stage-II and stage-III of fatigue crack growth curve.

1. Introduction

With the application of fracture mechanics concepts to fatigue failure and the development of sophisticated crack detecting techniques, the crack propagation data can be obtained in terms of crack length (a) and number of cycles (N). Attempts were made by earlier researchers to determine the fatigue crack growth rate (FCGR) from a vs. N plots either by graphical procedures or by computational methods and to establish some functional relationship between fatigue crack growth rate and different variables such as loading variables, K_{max}, ΔK, R, f, etc. as described in Eq. (1).

$$\frac{da}{dN} = \Phi(K_{max}, \Delta K, R, f, T \ldots) \qquad (1)$$

Because of the complexity of the fracture process, no general relationship has so far been established that would include all the variables. For predicting fatigue life, the rate at which a fatigue crack grows must be suitably described in terms of various crack driving parameters. Several empirical and phenomenological-based crack propagation models have been proposed till date for estimating fatigue life. The basic model for crack propagation rate was first proposed by Paris and Erdogan who assumed that fatigue crack propagation rate da/dN depended on stress intensity factor range ΔK. The mid-region of the curve (region II) followed a straight line fit on a log–log plot with scaling constants C and n. Here it followed the law $\frac{da}{dN} = C \cdot (\Delta K)^n$, known as Paris–Erdogan equation [1]. However, certain metals and alloys, like Titanium and its alloys, do not follow this rule [2]. Since the point of transition from stage-II to stage-III depends on fracture toughness of the material, stress intensity factor range and stress ratio. Forman et al. [3] modified the above equation and suggested the form as $\frac{da}{dN} = \frac{C_a \cdot \Delta K^{n_a}}{(1-R) \cdot K_C - \Delta K}$. Walker [4] suggested another form of the equation given by $\frac{da}{dN} = C_b[(1-R)^{c_1} K_{max}]^{n_b}$. Later Elber [5] introduced the concept of crack closure and suggested that crack growth rate is a function of ΔK_{eff} where $\Delta K_{eff} = K_{max} - K_{open}$. There are several other models developed till date, such as Collipriest [6], Dover [7], Suliva and Crooker [8], Liu [9], Yokobori [10], Xiulin [11], etc. However, no single model is applicable to all materials nor considers the influence of all parameters at a time.

The aim of developing a fatigue crack growth model is to predict a safe operating life while designing a structure/component subjected to cyclic loading. The service life of a structure/machine component under cyclic loading can be estimated by integrating the rate equation of the Paris type. However, direct integration becomes robust and complicated as the geometrical factor 'f(g)' in the expression of ΔK varies with crack length. Therefore, fatigue life may be estimated by numerical integration using different values of 'f(g)' held constant over a small crack length increment [12]. To overcome this difficulty, the authors have attempted to introduce a life prediction procedure by adopting an 'Exponential Model'. The model can predict the fundamental a–N curve to calculate life without integration of FCGR curve. It is worth mentioning that an exponential model is often used for calculation of growth of population/bacteria, etc. The basic equation of the model is:

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Nomenclature

a	crack length measured from edge of the specimen (mm)	m	specific growth rate
a_i	crack length corresponding to the 'ith' step (mm)	m_{ij}	specific growth rate in the interval i–j
a_j	crack length corresponding to the 'jth'step (mm)	n	exponent in the Paris equation
a_f	final crack length (mm)	n_a	exponent in the Forman equation
A', B', C' and D'	curve fitting constants in the 'Exponential Model'	n_b	exponent in the Walker equation
B	plate thickness (mm)	N	number of cycles or fatigue life
c_1	constant in the Walker equation	N_i	number of cycles corresponding to the 'ith' step
C	constant in the Paris equation	N_j	number of cycles corresponding to the 'jth' step
C_a	constant in the Forman equation	N_f	final number of cycles
C_b	constant in the Walker equation	N_f^F	fatigue life of Forman model (cycles)
COD	crack opening displacement	N_f^P	predicted fatigue life (cycles)
da/dN	crack growth rate	N_f^E	experimental fatigue life (cycles)
E	Young's modulus of elasticity (MPa)	P	population
F	remotely applied load (N)	P_0	initial population
F_{max}	maximum load	$P(t)$	population at any time 't'
F_{min}	minimum load	r	population growth rate
f(g)	geometrical factor	R	load ratio
f	frequency (Hz)	t	time
K	stress intensity factor (MPa$\sqrt{m}$)	T	temperature
K_C	plane stress fracture toughness (MPa$\sqrt{m}$)	w	plate width (mm)
K_{IC}	plane strain fracture toughness (MPa$\sqrt{m}$)	ν	Poison's ratio
K_{open}	opening stress intensity factor (MPa$\sqrt{m}$)	β_{IC}	fracture toughness correlation factor
K_{max}	maximum stress intensity factor (MPa$\sqrt{m}$)	σ_{ys}	yield point stress (MPa)
ΔK	stress intensity factor range (MPa$\sqrt{m}$)	σ_{ut}	ultimate stress (MPa)
ΔK_{eff}	effective stress intensity factor (MPa$\sqrt{m}$)		
l	dimensionless factor in 'Exponential Model' formulation		

$$P(t) = P_0 e^{rt} \tag{2}$$

where, $P(t)$ is the Population at any time 't'; P_0 is initial population; r is the Malthusian parameter or Intrinsic rate or specific growth rate.

An attempt has been made to suitably modify the model for crack growth behavior under constant amplitude loading and test its validity in the light of experimental results.

2. Experimental procedure

This study was conducted using 7020 and 2024 Al-alloys. The 7020 Al-alloy suitable for ground transport system was procured in the as-fabricated condition, while 2024 Al-alloy was procured in T3 heat-treated condition. The 7020 Al-alloy was subjected to T7 heat-treatment to obtain optimum mechanical properties. The chemical composition and the mechanical properties of the alloys are given in Tables 1 and 2, respectively.

Single-edge notched specimens having a thickness of 6.5 mm were used for conducting the fatigue test. The specimens were made in the LT plane, with the loading aligned in the longitudinal direction. The detail geometry of the specimen is illustrated in Fig. 1.

The experiments were performed using an *Instron*-8502 machine with 250 kN load cell capacity, interfaced to a computer for control and data acquisition. All tests were conducted in air and at room temperature. The test specimens were fatigue pre-cracked under mode-I loading to an a/w ratio (non-dimensional crack length or normalized crack length) of 0.3 and were subjected to constant load test (i.e. progressive increase in ΔK with crack extension) maintaining a load ratio of 0.1. The sinusoidal load cycles were applied at a frequency of 6 Hz. The crack growth was monitored with the help of a COD gauge mounted on the face of the machined notch. The following equations were used to determine stress intensity factor K [13].

$$K = f(g) \cdot \frac{F\sqrt{\pi a}}{wB} \tag{3}$$

$$\text{where, } f(g) = 1.12 - 0.231(a/w) + 10.55(a/w)^2 - 21.72(a/w)^3 + 30.39(a/w)^4 \tag{4}$$

3. Formulation and validation of the model

3.1. Model formulation

The differential equation describing an exponential growth is

$$\frac{dP}{dt} = rP \tag{5}$$

where P is population and t is time.

The solution of the above differential equation is

$$P(t) = P_0 \cdot e^{rt} \tag{6}$$

The equation is called the "law of growth", and the quantity r in this equation is referred to as the Malthusian parameter, also known as specific growth rate.

In the present case Eq. (6) is modified and rewritten as:

$$a_j = a_i e^{m_{ij}(N_j - N_i)} \tag{7}$$

$$m_{ij} = \frac{\ln\left(\frac{a_j}{a_i}\right)}{(N_j - N_i)} \tag{8}$$

where a_i and a_j *is the* crack length in ith step and jth step in 'mm', respectively; N_i and N_j *is number* of cycles in ith step and jth step, respectively; m_{ij} *is* specific growth rate in the interval i–j; i is the number of experimental steps and $j = i + 1$.

Fatigue crack growth behavior is strongly dependent on initial crack length and previous load history. Therefore, while using the exponential model described in Eq. (7) each previous crack length is taken as the initial crack length for the present step

Table 1
Chemical Composition of 7020-T7 and 2024-T3 Al-alloys

Materials	Al	Cu	Mg	Mn	Fe	Si	Zn	Cr	Others
7020-T7 Al-alloy	Main constituent	0.05	1.2	0.43	0.37	0.22	4.6	–	–
2024-T3 Al-alloy	90.7–94.7	3.8–4.9	1.2–1.8	0.3–0.9	0.5	0.5	0.25	0.1	0.15

Table 2
Mechanical properties of 7020-T7 and 2024-T3 Al-alloys

Material	Tensile strength (σ_{ut}) MPa	Yield strength (σ_{ys}) MPa	Young's modulus (E) MPa	Poisson's ratio (ν)	Plane strain fracture toughness (K_{IC}) MPa$\sqrt{m}$	Plane stress fracture toughness (K_C) MPa$\sqrt{m}$	Elongation
7020-T7 Al-alloy	352.14	314.7	70,000	0.33	50.12	236.8	21.54% in 40 mm
2024-T3 Al-alloy	469	324	73,100	0.33	37.0	95.31	19% in 12.7 mm

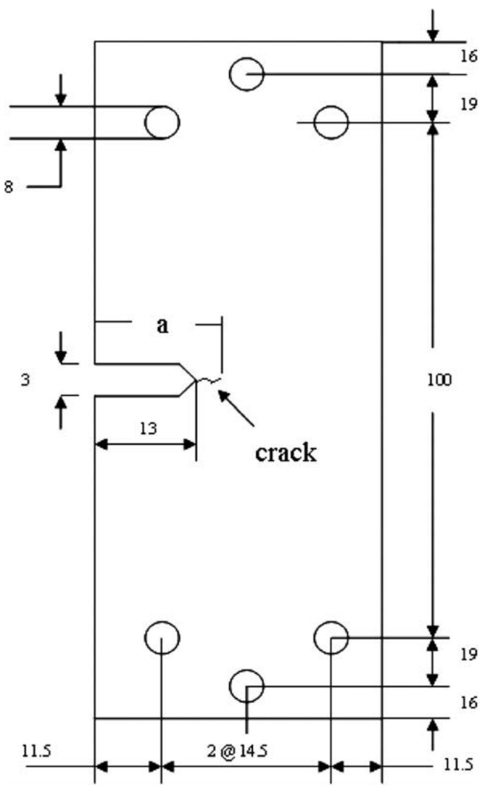

Fig. 1. Single edge notched specimen geometry.

and the specific growth rate 'm' is calculated for each step in incremental manner. Since it is an empirical model, experimental data are needed to determine the values of 'm' for each step, which is a controlling important parameter in the proposed model. The procedural steps for the model formulation are as follows:

1. The specific growth rate 'm' is calculated for each step from experimental a–N data according to the Eq. (8) and subsequently refined by curve fitting with calculated m and a values.
2. The specific growth rate is correlated with another parameter l which takes into account the two crack driving forces ΔK and $K_{\max}$ as well as material parameters K_C, E, σ_{ys} and is represented by Eq. (9).

$$l = \left[\left(\frac{\Delta K}{K_C}\right)\left(\frac{K_{\max}}{K_C}\right)\left(\frac{\sigma_{ys}}{E}\right)\right]^{\frac{1}{4}} \tag{9}$$

The different m and l values are fitted by 3rd degree polynomial for region-II and III for both the materials. The predicted 'm' values are calculated from the Eq. (10) for three specimens of each material.

$$m = A'l^3 + B'l^2 + C'l + D' \tag{10}$$

where, A', B', C', and D' are curve fitting constants whose average values for the two different materials are presented in Table 3. It may be noted that the plane stress fracture toughness (K_C) has been calculated and presented in Table 2 from plane strain fracture toughness (K_{IC}) by an empirical relation proposed by Irwin [14] as given in Eq. (11).

$$K_C^2 = K_{IC}^2(1 + 1.4\beta_{IC}^2) \tag{11}$$

$$\text{where, } \beta_{IC} = \frac{1}{B}\left(\frac{K_{IC}}{\sigma_{ys}}\right)^2 \tag{12}$$

3. The predicted number of cycles or fatigue life is calculated from Eq. (13).

$$N_j = \frac{\ln\left(\frac{a_j}{a_i}\right)}{m_{ij}} + N_i \tag{13}$$

The predicted values of specific growth rate (m_{ij}) of the tested specimens have been calculated by putting the average values of the curve fitting constants in Eq. (10). The fatigue life is calculated by using the model Eq. (13). This is done by taking the first experimental a and N values of the tested specimen as the initial values (a_i and N_i). N_j (for $j = 2$) is calculated from the initial values of a and N (a_i and N_i for $i = 1$). Subsequently the crack length is increased in steps of 0.05 mm and fatigue life is calculated till the final crack length a_f is reached.

3.2. Comparison of predicted results

The predicted results have been compared with experimental data and also the results obtained by using Forman model. The values of the constants 'C_a' and 'n_a' of the Forman equation have been calculated from experimental data by taking average values of three tested specimens for each material as given in Table 4. The predicted a–N curve obtained from the proposed exponential mod-

Table 3
Average values of curve fitting constants

Alloy	A'	B'	C'	D'
7020-T7	24871.3×10^{-6}	29217.67×10^{-6}	-6173.4×10^{-6}	311.153×10^{-6}
2024-T3	518820×10^{-6}	-47686×10^{-6}	975.443×10^{-6}	11.185×10^{-6}

Table 4
Values of Forman constants

Forman constants	7020-T7 Al-alloy				2024-T3 Al-alloy			
	Sample-1	Sample-2	Sample-3	Average	Sample-1	Sample-2	Sample-3	Average
$C_a \times 10^{-5}$	1.0	2.0	1.0	1.33	1.0	1.0	1.0	1.0
n_a	3.2472	3.1752	3.1637	3.1954	3.1088	3.2075	3.31188	3.2094

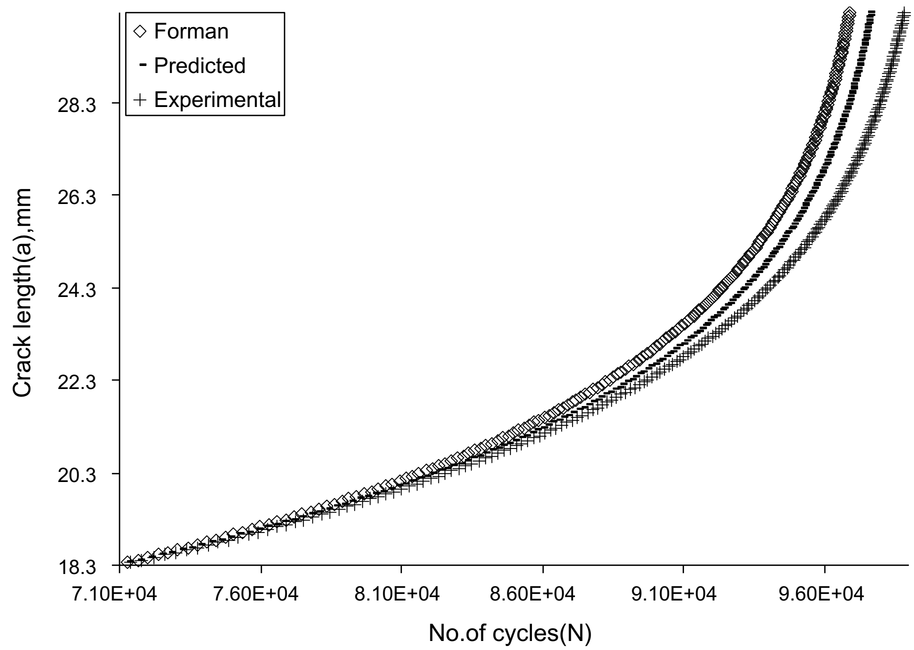

Fig. 2. Comparison of Forman, predicted and experimental number of cycle (7020-T7 alloy).

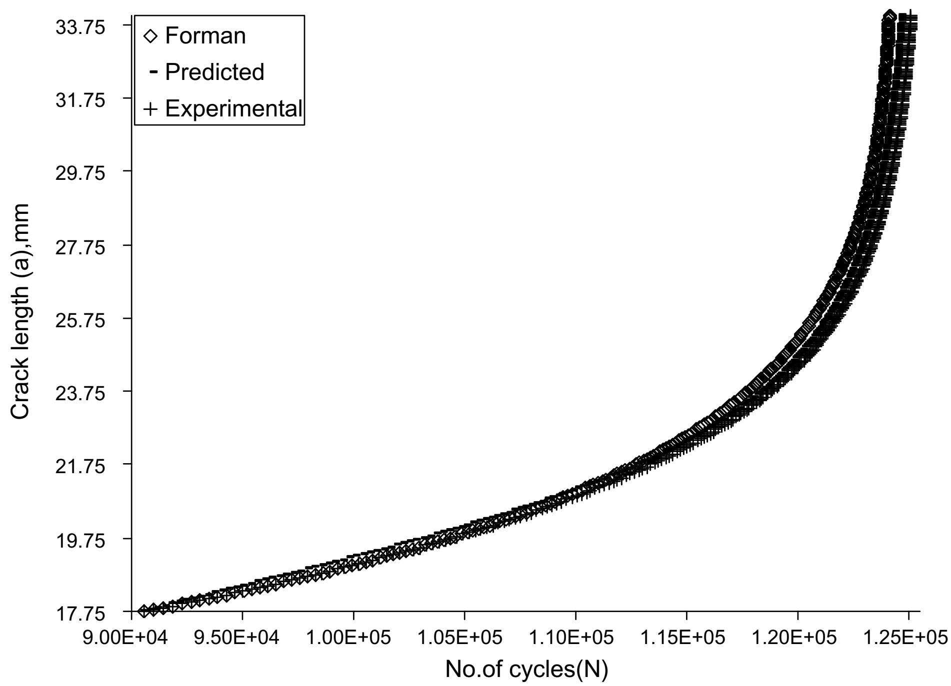

Fig. 3. Comparison of Forman, predicted and experimental number of cycle (2024-T3 alloy).

el and that obtained from the Forman model have been compared with the experimental results (Figs. 2 and 3) along with their percentage deviations presented in Table 5. The da/dN–ΔK curves are shown in Figs. 4 and 5 for the tested specimens for comparison. It

Table 5
Load scenarios and experimental results

Alloy	K_C MPa	F_{max} KN	F_{min} KN	a_i mm	a_f mm	$N_f^F \times 10^3$cy.	$N_f^P \times 10^3$cy.	$N_f^E \times 10^3$cy.	%age dev. in N_f^F	%age dev. in N_f^P
7020-T7	236.8	8.89	0.89	18.30	30.24	96.899	97.561	98.829	−1.953	−1.283
2024-T3	95.31	7.20	0.72	17.75	34.0	124.116	124.564	125.088	−0.777	−0.419

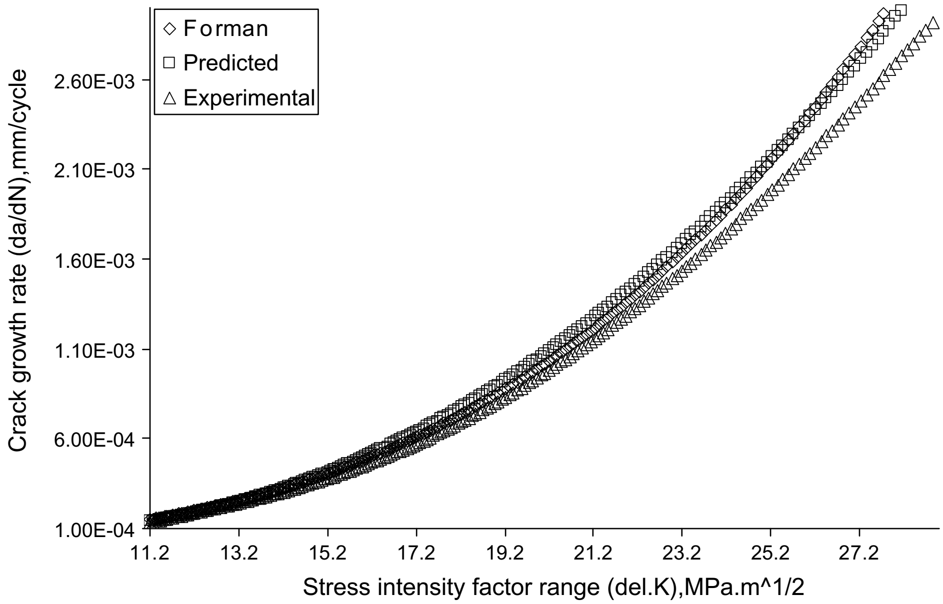

Fig. 4. Comparison of Forman, predicted and experimental crack growth rate (7020-T7 alloy).

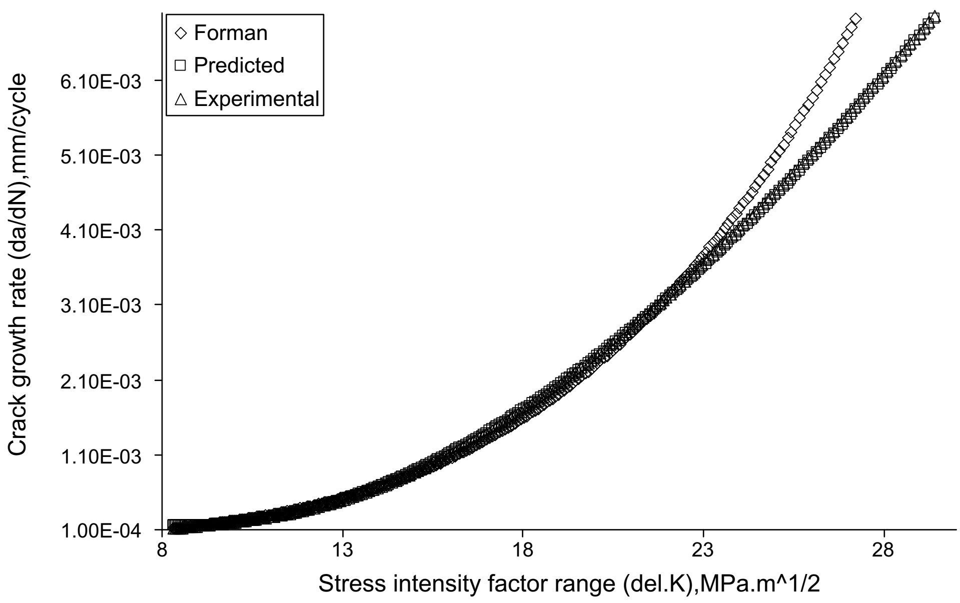

Fig. 5. Comparison of Forman, predicted and experimental crack growth rate (2024-T3 alloy).

can be seen that the predicted results are in good agreement with the test data and gives better accuracy in fatigue life in comparison to Forman model. The plots of log (da/dN)–log (ΔK) given in Figs. 6 and 7 for the two materials show the three regions of FCGR curve. However, the life prediction has been done for regions-II and III only.

4. Discussion

The basic aim of this work is to develop a fatigue crack propagation model so as to predict the life of the components without adopting complicated integration procedure. The proposed model not only covers the stage-II (i.e. Paris region) but also stage-III like

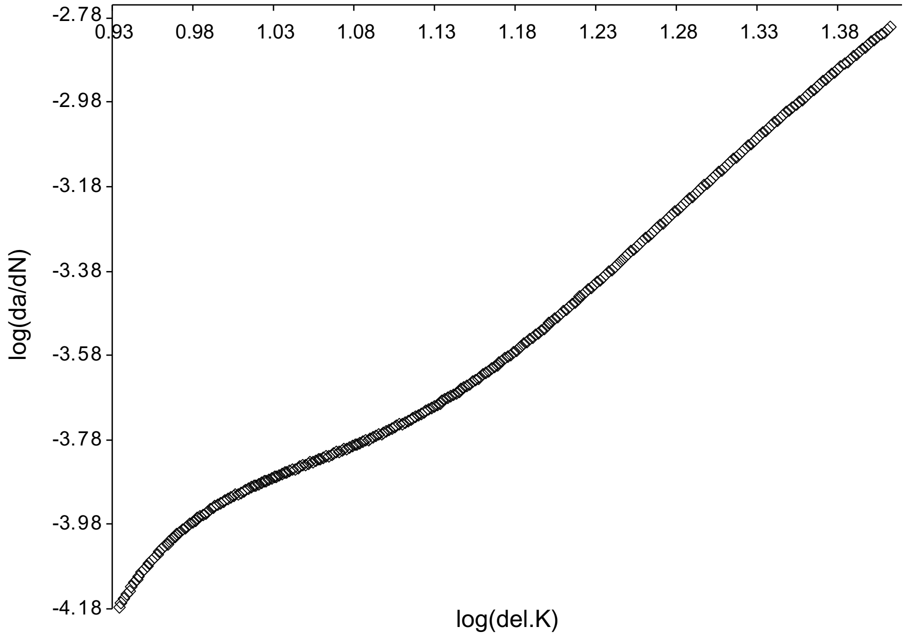

Fig. 6. Log (da/dN)–log (del K) curve (7020-T7 alloy).

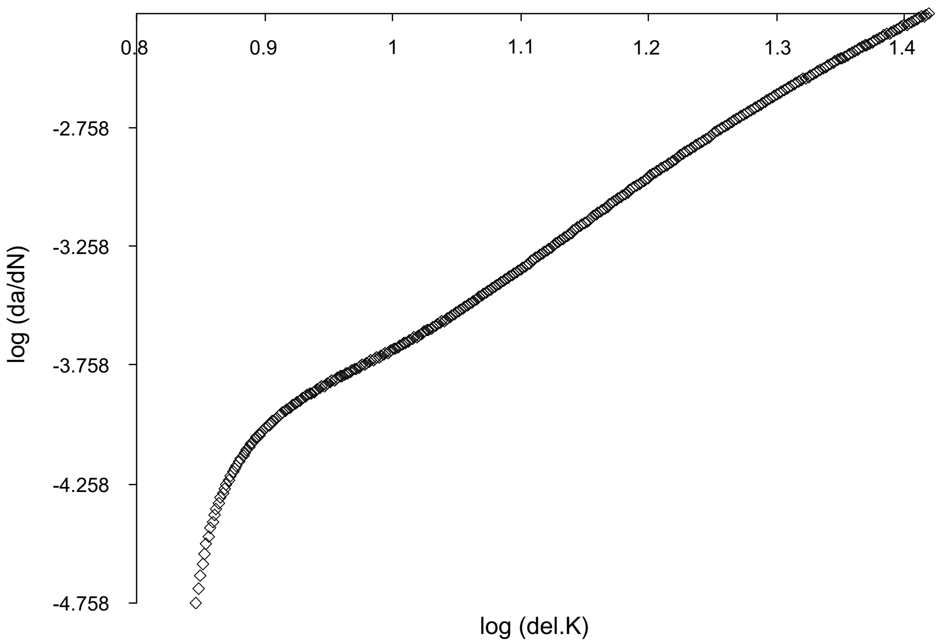

Fig. 7. Log (da/dN)–log (del K) curve (2024-T3 alloy).

Forman model. The only difficulty is to find out the value of the parameter 'm' (specific growth rate) and to correlate it with two crack driving forces ΔK and K_{max} and with the material parameters plane stress fracture toughness (K_C), modulus of elasticity (E) and yield stress (σ_{ys}) by curve fitting. As it is already mentioned that the curve fits well with 3rd degree polynomial for stage-II and stage-III, one has to determine the values of four constants, which depend on initial crack length and the materials used. Compared to the exponential model of Adib and Baptista [2], the proposed model seems to be better on three points. It avoids complicated numerical integration for life prediction. It also covers both the stages-II and III. Finally, it can as well be extended to overload induced retardation cases with some modification in the functional relationship of m and l.

It is necessary to discuss the significance of the specific growth rate 'm'. The value of m changes with change in loading condition as well as crack length. Since crack length a changes with the number of cycles N, m also changes with N. According to 'Unified

Approach', FCGR not only depends on the single crack driving force ΔK, but also on K_{max} to take into account the mean stress effects [15–20]. Further, fracture toughness (K_C) has been taken into consideration as the present modeling covers region III. Considering the two material parameters 'E' and 'σ_{ys}' the specific growth rate 'm' has been correlated with the parameter 'l' through dimensional groups ($\Delta K/K_C$), (K_{max}/K_C) and (σ_{ys}/E) as presented in Eqs. (9) and (10), respectively. As mentioned earlier, the correlation between 'm' and 'l' fits well by a 3rd degree polynomial with R^2 values in the region of 0.9998 for both the materials. Due to statistical nature of fatigue it is necessary to use the results of a number of fatigue tests at a given load level to find a single representative value of the number of cycles to failure N_f. Therefore, average values of specific growth rate, m for three samples of each material have been taken for the calculation of specific growth rate for the tested specimen. As far as the accuracy of the nature of growth rate is concerned, it is better in the sense that the values of 'm' are calculated by incremental manner. The fatigue life is calculated in a cumulative manner by adding the life of the previous steps. Because of the above mentioned facts, the proposed model can be extended to overload induced fatigue crack growth retardation and also variable amplitude loading (VAL). The authors have used earlier [21] an exponential model for evaluation of retardation parameters after mixed-mode overload by introducing a mode-mixity factor in the expression for l.

A typical crack growth data involves stress intensity range ΔK raised to a power of around 3. Any inaccuracy in the value of stress intensity factor is magnified in the life calculation. For example, a change in load level and hence the basic stress intensity solution of ±1% gives a change in life of −3.5% to +3.7%. The discrepancies may be even more dramatic for initial cracks loaded near the fatigue threshold limit [22]. In the present investigation, it is observed that the crack growth data (specific growth rate) involves stress intensity range as well as maximum stress raised to highest power of 3/4. Therefore, a change in load level of ±1% gives a change in life of ±0.775% and ±0.24% for 7020-T7 and 2024-T3 Al-alloys, respectively; whereas, in case of Forman model those changes correspond to ±1.067% and ±0.971% for 7020-T7 and 2024-T3 Al-alloys, respectively. Hence, it can be concluded that the accuracy of the proposed model is better than other empirical models.

5. Conclusion

(1) Exponential model of the form $a_j = a_i e^{m_{ij}(N_j - N_i)}$ can be effectively used to determine the fatigue life without going through numerical integration.
(2) The model covers both the stages of fatigue crack propagation (II and III).
(3) The accuracy of the 'Exponential model' is much better than other available empirical models and the curves predicted by this model are in good agreement with the experimental data.

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