Frequency-Domain Block Filtered-x NLMS Algorithm for Multichannel ANC

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Abstract

In this paper, we present an adaptive filtering algorithm for multichannel Active Noise Cancellation (ANC) system in frequency domain. This approach rests upon the frequency domain block filtered-x LMS algorithm and NLMS algorithm, utilization of which facilitates variable step size control for multichannel ANC. Computational complexity for the proposed algorithm is evaluated and the performance of the proposed algorithm is validated through computer simulations for multichannel ANC.

1. Introduction

Active noise cancellation (ANC) has gained a lot of research interest because of rapid increase of acoustical noise pollution and insufficiency of passive techniques for noise control. ANC uses the superposition principle, where the undesired noise is reduced by adding another noise with the same amplitude but opposite polarity, which is generated by actuators such as loudspeaker [1][2]. The filtered-x LMS algorithm (FXLMS) is the most common algorithm applied in both feedforward and feedback ANC due to its ease in implementation [2].

In the FXLMS algorithm shown in fig.1 primary path transfer function, P(z), defines the path from the noise source to the cancellation point and P(n) is its impulse response. ANC systems also have secondary path transfer function, S(z), which is defined as the path leading from the adaptive filter output to error sensor that measures the residual noise and S(n) is its impulse response. Most available ANC algorithms including FXLMS, require online or offline identification of secondary path. If there is only one reference microphone, one loudspeaker and one error microphone then the situation is termed as a single-channel ANC but in case of multichannel ANC more than one reference microphone or loudspeaker or error microphone are present

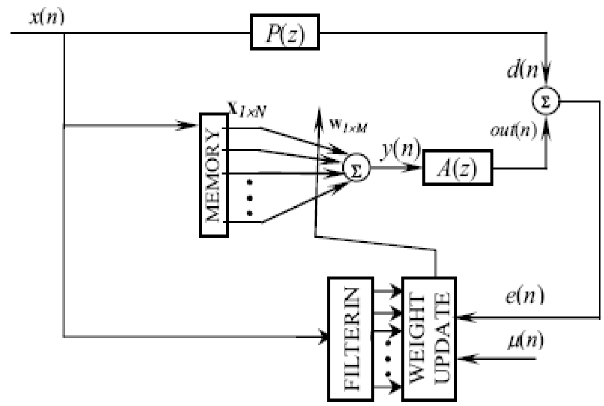

Figure 1. Block diagram of active noise canceller.

Several researchers have developed variations of the FXLMS algorithm to improve the canceller performance and robustness. Shen and Spanias [3], Reichard and Swanon [4] proposed block implementation of the FXLMS algorithm, both in time and frequency domain, which is exact implementation of FXLMS algorithm. In [5] Das,Panda and Kuo proposed a new generalized time domain block FXLMS (BFXLMS) algorithm for single channel ANC. In addition they proposed a reduced structure FBFXLMS algorithm without sacrificing performance.

In this paper, a simple and computationally efficient algorithm for multichannel ANC is proposed which is developed in the line of single channel FBFXLMS algorithm as reported in [5]. In addition NLMS [7] algorithm is employed to facilitate variable step size. The new algorithm is termed as Multichannel FBFXNLMS algorithm.

2. FBFXNLMS for multichannel ANC

The block diagram for active noise canceller is shown in fig.1. The weight update equation for FXNLMS algorithm is given by [5]

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$$\mathbf{w}(n+1) = \mathbf{w}(n) + 2\frac{\mu}{(\mathbf{x}(n)^T\mathbf{x}(n))}\mathbf{x}'_R(n)e(n) \quad (1)$$

where $\mathbf{w}(n) = [w_0(n)\ w_1(n)..........w_{N-1}(n)]$ is the weight vector of the adaptive filter, N is the filter length, μ is the step size parameter and $\mathbf{x}'_R(n) = [x'(n)\ x'(n-1)..........x'(n-N+1)]$

$x'(n) = S(n)*x(n)$, $e(n) = d(n) - S(n)*w(n)*x(n)$, where d(n) is the primary noise to be cancelled and * denotes linear convolution operation.

In case of block implementation weight update equation becomes

$$\mathbf{w}(k+N) = \mathbf{w}(k) + 2\frac{\mu}{\mathbf{x}(k+N)^T\mathbf{x}(k+N)}\mathbf{c}_\mathbf{R}(k+N) \quad (2)$$

where $\mathbf{c}_\mathbf{R}(k+N) = X'(k+N)\mathbf{e}_\mathbf{R}(k+N)$ (3)

$$X'(k+N) = [\mathbf{x}'_R(k+N)\ \mathbf{x}'_R(k+N-1)...\mathbf{x}'_R(k+1)]$$

$$\mathbf{x}'_R(k+N) = X(k+N)\mathbf{s}(k) \quad (4)$$

$$X(k+N) = [\mathbf{x}_\mathbf{R}(k+N)\ \mathbf{x}_\mathbf{R}(k+N-1)...\mathbf{x}_\mathbf{R}(k+1)]$$

$$\mathbf{x}_\mathbf{R}(k+N) = [x(k+N)\ x(k+N-1)...x(k+1)]$$

$$\mathbf{e}_\mathbf{R}(k+N) = \mathbf{d}_\mathbf{R}(k+N) - \hat{\mathbf{d}}_\mathbf{R}(k+N)$$

$$\hat{\mathbf{d}}_\mathbf{R}(k+N) = \left[\mathbf{Y}(k+N)\mathbf{s}(k)\right]^T$$

$$\mathbf{Y}(k+N) = \left[\mathbf{y}_R(k+N)\ \mathbf{y}_\mathbf{R}(k+N-1)\ ...\ \mathbf{y}_\mathbf{R}(k+1)\right]$$

$$\mathbf{y}_\mathbf{R}(k+N) = X(k+N)\mathbf{w}(k) \quad (5)$$

Convergence in the mean of the algorithm is guaranteed, provided that the step size μ is limited by [5] $0 \le \mu \le 1/(N\lambda_{\max})$ and $E[X'^T(k)X'(j)] = 0,\ k \ne j$ where $\lambda_{\max}$ is the maximum eigenvalue of the input signal autocorrelation matrix.

Using the overlap-save method, all the linear convolutions in (3), (4), (5) can be implemented as

$$\mathbf{y}_R(k+N) = X(k+N)\mathbf{w}(k)$$

$$= \mathbf{T_N}[\mathbf{O}_N\ \mathbf{I}_N]\left\{\begin{bmatrix}\mathbf{x}(k)\\ \mathbf{x}(k+N)\end{bmatrix}\begin{bmatrix}\mathbf{I}_N\\ \mathbf{O}_N\end{bmatrix}\mathbf{w}(k)\right\} \quad (6)$$

$$\mathbf{x}'_R(k+N) = X(k+N)\mathbf{s}(k)$$

$$= \mathbf{T}_N[\mathbf{O}_N\ \mathbf{I}_N]\left\{\begin{bmatrix}\mathbf{x}(k)\\ \mathbf{x}(k+N)\end{bmatrix}\begin{bmatrix}\mathbf{I}_N\\ \mathbf{O}_N\end{bmatrix}\mathbf{s}(k)\right\} \quad \mathbf{(7)}$$

$$\mathbf{c}_R(k+N) = X'(k+N)\mathbf{e}_R(k+N)$$

$$= \mathbf{T}_N[\mathbf{O}_N\ \mathbf{I}_N]\left\{\begin{bmatrix}\mathbf{x}'(k)\\ \mathbf{x}'(k+N)\end{bmatrix}\begin{bmatrix}\mathbf{I}_N\\ \mathbf{O}_N\end{bmatrix}\mathbf{e}_R(k+N)\right\} \quad \mathbf{(8)}$$

where the matrix I_N is an N×N identity matrix, the matrix O_N is N×N with all zero elements, and $\mathbf{T}_N$ is an N×N matrix which has ones on the secondary diagonal and zeros elsewhere.

FFT based implementation of all the three linear convolutions defined above can be done by defining F_{2N} and F_{2N}^{-1} as the 2N-point FFT and IFFT. The linear convolution in (6) may be implemented as

$$\mathbf{X}(k+N) = F_{2N}\begin{bmatrix}\mathbf{x}(k)\\ \mathbf{x}(k+N)\end{bmatrix}$$

$$\mathbf{W}(k) = F_{2N}\begin{bmatrix}\mathbf{I}_N\\ \mathbf{O}_N\end{bmatrix}\mathbf{w}(k)$$

$$\mathbf{x}_R(k+N) = [\mathbf{O}_N\ \mathbf{T}_N]F_{2N}^{-1}\left[\mathbf{X}(k+N)\otimes\mathbf{W}(k)\right] \quad (9)$$

where ⊗ denotes point-by-point multiplication. Similarly, (7) may be implemented using the 2N point FFT as

$$\mathbf{S}(k) = F_{2N}\begin{bmatrix}\mathbf{I}_N\\ \mathbf{O}_N\end{bmatrix}\mathbf{s}(k)$$

$$\mathbf{x}'_R(k+N) = [\mathbf{O}_N\ \mathbf{T}_N]F_{2N}^{-1}\left[\mathbf{X}(k+N)\otimes\mathbf{S}(k)\right] \quad (10)$$

The FFT-based implementation of (8) can be written as

$$\mathbf{X}'(k+N) = F_{2N}\begin{bmatrix}\mathbf{x}'(k)\\ \mathbf{x}'(k+N)\end{bmatrix} \quad (11)$$

$$E_R(K+N) = F_{2N}\begin{bmatrix}\mathbf{I}_N\\ \mathbf{O}_N\end{bmatrix}\mathbf{e}_R(k+N)$$

$$\mathbf{c}_R(k+N) = \left[\mathbf{O}_N\quad \mathbf{T}_N\right]$$

$$F_{2N}^{-1}\left[\mathbf{X}'(k+N)\otimes E_R(k+N))\right] \quad (12)$$

Removing (10) and (11) from above and taking $\mathbf{X}'(k+N) = \mathbf{X}(k+N)\otimes\mathbf{S}(k)$ we obtained the reduced structure FBFXNLMS algorithm which saves two FFT blocks.

In multiplechannel ANC, we assume, L number of reference sensors, P number of loudspeakers and Q numbers of error microphones are employed. So in total LP numbers of adaptive filters are present and their transfer functions are represented as $\mathbf{w}_{pl}$ and PQ number of secondary paths is represented as

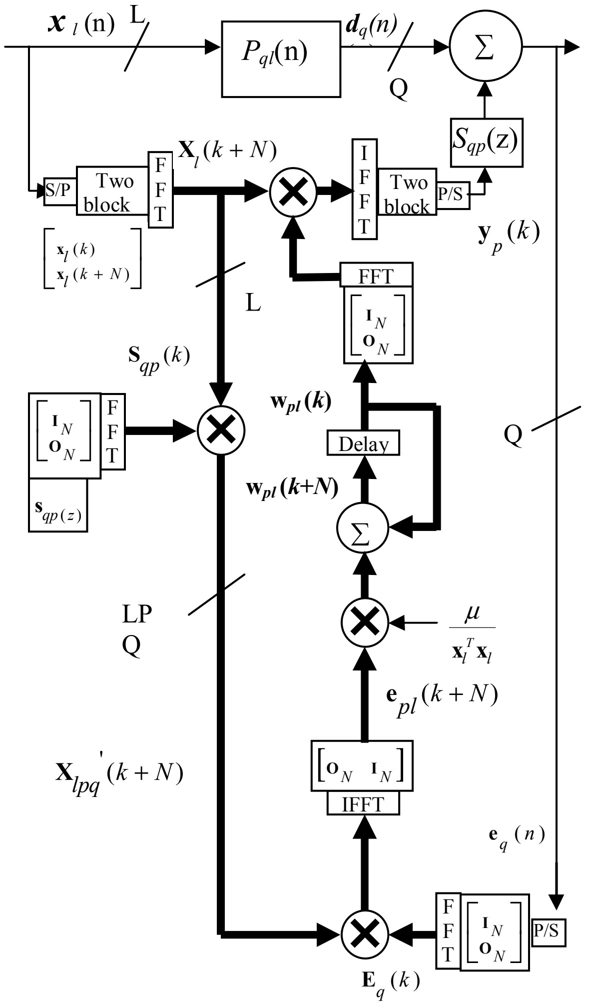


Figure 2. Block diagram for multichannel BFXNLMS algorithm active noise canceller.

$\mathbf{s}_{qp}$. Applying multiple error LMS algorithm, proposed by Elliott et. al.[1][2], multiple channel ANC problem can be solved by applying FBFXNLMS to all possible single channel paths in the multiple channel system. The weight update equation can be written as

$$\mathbf{w}_{lp}(n+1)=\mathbf{w}_{lp}(n)+2\frac{\mu}{\mathbf{x}_l^T\mathbf{x}}\sum_{q=1}^{Q}\mathbf{c}_{Rlq}(n) \qquad (13)$$

for1<l<L and 1<p<P and

$$\mathbf{c}_{Rlq}(k+N)=[\mathbf{O}_N\ \ \mathbf{T}_N]F_{2N}^{-1}\left[\mathbf{X}'_l(k+N)\otimes E_{Rq}(k+N)\right]$$

$$\mathbf{X}'_l(k+N)=\mathbf{X}_l(k+N)\otimes\mathbf{S}_{qp} \qquad (14)$$

$$\mathbf{S}_{qp}=F_{2N}\begin{bmatrix}\mathbf{I}_N\\ \mathbf{O}_N\end{bmatrix}\mathbf{s}_{qp}$$

$$E_{Rq}(K+N)=F_{2N}\begin{bmatrix}\mathbf{I}_N\\ \mathbf{O}_N\end{bmatrix}\mathbf{e}_{Rq}(k+N)$$ where $\mathbf{s}_{qp}$ is the transfer function of the secondary path connecting pth loudspeaker and qth error microphone. The block diagram for multichannel reduced structure FBFXNLMS algorithm is shown in fig.2.

3. Computational Complexity

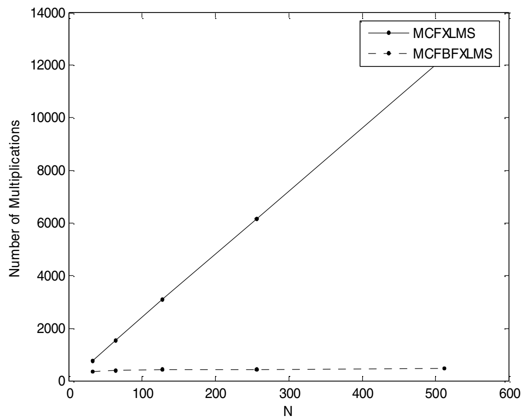
To obtain N outputs, LPN^2 multiplications and LPN(N-1) additions are required. For filtering N samples of reference signal through the secondary path of length N, $LPQN^2$ multiplications and LPQN(N-1) additions are required. For weight update, $LP(Q+1)N^2$ multiplications and $LP(Q+1)N^2$ additions are required. Therefore the total number of multiplications required is $2LP(Q+1)N^2$ and the total additions required is NLP(Q+1)(2N-1).

For FBFXNLMS algorithm, single channel ANC using overlap save method, the N-point FBFXNLMS algorithm involves the computation of (i) six 2N-point FFTs, (ii) three 2N point complex multiplications and (iii) N number of weight updates. For real-valued input data, total number of real multiplications is $12N\log_2 N+24N$ and the real additions is $24N\log_2(2N)+13N$.

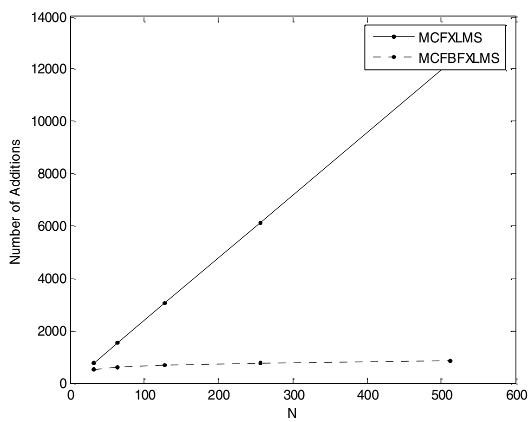
In case of multichannel ANC the number of 2N point FFT/IFFT required for (i) input signal transform is L,(ii) adaptive filter output signal transform is P, (iii) adaptive filter transform is LP, (iv) secondary path transfer function transform is PQ, (v) error signal transform is Q, (vi) transform of product of filtered input signal and error is LP. So total number of FFT is L+P+2LP+PQ+Q. Each FFT requires $2N\log_2(N)$ real multiplications and $4N\log_2(N)$ real additions . Also LP,LPQ, LPQ number of 2N point frequency domain complex multiplications are required for computing adaptive filter output, filtered input signal, weight update respectively. Each 2N point complex multiplication involves 8N real multiplications and 4N real additions. Also the number of real additions required for weight update is LPN+2LPQN. So total real multiplications required is $(L+P+2LP+PQ+Q)2N\log_2(N)+(LP+2LPQ)8N$ and real additions required is $(L+P+2LP+PQ+Q)4N\log_2(2N)+(LP+2LPQ)4N+LPN+2LPQN$. Computational complexity curves as plotted in fig.3 showing saving in computation.

4. Simulation and Results

In the experiments, we considered one reference



(a)



(b)

Figure 3. Comparison of Computational Complexity (a) Multiplications (b) Additions

Table 1. Computational Complexity per sample

N	Number of multiplication		Number of addition	
	FXNLMS	FBFXNLMS	FXNLMS	FBFXNLMS
32	768	340	756	532
64	1536	376	1524	604
128	3072	412	3060	676
256	6144	448	6132	748
512	12288	484	12276	820
1024	24576	520	24564	892

microphone, two loudspeakers and two error microphones. Memory size N is chosen to be 10. Mean squire error (MSE) in db given by $\text{MSE}=10\log_{10}(E(e^2(n)))$ is obtained through simulation taking uniform white noise with power 0.0844 (uniformly distributed random numbers between -0.5 and 0.5) as the input signal. In our

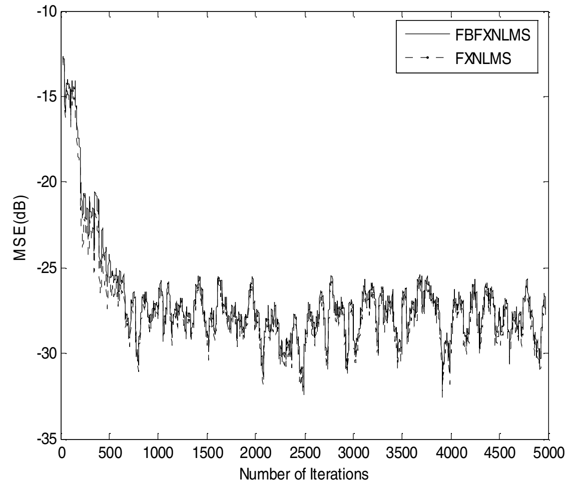


Figure 4. Convergence characteristics of proposed FBFXNLMS and FXNLMS algorithm for multichannel ANC

experiment the primary path transfer functions are $\mathbf{P}_{11} = z^{-5} - 0.3z^{-6} + 0.2z^{-7}$, $\mathbf{P}_{12} = z^{-5} - 0.4z^{-6} + 0.1z^{-7}$ And the secondary path transfer functions are minimum phase model as described below

$$\mathbf{s}_{11} = z^{-2} + 0.5z^{-3} \qquad \mathbf{s}_{21} = 1.1z^{-2} + 0.4z^{-3}$$

$$\mathbf{s}_{12} = z^{-2} + 0.6z^{-3} \qquad \mathbf{s}_{22} = 0.9z^{-2} + 0.3z^{-3}$$

Simulations are done for the proposed algorithm taking μ=0.06 and for comparison FXNLMS algorithm with μ=0.02. Simulation results are plotted in fig.4.From the results, it is evident that the proposed algorithm offers identical performance as the standard FXNLMS algorithm fro multichannel ANC but the real advantage of the proposed algorithm is large saving in computational complexity.

5. Conclusion

In this paper, a novel adaptive algorithm is developed for noise mitigation in multichannel ANC which employs normalized LMS algorithm to facilitate variable step size control. Detailed mathematical formulation of the algorithm for multichannel control structure is presented. The validity of the proposed algorithm is demonstrated through computer simulation. Also from the computational analysis it is found that proposed algorithm is superior to the standard FXNLM algorithm and this becomes more prominent with increase in number of channels.

10. References

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