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Manufacturing cell formation with production data using neural networks

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Abstract

Batch type production strategies need adoption of cellular manufacturing (CM) in order to improve operational effectiveness by reducing manufacturing lead time and costs related to inventory and material handling. CM necessitates that parts are to be grouped into part families based on their similarities in manufacturing and design attributes. Then, machines are allocated into machine cells to produce the identified part families so that productivity and flexibility of the system can be improved. Zero-one part-machine incidence matrix (PMIM) generated from route sheet information is commonly presented as input for clustering of parts and machines. An entry of '1' in PMIM indicates that the part is visiting the machine and zero otherwise. The output is generated in the form of block diagonal structure where each block represents a machine cell having more than one machines and a part family. The major limitations of this approach lies in the fact that important production factors like operation time, sequence of operations, and lot size of the parts are not accounted for. In this paper, an attempt has been made to propose a clustering methodology based on adaptive resonance theory (ART) for addressing these issues. Initially, a methodology considering only the operation sequence of the parts has been proposed. Then, the methodology is suitably modified to deal with combination of operation sequence and operation time of the parts to address generalized cell formation (CF) problem. A new performance measure is proposed to quantify the performance of the proposed methodology. The performance of the proposed algorithm is tested with benchmark problems from open literature and the results are compared with the existing methods. The results clearly indicate that the proposed methodology outperforms the existing methods in most cases.

Keywords: Cell formation; Grouping efficiency; Exceptional elements

1. Introduction

The primary concern in cellular manufacturing is to adopt group technology (GT) for identification of machine cells and part families so that performance measures related lead time, inventory and material handling can be enhanced. For constructing better manufacturing layout the cell formation problem is treated as an important issue since it makes a distinction over the conventional layouts (Vitanov, Tjahjono, & Marghalany, 2007). Usually, part-machine incidence matrix (PMIM) developed from route sheet information (Fig. 1) is presented as input to any clustering algorithm. In PMIM the '1's represent the visit of a part to a particular machine and the zeros represent non visits. From the output matrix, part families and machine cells are identified from the diagonal blocks. If '1's appear in off-diagonal blocks, they represent inter-cell movements of parts. Later researchers considered some real life production data like operational time of the parts and operational sequence of the parts to be processed in various machines instead of considering '1's in the input matrix as illustrated with an example in Table 1. Several methods viz., array manipulation, hierarchical clustering, non-hierarchical clustering, mathematical programming, graph theory, heuristics and metaheuristics exist in the literature to solve cell formation (CF) problem (Jeffrey Schaller, 2005; Venugopal & Narendran, 1992). These methods are found to produce good solutions for well structured matrices where part families and machine cells exist naturally (Gupta, 1991). However, they fail to produce so for ill structured matrices and end up with many exceptional elements.

Adaptive resonance theory (ART1), a class of neural networks, has been proposed by Carpenter and Grossberg (1987). Dagli and Huggahalli (1995) have demonstrated the ability of a neural network in solving cell formation problem. The iterative activation and competition model proposed by Moon (1990) exhibited a significant advantage over earlier algorithms when PMIM is presented as input. The major demerit with these approaches is that they do not take into account the other important real time production factors such as operational times, sequence of operations, and production lot sizes that have significant bearing on smooth flow of materials in group technology layout. When actual production factors are considered, the input matrix consists of non-binary and real valued elements and finds difficulties in representation while solving CF problem. However, two popular algorithms viz., the

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Parts / Machines	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43
A																																					√					√	
B		√								√																		√				√					√	√		√		√	
C						√	√												√															√	√	√							
D					√				√					√							√		√						√												√		√
E					√			√	√					√	√	√		√			√		√						√				√								√		√
F	√	√																																									
G	√																																										
H	√	√		√	√	√	√	√				√	√	√		√							√									√	√	√	√	√	√		√	√		√	√
I		√		√						√																	√	√				√					√	√		√		√	
J	√											√	√												√	√					√								√				
K			√																	√	√			√						√													
L											√						√					√		√						√													
M			√																					√																			
N		√				√	√																											√									
O					√				√					√	√	√							√																		√		√
P		√								√																						√			√		√	√				√	

Fig. 1. Route sheet. (Courtesy: J.L. Burbidge (1989). Production flow analysis for planning group technology, Oxford University press, New York.)

Table 1
Machine Part incidence matrix (3 × 3) with sequence data

Machine/part	M1	M2	M3
P1	1	2	0
P2	3	1	2
P3	2	0	1

clustering algorithm (Nair & Narendran, 1999) and fuzzy ART algorithm (Suresh, Slomp, & Kaparthi, 1999) found in the literature have been proved to produce satisfactory results for the CF problem with non-binary data. In order to evaluate the goodness of the cell formation, a good number of performance measures have been proposed in the literature (Harhalakis, Nagi, & Proth, 1990; Kumar & Chandrasekharan, 1990; Nair & Narendran, 1996; Zolfaghari & Liang, 2003).

In this work, an attempt has been made to use the operation sequence of the parts known as ordinal level data and the operation time of the parts known as ratio level data or workload data which are obtained through the route sheets to group the parts into part families and machines into machine cells. The proposed algorithm employs the principle of modified ART1 network found in the literature (Venkumar & Haq, 2005). Basically, the ART1 network classifies a set of binary vectors into groups based on their similarity. The ART1 recognizes patterns and clusters the binary vectors with the recognized pattern based on the comparison mechanism (Kao & Moon, 1991). The proposed algorithm first converts the given non-binary data into a zero-one binary matrix known as part machine precedence matrix (PMPM) and feed the ART1 network with PMPM as the input matrix.

2. The overview of ART1

The ART1 network is an unsupervised vector classifier that accepts input vectors that are classified according to the stored patterns if they resemble most. It also provides a mechanism of adaptive expansion of the output layer of neurons until an adequate size is reached based on the number of classes inherent in the observation. The ART1 network can adaptively create a new class corresponding to an input pattern if it is determined to be sufficiently different from existing clusters. This determination called the vigilance test is incorporated into the network. Thus, the ART1 architecture allows the user to control the degree of similarity of patterns placed in the cluster. In this work, ART1 network is adapted to group the binary matrix which is given in the form of part-machine precedence matrix for the considered CF problem. The ART1 network has two layers. One is the input layer (also called as comparison layer) and the other one is the output layer (also called as recognition layer). Every input (bottom) neuron is connected to every output (top) layer neurons. There are bottom-up weights (b_{ij}) associated from the input neurons to the output neurons and top-down weights (t_{ji}) associated from the output neurons to the input neurons. The bottom-up weights are used for cluster competition and top-down weights are used for cluster verification.

In this work, an ART1 based algorithm is proposed to handle the cell formation problem with operation time and operation sequence of the parts. In Section 3, the algorithm based on ART1 is proposed for solving cell formation problem with only operation sequence information as input data. In Section 5, the ART1 based cell formation algorithm with combined data operation sequence and time is proposed.

3. The ART1 based algorithm for cell formation with operation sequence

The input is considered from the route sheet of the parts. A typical route sheet from Burbidge (1989) is given in Fig. 1. Table 1 illustrates the matrix with sequence data. There are three dissimilar machines (M1, M2 and M3) and three parts (P1, P2 and P3). Part 1 and Part 3 are having two operations whereas Part 2 has three operations. The sequence or the order of the operations of each part varies. Thus P1 visits M1 for its first operation and M2 for its second operation. P2 visits M2 for its first operations and M3 for its second operations and M1 for its third operation. P3 visits M3 for its first operation and M1 for its second operation. Since the final products are dissimilar each part has different sequence of operations. If the requirement of machines are only considered then all the non zero values become ones and the zeros remain unchanged which is known as part machine incidence matrix (PMIM). In this section the methodology uses ART1 where the input to the algorithm contains only zero-one binary elements. In Section 5, a modified ART 1 is proposed to handle combination of both operational time and sequence of the parts, expressed in non zero values measured in ratio scale (known as ratio level data). Since it is not possible to make use of ART1 as proposed by Carpenter and Grossberg (1987), ART1 is modified such that Euclidean distance measure is included to handle the ratio level data. This is one of the main contributions of the work presented in this paper.

The input to the algorithm proposed in this section is the sequence based part-machine incidence matrix (PMIM) of size '$N \times M$' for N parts and M machines cell formation problem.

3.1. Phase 1. Formulation of part-machine precedence matrix

Step 1: Using the given PMIM with sequence data, a machine–machine precedence matrix (MMPM) of size $M \times M$ is constructed for every part. Each row of a MMPM represents a machine and the '1's in the row indicate the machines which are required for the part j subsequently. The row corresponding to the first machine to be visited by the part, the '1's are given to all the machines required by the part, thus it holds the maximum number of ones in the MMPM of the particular part. The number of '1's is decreased by '1' to the subsequent machines required by the part. For the rows corresponding to the machine which are not required by the part, all the elements are assigned with zero.

Step 2: Using the 'N' number of MMPMs, a single part-machine precedence matrix (PMPM) of size '$N \times (M \times M)$' is constructed. Each row of the PMPM corresponds to a part and the element of the row is obtained by placing all the rows of the MMPM in a linear sequence.

3.2. Phase 2. Grouping of parts into part families using ART1

The PMPM obtained from the phase1 is given as input to the ART1 network.

Step 1. Before starting the network training process, the bottom-up weights b_{ij} and top-down weights $t_{ji}$are set to initial values by using the Eqs. (1) and (2) respectively.

$$b_{ij} = \frac{1}{(1+N)} \quad \text{for all } i \text{ and } j \tag{1}$$

$$t_{ji} = 1 \quad \text{for all } i \text{ and } j \tag{2}$$

The vigilance threshold ρ is suitably selected such that $0 < \rho < 1$.

Step 2. Apply new input vector X_i.

Step 3. Compute matching scores using Eq. (3).The output μ_j of every output node j equals

$$\mu_j = \sum_i b_{ij}(t)x_i \text{ for } j = 0, 1, \ldots, \quad (\text{M-1}) \tag{3}$$

Step 4. Select best matching exemplar i.e. node (θ) with maximum output $\mu_\theta = \max(\mu_j)$. Outputs of other neurons are suppressed. In case of tie choose the neuron with lower j.

Step 5. Vigilance test i.e. test of similarity with best matching exemplars.Compute $\|X\| = \sum_i x_i$ number of 1's in the input vector.Compute $\|T \cdot X\| = \sum_i t_{i\theta} \cdot x_i$ number of perfectly matching 1's between input vector and best matching exemplar.

Step 6. Similarity check. If their similarity $\frac{\|T \cdot X\|}{\|X\|} > \rho$ then go to step 7.

Step 7. Disable the best exemplar temporarily output of the best matching node selected in the step 4 is temporarily set to zero; other outputs have already been suppressed.Then go to step 3. In step 3, a new neuron in the output layer gets selected to represent the new class.

Step 8. Update best matching exemplar using Eqs. (4) and (5).

$$t_{i\theta}(t+1) = t_{i\theta}(t) \cdot x_i \tag{4}$$

$$b_{i\theta}(t+1) = \frac{t_{i\theta}(t) \cdot x_i}{0.5 + \sum_i t_{i\theta}(t)x_i} \tag{5}$$

Step 9. Repeat the step 2 after enabling any nodes disabled in step 6.

The output of this phase will be the optimal number of part families and the list of parts within each part family.

3.3. Phase 3. Grouping of machines into machine cells

Step 1. Each machine is allocated to a cell corresponding to a particular part family where the total number of operations required by all the parts in the family put together is maximum.

Step 2. The columns of the output are rearranged into block diagonal form such that the number of inter-cell movements are kept minimum.

4. Numerical illustrations

Table 2 shows the sequence based PMIM of an example problem where seven parts are to be processed using five machines. For every part, a MMPM is constructed. Table 3 shows the MMPM for the parts P1and P2. Table 4 shows the PMPM constructed as per step 2 of phase I of the algorithm. Table 5 shows the output of the algorithm. There are two part families and machine cells. The parts p2, p3, p4 and p6 are associated with the machines m1, m2 and m4 in one family and the parts p1, p5 and p7 are in another family associated with the machines m3, m4 and m5. It is observed from the output matrix that parts p2, p4, p6 and p7 have one exceptional element each (i.e. ones in off-diagonal blocks) and so each of the respective parts have one inter-cell move The part p3 has two exceptional elements but one inter-cell move because both the exceptional elements belong to the same cell. Hence, there are six exceptional elements and five inter-cell moves. The group technology efficiency (GTE) is calculated using Eq. (10) and found to be 64.3% (Nair & Narendran, 1998).

5. The ART based algorithm for cell formation with combined operation sequence and time

The Sections 3 and 4 deal with the cell formation problem considering only the sequence data where the proposed ART1 takes into account only the binary form of the matrix and produces the results. In this section, the operational time of the parts is also taken into account and thereby a new matrix known as Matrix of Combined data as described in the algorithm (steps 1–3) is constructed after combining both sequence of parts and operational time of parts. So, the input to algorithm will be a matrix containing ratio level data and hence the ART1 proposed in Section 3 phase 2 will be further modified to consider the operational time also as given in steps (5–15).

The input to the algorithm is the matrices showing operational time of the parts and operational sequence of the parts based part-machine incidence matrix (PMIM) of size $N \times M$.

Step 1. Using the given PMIM with the sequence data, for every part, a machine–machine precedence matrix (MMPM) of size $M \times M$ is constructed. Each row of a MMPM repre-

Table 2
Part machine incidence matrix (7×5) with sequence data

	m1	m2	m3	m4	m5
p1	1	2	0	3	0
p2	0	1	2	0	3
p3	2	0	0	1	3
p4	0	1	2	0	3
p5	1	2	0	3	0
p6	3	0	1	0	2
p7	0	3	0	2	1

Table 3
Machine–machine precedence matrix for parts

	For part-1						For part-2				
	m1	m2	m3	m4	m5		m1	m2	m3	m4	m5
m1	1	1	0	1	0	m1	0	0	0	0	0
m2	0	1	0	1	0	m2	0	1	1	0	1
m3	0	0	0	0	0	m3	0	0	1	0	1
m4	0	0	0	1	0	m4	0	0	0	0	0
m5	0	0	0	0	0	m5	0	0	0	0	1

Table 5
Output matrix of size (7 × 5)

	m3	**m5**	**m1**	**m2**	**m4**
p2	2	3	0	1	0
p3	0	3	2	0	1
p4	2	3	0	1	0
p6	1	2	3	0	0
p1	0	0	1	2	3
p5	0	0	1	2	3
p7	0	1	0	3	2

sents a machine and the ones in the row indicate the machine ids subsequently required by the part under consideration. Hence, the row corresponding to the first machine to be visited by the part has ones in case of all the machines subsequently required by the part and thus, it holds the maximum number of ones in the MMPM of the particular part. Similarly, the row corresponding to the machine where last operation in the operation sequence of the part is to be carried out contains minimum number of ones (i.e. only one). The rows corresponding to the machine which are not required by the part, all the elements are assigned with zero.

Step 2. Using the n number of MMPMs a single part machine precedence matrix of size $N \times (M \times M)$ is constructed. Each row of the PMPM corresponds to a part and the element of the row is obtained by placing all the rows of the MMPM in a linear sequence.

Step 3. Multiply all the ones present in the PMPM by the respective workload data from the work load matrix. The new matrix is a combination of ordinal and ratio level data which is named as matrix of combined data (MCD).

Step 4. The MCD will be the input to the modified ART1 which is given in the steps (5–15).

Step 5. Set nodes in the input layer equal to N (number of parts) and nodes in output layer equal to $(M \times M)$ where M is the number of machines. Set vigilance threshold (ρ).

Step 6. Initialize top-down connection weights.Top-down weights $wt_{ji}(0) = 0$ for $i = 1, 2,\ldots N$ and $j = 1,2,\ldots(M \times M)$.

Step 7. Let $q = 1$. The first input vector X_1 (first row of the workload matrix) is presented to the input layer and assigned to the first cluster. Then, first node in the output layer is activated.

Step 8. The top-down connection weights for the present active node are set equal to the input vector.

Step 9. Let $q = q + 1$. Apply new input vector X_q (input vectors are the rows of the PMPM).

Step 10. Compute Euclidean distance between X_q and the exemplar stored in the top-down weights (wt_{ji}) for all the active nodes i as given in the Eq. (6). This distance function is used to calculate similarity between the stored pattern and the present input pattern. If the similarity value is less than or equal to ρ (vigilance threshold), the present input is categorized under the same cluster as that of stored pattern.

$$e_i = \sqrt{\sum_{j=1}^{M} (x_{qj} - wt_{ji})^2} \tag{6}$$

Step 11. Perform vigilance test. find out minimum Euclidean distance.

Step 12. If min e_i leq ρ (threshold value), select output node for which Euclidean distance is minimum. If tie occurs, select the output node with lowest index number. Suppose output node k is selected then allocate the vector X_q to the node k (cell) and activate node k. Make increment to the number of parts in the active node k by one. If e_is for all active nodes are greater than ρ, then go to step 13.

Step 13. Start a new cell by activating a new output node.

Step 14. Update top-down weights of active node k.The decision for belongingness of an input vector to a node (cluster) is determined using similarity between previously stored exemplar with present input pattern. In other words, top-down weights play the role of storing exemplars (for active nodes) for comparison purpose. Therefore, top-down weights must contain relevant information of all the input vectors already classified under an active node (cluster) in aggregate nature. The top-down weights are updated each time when a new input vector is presented and clustered to an active node. When a vector is selected (to be allocated to an output node), its top-down weights are updated using more information of the previously stored exemplar and a relatively less information of the input vector (pattern) as shown in Eq. (7).

$$wt_{jk} = \left(\frac{n}{m} \cdot wt_{jk}\right) + \left(\frac{1}{m} \cdot x_{qj}\right) \tag{7}$$

Step 15. Go to step 5 and repeat till all the rows are assigned in the output nodes (cells).

Step 16. Check for single ton part family. If a single ton is found in any part family, then perform the following operations to merge the part family with one part into any other part families.

- Determine average of processing time in each part family.
- Calculate the Euclidean distance between the part families.

Table 4
Part machine precedence matrix for the problem size 7 × 5

	1	2	3	4	5	6	7	8	9	0	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
P1	1	1	0	1	0	0	1	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
P2	0	0	0	0	0	0	1	1	0	1	0	0	1	0	1	0	0	0	0	0	0	0	0	0	1
P3	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	1	1	0	0	0	0	1
P4	0	0	0	0	0	0	1	1	0	1	0	0	1	0	1	0	0	0	0	0	0	0	0	0	1
P5	1	1	0	1	0	0	1	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
P6	1	0	0	0	0	0	0	0	0	0	1	0	1	0	1	0	0	0	0	0	1	0	0	0	1
P7	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	1	0	0	1	0	1	1

- Merge the part family containing single part with another part family in such a way that Euclidean distance between them is minimum than other part families.

Step 17. Allocate machines to the part families using following procedure.
- The number of operations of a part in a particular machine is computed. If the part has maximum number of operations in machine i then the machine i is allocated to that part family where the part exists.
- In case tie occurs the machine is allocated with the part family where minimum inter-cell moves are possible.
- If again tie occurs the machine is allocated with the small identification number of part family.

6. Measure of performance

There are some popular measures like grouping efficiency and group efficacy (Kumar & Chandrasekharan, 1990) for measuring the goodness of the block diagonal structure of the output matrix in CF problems. However, all these measures treat all the operations equally and are suitable only for the binary matrix. These measures cannot be adopted for generalized cell formation problem where operational sequence and time of the parts are considered.

Therefore, group technology efficiency (GTE) given by Harhalakis et al. (1990) and modified grouping efficiency by Mahapatra and Sudhakara Pandian (2007) can be conveniently used to measure the performance considering sequence of parts and operational times of the parts respectively. Group technology efficiency is defined as the ratio of the difference between the maximum numbers of inter-cell travels possible and the numbers of inter-cell travels actually required by the system to the maximum numbers of inter-cell travels possible.

The maximum numbers of inter-cell travels possible in the system is

$$I_{\mathrm{p}} = \sum_{j=1}^{N}(n-1) \tag{8}$$

$$I_{\mathrm{r}} = \sum_{j=1}^{N}\sum_{w=1}^{n-1} t_{njw} \tag{9}$$

$$t_{njw} = 0 \text{ if the operations w, w+1 are performed in the same cell}$$
$$= 1 \text{ otherwise.}$$

$$\mathrm{GTE} = \frac{I_{\mathrm{p}} - I_{\mathrm{r}}}{I_{\mathrm{p}}} \tag{10}$$

In this work the modified grouping efficiency (Mahapatra & Sudhakara Pandian, 2007) is redefined as grouping efficiency for ratio level data as given in Eq. (11) which is given by the ratio between the total processing time inside the cells to the summation of exceptional elements, total processing time of the cells and total value of the voids (given by the voids factor) present in the cells. Voids factor for cell k is calculated by multiplying the number of voids in cell k to the average time of machine i in cell k. The void factor of cell k is multiplied by the total processing time of cell k to give the value of voids present in cell k.

$$\mathrm{GER} = \frac{\sum_{k=1}^{c} T_{\mathrm{pt}k}}{N_{\mathrm{e}} + \sum_{k=1}^{c} T_{\mathrm{pt}k} + \sum_{k=1}^{c}\left(N_{\mathrm{v}i} \times \frac{\sum T_{ik}}{\sum N_{\mathrm{p}}}\right)} \tag{11}$$

But in case of GER the operation sequence is not addressed to make it a generalized one. GTE is used to measure the performance of the output matrix with the information of sequence of parts. There is no measure found in the literature to include both the information combined. Hence, a new measure of grouping efficiency termed as ratio–ordinal combined efficiency (ROCE) has been proposed in this work to find out the goodness of the grouping in the cell formation problem that deals with both operation sequence and time in the input matrix with due consideration of equal weightages to both the data. Ratio–ordinal combined efficiency is calculated using the Eq. (12).

Ratio–ordinal combined efficiency (ROCE) is defined as the weighted average of grouping efficiency for ratio level data (GER) and group technology efficiency (GTE).

$$\mathrm{ROCE} = q(\mathrm{GER}) + (1-q)(\mathrm{GTE}) \tag{12}$$

T_{ptk}	total processing time inside cell k
T_{ik}	total time taken by ith machine in cell k
N_{p}	number of parts having operation in ith machine
$N_{\mathrm{v}i}$	number of voids in ith machine
N_{e}	total number of exceptional elements
I_{p}	maximum number of inter-cell travel possible in the system
I_{r}	number of inter-cell travel actually required by the system
n	number of operations ($w = 1,2,3,\ldots,n$)

The problem of size 12×10 from (George, Rajendran, & Ghosh, 2003) is considered for illustrating the proposed performance measure ROCE. Initially the input matrix (12×10) is fed into the algorithm. The output generated is given in Tables 6 and 7. Table 6 is the output matrix with operation sequence data. Table 7 represents the output matrix in terms of operation time. The total number of exceptional elements of the matrix ($N_{\mathrm{e}} = 4$). The total processing time inside each cell has been calculated and found to be ($T_{\mathrm{pt}k}$) ⇒ ($T_{\mathrm{pt1}} = 11.4$), ($T_{\mathrm{pt2}} = 10.05$), ($T_{\mathrm{pt3}} = 4.01$). The number of voids in each column is a count of zeros present in the respective columns inside the cells ($N_{\mathrm{v}i}$) ⇒ ($N_{\mathrm{v1}} = 1$, $N_{\mathrm{v4}} = 1$, $N_{\mathrm{v5}} = 1$, $N_{\mathrm{v8}} = 2$, $N_{\mathrm{v9}} = 1$, $N_{\mathrm{v10}} = 1$). The average of the operational time of parts in each column is calculated which is multiplied by the respective $N_{\mathrm{v}i}$ values to get the void factor. The maximum number of inter-cell travel possible in the system (I_{p}) is found to be 25 and the number of inter-cell travel actually required (I_{r}) by the system is 4. The maximum number of operations (n) is given as 5. The values of GER, GTE and ROCE are calculated using by substituting these values in Eqs. (10)–(12). Hence the value of GER is 0.7645, GTE = 0.84 and ROCE = 0.8023.

These values are calculated for the output of the existing methods and presented in Table 8.

It is observed that the results of the proposed performance measure outperforms the existing methods such as ACCORD (Nair &

Table 6
Output sequence matrix for the problem of size 12×10

	1	3	6	2	5	8	10	4	7	9
1		1	3					2		
5	3	5	1			2		4		
9	4	1	2			3				
10	3	1	2							
2				1	3	4	2			
3				2	4	1	3			
8				1			2			
12				3	2	1				
7				1	3		2			
4								1	3	2
6								1	3	2
11									1	

Table 7
Output matrix with operation time for the problem of size 12 × 10

	m1	m3	m6	m2	m5	m8	m10	m4	m7	m9
p1	0	0.96	0.63					0.95		
p5	0.63	0.97	0.61			0.94		0.89		
p9	0.54	0.92	0.72			0.92				
p10	0.39	0.61	0.72							
p2				0.86	0.54	0.04	0.67			
p3				0.88	0.49	0.08	0.73			
p7				1.2	0.81	0	0.83			
p8				1	0	0	0.62			
p12				0.7	0.72	0.02	0			
p4								0.07	0.83	0.72
p6								0.11	0.99	0.76
p11								0	0.71	0

Table 8
Comparison of the results of the proposed method over existing methods for the problem of size 12 × 10

Factors considered	ACCORD	Analytical iterative approach	Proposed method
Exceptional elements	5	5	4
Grouping efficiency	0.881	0.881	0.897
Grouping efficacy	1.026	1.026	1.026
Grouping efficiency (GER) (%)	69.24	69.24	76.45
Group technology efficiency(GTE) (%)	80.00	80.00	84.00
Ratio–ordinal combined efficiency (ROCE) (%)	74.62	74.62	80.23

Narendran, 1999) and analytical iterative approach (George et al., 2003).

7. Results and discussions

In this study, an efficient algorithm is proposed based on ART1 for generalized cell formation problem. The algorithm is coded in C^{++} and run on an IBM Pentium IV PC with 2.4 GHz Processor. Table 9 shows different size problems selected from open literature (George et al., 2003 & Nair & Narendran, 1998) for testing the proposed algorithm. For all trial data sets shown in Table 10, the input matrix is generated with uniformly distributed random numbers in the range of 0.5–5 for operational time and 1–9 for operational sequence. The problem sizes considered in this work range from 5 × 4 to 90 × 35.

The computational time required to obtain solution is reported for few sample problems in Table 11. The results are compared with the results produced by CASE algorithm (Nair & Narendran, 1998) as shown in Table 9. In addition a new weighted average performance measure ROCE is proposed that measures the performance of the algorithm proposed and tested with fifteen trial data sets. The results are found to be consistent for all the data sets tested which are shown in Table 10. Table 12 shows the block diagonal matrix produced by the proposed algorithm for the 40 × 25 example problem found in the literature (Nair & Narendran, 1998). The result of the example problem of size (12 × 10) obtained by the proposed algorithm outperforms other two methods as shown in Table 8. The weights for the exceptional elements (ones outside the cells) are given as one. Since in reality, the voids are not influencing the system as much as that of the exceptional elements, the weights for the voids (zeros inside the cells) are proportionally taken to the average values of the respective columns where the voids exist. As for as sequence matrix is concerned the inter-cell moves are calculated using the Eqs. (8) and (9). If the operation of a part is allotted in the same cell where the pervious operation of the part is taken place, then the inter-cell move is considered as zero. The total possible inter-cell moves are calculated just by taking summation of the difference between one and maximum operation of each part. It is the decision maker's choice to fix the value of the weighting factor q while calculating the proposed performance measure ROCE. In this work the value of q is considered as 0.5 for illustrating the performance measure by giving equal weightage to both GER and GTE. The proposed algorithm provides solution in a single iteration only. The advantage of the

Table 10
Performance of the proposed algorithm on test data sets

S. No.	Problem size	Exceptional elements	Inter-cell moves	GTE	GER	ROCE
1	5 × 4	0	0	100.00	83.48	91.74
2	5 × 5	1	1	85.71	81.15	83.43
3	7 × 5	6	5	64.30	72.01	68.16
4	8 × 6	2	2	84.61	70.15	77.38
5	19 × 12	8	9	83.93	65.08	74.51
6	20 × 12	11	10	78.00	59.56	68.78
7	20 × 20	3	3	94.00	84.25	89.13
8	30 × 15	21	17	76.71	60.02	68.37
9	37 × 20	25	25	71.59	60.99	66.29
10	50 × 25	49	46	69.13	58.39	63.76
11	55 × 20	15	19	81.20	66.03	73.62
12	60 × 28	39	38	70.50	57.20	63.85
13	65 × 30	58	52	76.68	59.59	68.14
14	80 × 32	53	59	74.57	62.28	68.43
15	90 × 35	54	56	77.69	62.26	69.98

Table 11
CPU time for the proposed algorithm

S. No.	Problem size	CPU time (s)
1	5 × 4	0.060213
2	30 × 15	0.396320
3	90 × 35	1.854945

Table 9
Comparison of the proposed algorithm with CASE

Problem size	No. of cells	CASE			Proposed algorithm		
		Exceptional elements	Inter-cell moves	Group technology efficiency	Exceptional elements	Inter-cell moves	Group technology efficiency
7 × 7	2	2	4	69.25	2	4	69.25
	3	3	6	53.85	3	6	53.85
20 × 8	3	10	17	58.54	10	17	58.54
20 × 20	4	NA	NA	NA	12	15	74.58
	5	15	19	67.80	16	18	69.49
40 × 25	5	NA	NA	NA	26	22	72.04
	8	35	31	66.67	35	31	66.67

NA, not available.

Table 12
Output matrix by the proposed ART1 based algorithm for example problem of size (40×25)

	4	5	7	12	16	18	19	23	1	2	17	24	3	11	20	25	8	9	10	6	13	14	15	21	22
1	5		3		4	2													**1**						**6**
4				1				2																	
5	3			2		1																			
6				3	2			1																	
7	3		2		4	1													**5**						
8		1			3		2																		
15		3			1		2			**4**									**5**						
16	1		3		2	4																			
17			1			2													**3**						
20								1																	
23		2			3		1																		
24		1			2																				
26	2			3				1															**4**		
29		3																		**2**				**1**	
30	4		2		3	1																			
34				2				1				**3**													
37			3	2				1																	
39				1																					
40					1				**2**										**3**						
2									2	3	4					**1**									
12									1		3	2	**4**			**5**									
31		**2**					**3**				1														
36									2	3	1			**4**											
3													2	3	1										
9													3	4	1	2									
13													3	2	1										
14		**1**											4	2	3										
22											**1**		3				**4**	**2**							
33														1	3	2									
10																3	2	1							
11																	2				**3**			**1**	
19																	1	3	2						
21																	1	3	2						
28																	2	1	3						
38																	2	3							**1**
18																						3	2		1
25																				1			3	2	
27				**1**																				3	2
32																					2	1	3		4
35																				2			4	1	3

proposed algorithm lies in its ability to generate quality solution for large size problems.

In modified ART1, the vigilance threshold (ρ) value greatly influences the number of cells obtained. The vigilance threshold value for each problem is varied from 1 to 9. It is found that the number of cells equals to the total number of machines if the vigilance threshold value is set at zero. As the vigilance threshold value increases, the number of cells is reduced as shown in Fig. 2. If the vigilance threshold value is further relaxed, the algorithm produces only one cell. Therefore, vigilance threshold value plays a vital role for obtaining quality solution. For each sample problem, it is incremented in the step of 0.5 starting from zero till desired solution is obtained.

The algorithm also takes care of avoiding cells with singleton part family that is encountered at times. The algorithm is flexible in such a way that the maximum number of parts to be accommodated in a family can be limited. From the Table 8, it is observed that the grouping efficiency and grouping efficacy measures found in the literature produce the values almost same in case of all the three methods compared. Hence the proposed grouping efficiency is evidently suitable to measure the performance of cell formation algorithm taking into account workloads on machines, weighting factor for voids, inter-cell moves and exceptional elements.

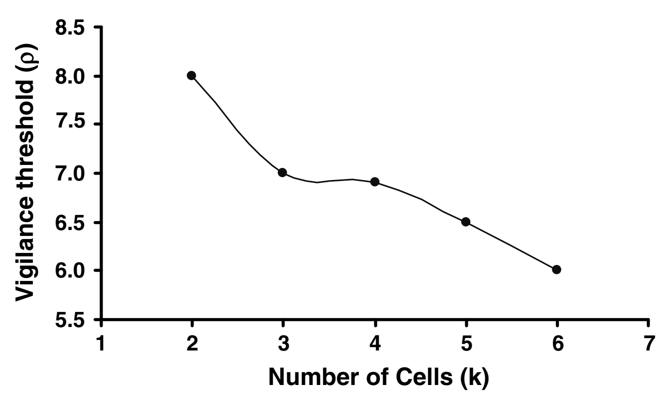

Fig. 2. Effect of vigilance threshold.

8. Conclusions

- In this work, ART1 based algorithm has been developed to solve the cell formation problem taking into account the production sequence data and operation time of the parts. In the proposed algorithm, a novel method of converting non-binary data into binary data is proposed for the convenience of dealing with ratio-level and ordinal level data in cell formation.

- The ART based algorithms have been proposed to address the issue of consideration of some important real life production factors and tested with problems of different sizes from open literature. The resulting solutions are compared with the solutions obtained by other existing methods. It is observed that the solutions obtained by proposed algorithm either outperform existing methods or remain the same.
- Since the algorithm uses simple network architecture, it helps to reduce computational burden compared to other algorithms. However the limitation of ART 1 lies in the order of presentation of the input to the network.
- As performance measures for resulting cells in cell formation problem considering both ratio-level and ordinal level data hardly exist in the literature, a new performance measure known as ROCE has been proposed for this purpose.
- The methodology of converting the non binary data into a suitable binary data and subsequently by feeding to the ART1 networks to solve the CF problem can be suitably modified or extended to solve the CF problem with other production data like batch sizes, machine capacity etc. for different objective criteria.
- The work can be further extended in future incorporating production data like machine capacity, production volume, layout considerations and material handling systems enhancing it to a more generalized manufacturing environment.

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