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DYNAMIC BEHAVIOR OF THREE-FLUID CROSSFLOW HEAT EXCHANGERS

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Abstract

A transient temperature response of three-fluid heat exchangers with finite and large capacitance of the separating sheets is investigated numerically for step, ramp, exponential and sinusoidal perturbations provided in the central (hot) fluid inlet temperature. The effect of two-dimensional longitudinal conduction in the separating sheet and of axial dispersion in the fluids on the transient response has been investigated. A comparison of the dynamic behavior of four possible arrangements of three-fluid crossflow heat exchangers has also been presented.

Keywords: axial dispersion, finite difference, longitudinal conduction, three-fluid heat exchanger, transient behavior.

Introduction

The thermo-hydraulic theory of two fluid exchangers is well developed and available in the standard literature [1]. The well-established algorithm for the thermal design of a two fluid heat exchanger, however, has no equivalent when the physical situation implies more than one thermal communication, such as heat losses to the ambient and the introduction of a third fluid.

Most heat exchanger applications in the process, power, transportation, thermal energy recovery, electronics and the aerospace industries involve transfer of thermal energy between two fluids through one thermal communication. However, in recent years some processes with heat transfer between three fluids have become important. Three fluid and multi-fluid heat exchangers are widely used in cryogenics and different chemical processes, such as air separation, helium separation from natural gas, purification and liquefaction of hydrogen, and ammonia gas synthesis. Three fluid heat exchangers allow a more compact and economical design also in various other applications.

A wide literature is available on the steady state behavior of three fluid heat exchangers. A pioneering effort [2] in analyzing crossflow problem considering it to be a case of heat transfer with three heat agents or streams has been given. There was another work by Sorlie [3] among the first few, developing a general theory for two temperature effectiveness of three-fluid heat exchangers of parallel and counter flow type. Extending the work of Sorlie [3], an analytical relationship was developed by Aulds and Barron [4] between the design variables for a general three-fluid heat exchanger with three thermal communications. Due to the complexity involved with the addition of other operating conditions, a numerical method was used by Barron and Yeh [5] for obtaining the temperature distribution and heat exchanger effectiveness of counter-current three-fluid heat exchangers that included the effect of longitudinal conduction of both the separating walls. Sekulic and Kmecko [6] analyzed the performance of three-fluid parallel stream heat exchangers on the basis of effectiveness and compared four possible arrangements of combining the streams with two thermal communications. Willis and Chapman [7] made an effort to present the performance of a three-fluid crossflow heat exchanger graphically in terms of the temperature effectiveness. An exact analytical solution of three-fluid crossflow heat exchangers was first tried by Baclic et al. [8] for unmixed flow arrangements using Laplace transforms. Sekulic and Shah [9] gave a very comprehensive review of methodologies for analyzing the steady state performance of three fluid heat exchangers. The effect of longitudinal conduction in wall on the thermal performance of, three-fluid crossflow heat exchanger was numerically calculated by Yuan and Kou [10]. Later, three arrangements of different repetitive pattern were analysed [11] in terms of overall heat recovery and uniformity of preheating on three fluid streams. The effect of wall longitudinal conduction on thermal performance of three-fluid crossflow heat exchangers under steady state was again investigated by Yuan and Kou [12], and the three arrangements of the fluid streams were compared. Yuan and Kou [13] further investigated the entropy generation in a three fluid crossflow heat exchanger in the presence of wall longitudinal conduction using a numerical technique.

Although heat exchangers mostly operate under steady state conditions, steady state analysis is not adequate for situations like start-up, shutdown, failure and accidents. The transient response of heat exchangers needs to be known for designing control strategies and for taking care of thermal stresses in mechanical design. This has motivated for the determination of transient temperature fields and a few analytical and semi-analytical works [14, 15] have also been performed on dynamic behavior of three-fluid heat exchangers.

In the present work, the transient temperature response of the three-fluid crossflow heat exchanger having large and finite core capacity with all the fluids unmixed is investigated numerically for step, ramp, exponential and sinusoidal perturbations provided in the central fluid inlet temperature. The four possible arrangements [9] for three-fluid crossflow heat exchangers (figure 1) have also been compared.

Mathematical Modelling

A direct-transfer, three-fluid, crossflow plate-fin heat exchanger is shown schematically in figure 2(a). For the mathematical analysis the two separating sheets having one fluid on either side are taken separately. The following assumptions are made for the analysis.

1. All the fluids are single phase, unmixed and do not contain any volumetric source of heat generation.
2. The thermo-physical properties of the fluid streams and the walls are constant and uniform.
3. The central fluid is either the hottest or the coldest fluid.
4. The exchanger shell or shroud is adiabatic and the effects of the asymmetry in the top and bottom layers are neglected.
5. Flow is well mixed in any of the passages, so that variation of temperature and velocity in the fluid streams in a direction normal to the separating plate (z-direction) is negligible.
6. The primary and secondary areas of the separating plates have been lumped together, so that the variation of wall temperature is also two-dimensional.

7. Transverse conduction through fins between adjacent separating sheets is neglected. This implies there will be a temperature extremum in the fin temperature profile [16].
8. The thermal resistances on both sides, comprising film heat transfer coefficient of primary and secondary surface and fouling resistance, are constant and uniform.
9. Heat transfer area per unit base area and surface configurations are constant.
10. Transverse thermal resistance of the separating sheets in a direction normal to it (z-direction) is negligible.

Due to the introduction of a third fluid the process of energy exchange in a three-fluid heat exchanger is more complex compared to that in a conventional heat exchanger. The central fluid stream exchanges heat simultaneously with two adjacent streams. The exact distribution of this thermal energy plays an important role in steady state as well as in the dynamic behavior of the heat exchanger. This distribution depends upon the conditions of all the three fluids and the total area associated with them. As the thermo-physical properties of the top and the bottom fluid streams may be different in a general situation, it is likely that the two separating sheets will have different temperatures, and the fins in the central passage will have an asymmetric temperature profile. This indicates that the central stream may transfer heat to the top and the bottom separating sheets at different rates. To take care of this phenomenon it is assumed that part of the secondary surface is associated with the top separating sheet (w1), and the rest is associated with the bottom separating sheet (w2). This idealization is depicted in Fig. 2 (a) and 2 (b).

Assuming $(\eta hA)_{b\text{-}w1}$ and $(\eta hA)_{b\text{-}w2}$ are the convective conductances associated with the top and the bottom separating sheet respectively, the following relationship can be obtained.

$$\frac{1}{(\eta hA)_{b-w1}}+\frac{1}{(\eta hA)_{b-w2}}=\frac{1}{(\eta hA)_{b}} \tag{1}$$

A non-dimensional parameter ϕ as defined in eq. (2) may be introduced.

$$\frac{(\eta hA)_{b-w1}}{(\eta hA)_{b}}=\frac{1}{\phi}\text{, and }\frac{(\eta hA)_{b-w2}}{(\eta hA)_{b}}=\frac{1}{(1-\phi)} \tag{2}$$

Proceeding with the same logic it may be assumed that the total thermal capacity of the separating sheets is also distributed amongst the upper and the lower sheet in the ratio ψ and (1-ψ) respectively.

$$(Mc)_{w1} + (Mc)_{w2} = (Mc)_w \tag{3}$$

Then,

$$\frac{(Mc)_{w1}}{(Mc)_w} = \psi \text{ , and } \frac{(Mc)_{w2}}{(Mc)_w} = (1-\psi) \tag{4}$$

Based on the above assumptions and idealizations, the conservation of energy for the three fluid streams and the two separating sheets can be expressed in non-dimensional form for an infinitesimal small control volume as follows.

For fluid streams a, b and c one gets Eq. (5), (6) and (7) respectively.

$$\frac{V_a}{R_{ab}}\frac{\partial T_a}{\partial \theta} = T_{w1} - T_a - \frac{E_{ab}}{R_{ab}}\frac{\partial T_a}{\partial Y} + \frac{N_a}{Pe_a}\frac{E_{ab}}{R_{ab}}\frac{\partial^2 T_a}{\partial Y^2} \tag{5}$$

$$V_b\frac{\partial T_b}{\partial \theta} = \frac{1}{\phi}(T_{w1} - T_b) + \frac{1}{(1-\phi)}(T_{w2} - T_b) - \frac{\partial T_b}{\partial X} + \frac{N_a}{Pe_b}\frac{\partial^2 T_b}{\partial X^2} \tag{6}$$

When fluid c is moving in x-direction

$$\frac{V_c}{R_{cb}}\frac{\partial T_c}{\partial \theta} = T_{w2} - T_c - \frac{E_{cb}}{R_{cb}}\frac{\partial T_c}{\partial X} + \frac{N_a}{Pe_c}\frac{E_{cb}}{R_{cb}}\frac{\partial^2 T_c}{\partial X^2} \tag{7a}$$

When fluid c is moving in y-direction

$$\frac{V_c}{R_{cb}}\frac{\partial T_c}{\partial \theta} = T_{w2} - T_c - \frac{E_{cb}}{R_{cb}}\frac{\partial T_c}{\partial Y} + \frac{N_a}{Pe_c}\frac{E_{cb}}{R_{cb}}\frac{\partial^2 T_c}{\partial Y^2} \tag{7b}$$

Similarly for the two separating sheets w1 and w2, heat conduction equations are given in Eq. (8) and (9) respectively.

$$\psi\frac{\partial T_{w1}}{\partial \theta} = R_{ab}(T_a - T_{w1}) + \frac{1}{\phi}(T_b - T_{w1}) + \lambda_x N_a\frac{\partial^2 T_{w1}}{\partial X^2} + \lambda_y N_a\frac{\partial^2 T_{w1}}{\partial Y^2} \tag{8}$$

$$(1-\psi)\frac{\partial T_{w2}}{\partial \theta} = \frac{1}{(1-\phi)}(T_b - T_{w2}) + R_{cb}(T_c - T_{w2}) + \lambda_x N_a\frac{\partial^2 T_{w2}}{\partial X^2} + \lambda_y N_a\frac{\partial^2 T_{w2}}{\partial Y^2} \tag{9}$$

Here Pe is axial dispersive Peclet number given by $\frac{(mc)L}{A_c.D}$, where D is diffusion coefficient of the fluid representative of the effect of axial dispersion.

It may be noted that five non-dimensional parameters for the steady state performance of a three-fluid heat exchanger were specified in earlier work [9]. On the other hand one needs nine non-dimensional parameters namely NTU, E_{a-b}, E_{c-b}, R_{a-b}, R_{c-b}, $V_{a,b,c}$ and $T_{c,in.}$ The extra parameters are necessary in the present case to take care of the thermal capacity of the fluid streams and the separating sheets.

Further, one can introduce the number of transfer units (NTU) for the three-fluid heat exchanger and replace Na in terms of NTU. Conventionally NTU for three-fluid heat exchanger is defined considering the thermal interaction of the central fluid stream with any one of the streams [9]. According to this convention NTU can be defined as follows.

$$NTU = \left[(mc)_a \left\{ \frac{1}{(\eta hA)_{b-w1}} + \frac{1}{(\eta hA)_a} \right\} \right]^{-1}$$

or $$\frac{1}{NTU} = C_a \left[\frac{\phi}{(\eta hA)_b} + \frac{1}{(\eta hA)_a} \right] \tag{10}$$

From the definition of R_{a-b} and E_{a-b} it can be shown that

$$Na = E_{a-b}.NTU \left[\phi + \frac{1}{R_{a-b}} \right] \tag{11}$$

The Eq. (5)-(9) are subjected to following initial and boundary conditions

$$T_a(X,Y,0)=T_b(X,Y,0)=T_c(X,Y,0)=T_{w1}(X,Y,0)=T_{w2}(X,Y,0)=0 \tag{12}$$

$$\left.\frac{\partial T_{w1}(X,Y,\theta)}{\partial X}\right|_{X=0} = \left.\frac{\partial T_{w1}(X,Y,\theta)}{\partial X}\right|_{X=N_a} = \left.\frac{\partial T_{w1}(X,Y,\theta)}{\partial Y}\right|_{Y=0} = \left.\frac{\partial T_{w1}(X,Y,\theta)}{\partial Y}\right|_{Y=N_a} = 0 \tag{13}$$

$$\left.\frac{\partial T_{w2}(X,Y,\theta)}{\partial X}\right|_{X=0} = \left.\frac{\partial T_{w2}(X,Y,\theta)}{\partial X}\right|_{X=N_a} = \left.\frac{\partial T_{w2}(X,Y,\theta)}{\partial Y}\right|_{Y=0} = \left.\frac{\partial T_{w2}(X,Y,\theta)}{\partial Y}\right|_{Y=N_a} = 0 \tag{14}$$

$$T_a(X,0,\theta) = T_{a,in} = 0 \tag{15}$$

$$\left.\frac{\partial T_a(X,Y,\theta)}{\partial Y}\right|_{Y=N_a} = 0 \tag{16}$$

$$T_b(0,Y,\theta) = T_{b,in} = \overline{\phi}(\theta) \tag{17}$$

$$\left.\frac{\partial T_b(X,Y,\theta)}{\partial X}\right|_{X=N_a} = 0 \tag{18}$$

$$T_{c,in}(X,Y,\theta) = T_{c,in} \tag{19}$$

where $T_{c,in}$ is $T_c(0,Y,\theta)$, $T_c(N_a,Y,\theta)$, $T_c(X,N_a,\theta)$ and $T_c(X,0,\theta)$ for the arrangements C1, C2, C3 and C4 respectively.

When fluid c flows in x-direction

$$\left.\frac{\partial T_c(X,Y,\theta)}{\partial X}\right|_{X=Z} = 0\,, \tag{20a}$$

where Z=Na and 0 for the arrangements C1 and C2 respectively.

When fluid c flows in y-direction

$$\left.\frac{\partial T_c(X,Y,\theta)}{\partial Y}\right|_{Y=Z} = 0\,, \tag{20b}$$

where Z=0 and Na for the arrangements C3 and C4 respectively.

Equations (5)-(9) along with the boundary conditions (12)-(20) give the complete formulation of a three-fluid crossflow heat exchanger. Solution of this set of equations will give the dynamic performance of heat exchanger once the nine basic input parameters NTU, $E_{a\text{-}b}$, $E_{c\text{-}b}$, $R_{a\text{-}b}$, $R_{c\text{-}b}$, $V_{a,b,c}$ and $T_{c,in}$ along with the two additional parameters $Pe_{a,b}$ and $\lambda_{x,y}$ are specified. Here $\overline{\phi}(\theta)$ is the perturbation given to the inlet temperature of the central fluid, $T_{b,in}$. In the present investigation, the following perturbations have been considered.

$$\bar{\phi}(\theta)=\begin{cases}1; & \text{for step input}\\ \begin{bmatrix}\alpha\theta, \ \theta\le 1\\ 1, \ \theta>1\end{bmatrix}; & \text{for ramp input}\\ 1-e^{-\alpha\theta}; & \text{for exponential input}\\ \sin(\alpha\theta); & \text{for sinusoidal input}\end{cases} \tag{21}$$

where α is assumed to be unity.

Method of Solution

The conservation equations are discretized using the finite difference technique. Forward difference scheme is used for time derivatives, while upwind scheme and central difference scheme are used for the first and second order space derivatives respectively [17]. The difference equations along with the boundary conditions are solved using Gauss-Seidel iterative technique. The convergence of the solution has been checked by varying the number of space grids and size of the time steps. The solution gives the two-dimensional temperature distribution for all three fluids as well as for the separator plate. Additionally one may calculate the mean exit temperatures as follows.

$$\bar{T}_{a,ex}=\frac{\int_0^{Na} T_{a,ex}.u\,dy}{\int_0^{Na} u\,dy}, \quad \bar{T}_{b,ex}=\frac{\int_0^{Na} T_{b,ex}.v\,dx}{\int_0^{Na} v\,dx} \quad \text{and} \quad \bar{T}_{c,ex}=\frac{\int_0^{Na} T_{c,ex}.w\,dy'}{\int_0^{Na} w\,dy'} \tag{22}$$

where y'=x or y depending upon the direction of flow of fluid c.

Results and Discussions

To check the validity of the numerical scheme, the solution for the steady state condition for the arrangement C4 at $E_{ab}=E_{cb}=1$, $R_{ab}=R_{cb}=1$, $V=\lambda=0$, $Pe=\infty$, $T_{a,in}=0$, $T_{b,in}=1$ and $T_{c,in}=0.5$, was compared with the analytical solutions [8]. Excellent agreement was observed as shown in figure 3.

The transient behavior of three-fluid crossflow heat exchanger has been studied for different excitations given to the central (hot) fluid inlet temperature. Performance of the heat exchanger was studied over a wide range of parameters as well as for sufficient time duration so that steady state conditions are obtained for each individual excitation. Some of the salient results are discussed in the next sections.

Case I: Heat Exchanger with Large Core Capacity in the Absence of Core Longitudinal Conduction and Axial Dispersion

Though the formulation has been done for a generalized case, an example has been taken where the core capacity is large. This makes $V_a=V_b=V_c \approx 0$, and the results are applicable to gas-to-gas heat exchangers.

Figures 4 (a), (b) and (c) exhibit the performance of the heat exchanger at NTU = 1, 5 and 10 for step, ramp and exponential inputs respectively. In all the cases, the steady state exit temperature is reached by the three streams within a small time interval at NTU=1. However, the difference of mean exit temperatures (at steady state) is the maximum in this case with the bottom and the upper layers having the maximum and the minimum values of temperature respectively. This is a common trend for step, ramp and exponential excitations as depicted in figure 4 (a), (b) and (c) respectively.

The time required to reach the steady state increases with the increase of NTU. Nevertheless with the increase of NTU, equalization of temperature between different streams takes place as more area for heat transfer is available. The temperature difference between the coldest stream and the hottest stream decreases at any time interval. Further, the exit temperatures of the two extreme fluid layers become identical from the very beginning even at NTU=5. It may be noted that, although the inlet temperature of fluid a is lower than that of fluid b, its exit temperature is higher than that of fluid b at higher NTUs.

For a sinusoidal variation of the inlet temperature of the central fluid, two different NTUs 1 and 5 have been considered, and the results are depicted in figure 4(d). At lower NTU, steady state is reached early even in case of sinusoidal excitation like the previous cases. The time lag of oscillations increases with the increase in NTU. All the temperature responses have a periodic nature whose amplitude and frequency vary with time and attain a constant value once steady state is reached. At steady state the mean exit temperatures of all the three fluid streams have a steady periodic nature, which can be described by a suitable sine-function. This fact may be appreciated from figure 5. Here all the mean exit temperatures of the three fluids have been plotted as functions of the inlet temperature of the central fluid. Near the steady state, three Lissajous figures of elliptic shapes are obtained for the three exit temperatures. This indicates that the mean temperatures at outlet are also experiencing a sinusoidal variation with the same frequency as $T_{b,in}$ but with different amplitudes. Therefore, for a sinusoidal excitation of the central fluid, figure 5 provides a very convenient and concise way of determining the response of all the three fluid streams under steady state conditions.

The effect of the conductance ratio on the mean exit temperature of the three fluids is depicted in figure 6. Although the conductance ratio does not have any effect on the steady state performance of the heat exchanger, its effect should not be neglected during the transients. From figure 6, it may be noted that a lower value of the conductance ratio increases the time to reach the steady state and the effect is more pronounced in the central fluid stream.

Case II: Heat Exchanger with Finite Core Capacitance, Core Longitudinal Conduction and Axial Dispersion

In the second case the transient behavior has been investigated with the introduction of the heat capacity ratio (V), two-dimensional longitudinal conduction in the separating sheets and axial dispersion in the fluids.

Because the step, ramp and exponential responses give similar behavior as observed earlier, only a step excitation has been considered for further analysis. Similar to the previous case, arrangement C4 only has been considered for the effect of various parameters.

The effect of longitudinal conduction in a heat exchanger core has been studied next. The exit temperature responses of the three fluids are depicted in figure 7. Initially, the temperature of the hotter fluids b and c decreases with an increase in longitudinal conduction, showing a reduction in the mean exit temperatures. The effect is not comparably significant on the lower temperature fluid (fluid a). However, in the later part of the transient period, performance of the fluids in the upper and lower layers deteriorates. The change in the performance of the middle layer fluid is insignificant near steady state.

The effect of axial dispersion on the step response of the three fluids for arrangement C4 has been depicted in figure 8. It shows that the increase of axial dispersion (decrease in Pe) adversely affects the performance of the heat exchanger. The mean exit temperature of the hotter fluids (fluids b and c) increases, and there is reduction in that of the cold fluid (fluid a). The difference is larger at smaller Pe, and it decreases with an increase in Pe. Practically, above Pe=20, the effect of axial dispersion is not much significant, and the difference in the performance is very small.

Comparison of different arrangements

A dynamic analysis of different arrangements of three-fluid single-pass crossflow heat exchanger for step disturbances in the central fluid stream has been conducted and is depicted in figure 9. It shows that for the central (hot) fluid, the mean exit temperature can be given in decreasing order as C2>C3>C4>C1 [Fig. 9(b)]. For the upper and the lower fluids, the mean exit temperature decreases as C4<C1<C2<C3 [Fig. 9 (a) and (c)]. However, the differences in the mean exit temperature between arrangements C2 and C3, and between C1 and C4, are very small for all the three fluids. The selection of the arrangements can be done on the basis of the relative importance of the fluids. For the central (hot) fluid, arrangements C2 and C3 are better where all the fluid combinations are either cross or counter. However, arrangements C1 and C4 are better for the cold fluids (fluid a and fluid c). Between the cross co-current (C4) and the cross counter current (C3) heat exchangers, C4 performs better if the cold fluids are the desired fluids. This has also been proved earlier by the steady state analysis [6, 18]. The behavior for ramp and exponential inputs are similar to that for a step input because of the specific nature of the input functions.

Conclusion

In the analysis, a typical case of three-fluid crossflow heat exchanger with large core capacity is analysed based on a finite difference technique. The analysis has been extended to finite core capacity in the presence of core longitudinal conduction and axial dispersion. In general, it has been seen that at low NTU the heat exchanger reaches the steady state within a small interval of time, whereas at high NTU the temperature difference between the streams decreases. In the case of a sinusoidal excitation, the exit temperatures initially show unsteady periodic behavior. Gradually this becomes steady while the frequency ultimately reaches the frequency of the input excitation. It has been shown that the functional dependence of exit temperatures on the inlet excitation can be depicted with the help of Lissajous plots under steady state. Different arrangements for single pass crossflow heat exchangers have also been compared.

Although the numerical solution considered above has been applied to a few typical examples, other cases of transient heat transfer in three-fluid heat exchangers can also be analyzed. Moreover, from the given analysis one can determine the transient behavior of the core temperature at different time instants, which may be needed for mechanical design and calculation of thermal stresses.

Nomenclature

A, A_{HT} - heat transfer area, m^2

A_c - flow area, m^2

C - heat capacity rate (mc), W/K

c, Cp - specific heat, J/kg-K

C1, C2, C3, C4 - crossflow arrangements

D - diffusion coefficient, W/ m-K

$E_{a,c-b}$ - capacity rate ratio $= \frac{(mc)_{a,c}}{(mc)_b}$

h - heat transfer coefficient, W/ m^2-K

k - thermal conductivity of the separating sheet, W/m

L - heat exchanger length, m

M - mass of the separating sheet, kg

m - mass flow rate of fluid, kg/s

Na - $\left[\frac{\eta hA}{mc}\right]_b$

NTU - number of transfer units, $\frac{(UA)_{a-b}}{(mc)_a}$

Pe - axial dispersive Peclet number $= \frac{(mc)L}{A_c.D}$

Q, q - rate of heat transfer, W

$R_{a,c-b}$ - conductance ratio $= \frac{(\eta hA)_{a,c}}{(\eta hA)_b}$

Ru - $\frac{(UA)_{a-b}}{(UA)_{c-b}}$

T - dimensionless temperature $= \frac{t \quad - t_{a,in}}{t_{b,in} - t_{a,in}}$

t - temperature

$\overline{T}$ - dimensionless mean temperature

$\bar{t}$ - mean temperature

U - overall heat transfer coefficient, W/ m^2-K

u, v, w - velocity of flow, m/s

V - heat capacity ratio $= \frac{LA_c \rho c}{Mc_w}$

$$X = \left(\frac{\eta hA}{mc}\right)_b \frac{x}{L_x} = Na \frac{x}{L_x}$$

$$Y = \left(\frac{\eta hA}{mc}\right)_b \frac{y}{L_y} = Na \frac{y}{L_y}$$

Greek letters

δ - equivalent thickness of the separating sheet, m

η - overall surface efficiency

θ - $\frac{(\eta hA)_b \tau}{(Mc)_w}$, dimensionless time

λ - longitudinal heat conduction parameter, $\lambda_x = \frac{k\delta L_y}{L_x (mc)_b}$, $\lambda_y = \frac{k\delta L_x}{L_y (mc)_b}$

μ - dynamic viscosity, N-s/m^2

ρ - density, kg/ m^3

τ - time, s

$\bar{\phi}(.)$ - perturbation in hot fluid inlet temperature

$$\phi = \frac{(\eta hA)_{b-w1}}{(\eta hA)_b}$$

$$\psi = \frac{(Mc)_{w1}}{(Mc)_w}$$

Subscripts

a, b, c - fluid streams a, b and c

c, h - cold and hot side

ex - exit

i, in - inlet

max - maximum

mean - mean value

min - minimum

o, out - exit/outlet

w - separating wall

1, 2 - state 1 and 2

References

[1] Shah, R.K., and Sekulic, D.P., 1998, "Heat Exchangers" *Handook of Heat Transfer,* Rosenhow, Hartnett and Cho eds., McGraw Hill, New York, 3, pp.17.63-17.76.

[2] Rabinovich, G.D., 1962, "On a Particular Case of Stationary Heat Transfer With Cross Flow of Heat Agents," Int. J. Heat Mass Transfer, **5**, pp. 409-412.

[3] Sorlie, T., 1962, "Three-Fluid Heat Exchanger Design Theory - Counter and Parallel Flow," Technical Report No. 54, Dept. of Mech. Engg., Stanford University, Stanford, California.

[4] Aulds, D.D, and Barron, R.F., 1967, "Three-Fluid Heat Exchanger Effectiveness," Int. J. Heat Mass Transfer, **10**, pp. 1457-1462.

[5] Barron, R.L. and Yeh, S.L., 1976, "Longitudinal conduction in a Three-Fluid Heat Exchanger," ASME Paper, 76-WA, HT-9, pp. 2-7.

[6] Sekulic, D.P. and Kmecko, I., 1995, "Three-Fluid Crossflow Heat Exchanger Effectiveness – Revisited," J. Heat Transfer, **117**, pp. 226-229.

[7] Willis Jr., N.C. and Chapman, A.J., 1968, "Analysis of Three-Fluid Crossflow Heat Exchanger," J. Heat Transfer, **90**, pp. 333-339.

[8] Baclic, B.S., Sekulic, D.P. and Gvozdenac, D.D., 1982, “Performance of Three-Fluid Single Pass Crossflow Heat Exchanger,” *Heat Transfer*, U. Grigull, E. Hahne, K. Stephan and J. Straub eds., Hemisphere, Washington, 6, pp. 167-172.

[9] Sekulic, D.P. and Shah, R.K., 1995, “Thermal Design Theory of Three-Fluid Heat Exchangers,” Ad. Heat Transfer, **26**, pp. 219-328.

[10] Yuan, P. and Kou, H.S., 1998, “The Effect of Longitudinal conduction in a Three-Fluid Crossflow Heat Exchanger,” Numerical Heat Transfer, Part A, **34**, pp. 135-150.

[11] Kou, H.S. and Yuan, P., 2001, “Thermal Analysis of Plate-Fin Crossflow Heat Exchanger Including Three Fluids with Different Arrangements,” Int. Jr. of Transport Phenomena, **3**, pp. 29-41.

[12] Yuan, P. and Kou, H.S., 2001, “The Comparison of Longitudinal Wall Conduction Effect on The Crossflow Heat Exchangers Including Three Fluid Streams With Different Arrangements,” Applied Thermal Engineering, **21**, pp. 1891-1907.

[13] Yuan, P. and Kou, H.S., 2001, “Entropy Generation On a Three-Gas Crossflow Heat Exchangers With Longitudinal Wall Conduction,” Int. Comm. Heat Mass Transfer, **28**(6), pp. 803-813.

[14] Sekulic, D.P. and Herman, C., 1987, “Transient Temperature Fields in a Three-Fluid Heat Exchanger,” In Proc. XVII Int. Congr. Refrig., Vienna, pp. 833-837.

[15] Bielski, S., and Malinowski, L., 2003, “A Semi-Analytical Method for Determining Unsteady Temperature Fields in a Parallel-Flow Three-Fluid Heat Exchanger,” Int. Comm. Heat Mass Transfer, **30 (8)**, pp. 1071-1080.

[16] Prasad, B.S.V., 1997, “Fin Efficiency and Mechanisms of Heat Exchange through Fins in Multi-Stream Plate-Fin Heat Exchangers: Development and Application of a Rating Algorithm Passages,” Int. J. Heat Mass Transfer, **40**(18), pp. 4279-4288.

[17] Ozisic, M.N., 1994, *Computational Methods in Heat Transfer*, CRC Press, London.

[18] Ellis, W.E., 1968, "Comparative Evaluations of Alternative Flow Configurations of Three-Fluid, Cross Flow, Heat Exchangers," M.S. Thesis, Dept. of Mech. Eng., Rice University, Houston, TX.

List of Figure Captions

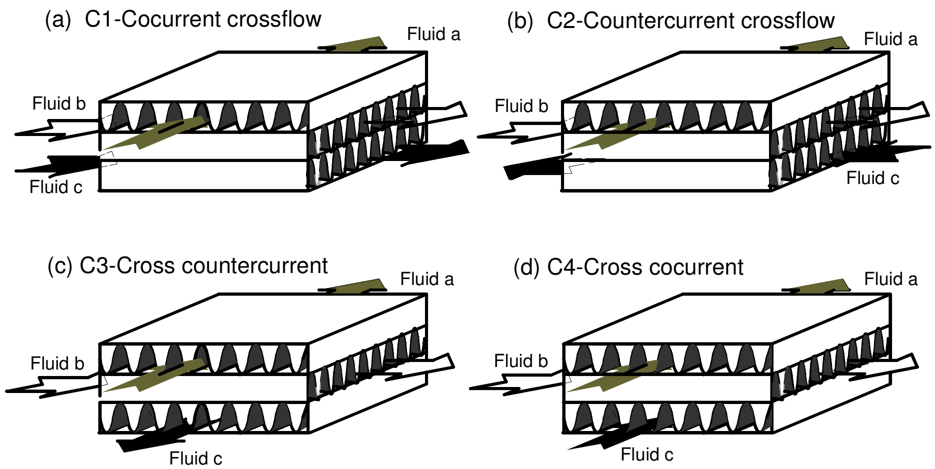


Fig.1, Mishra, JHT

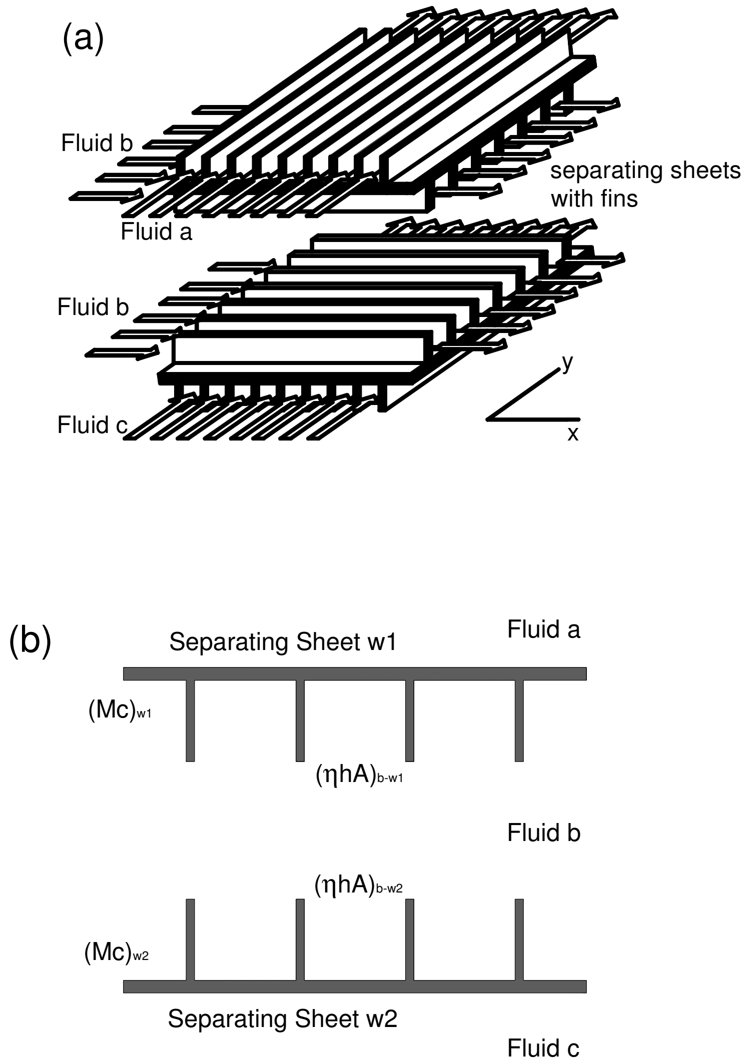


Fig.2, Mishra, JHT

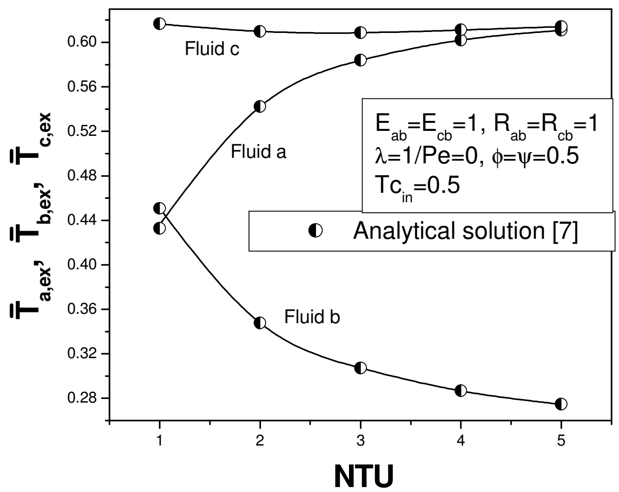


Fig.3, Mishra, JHT

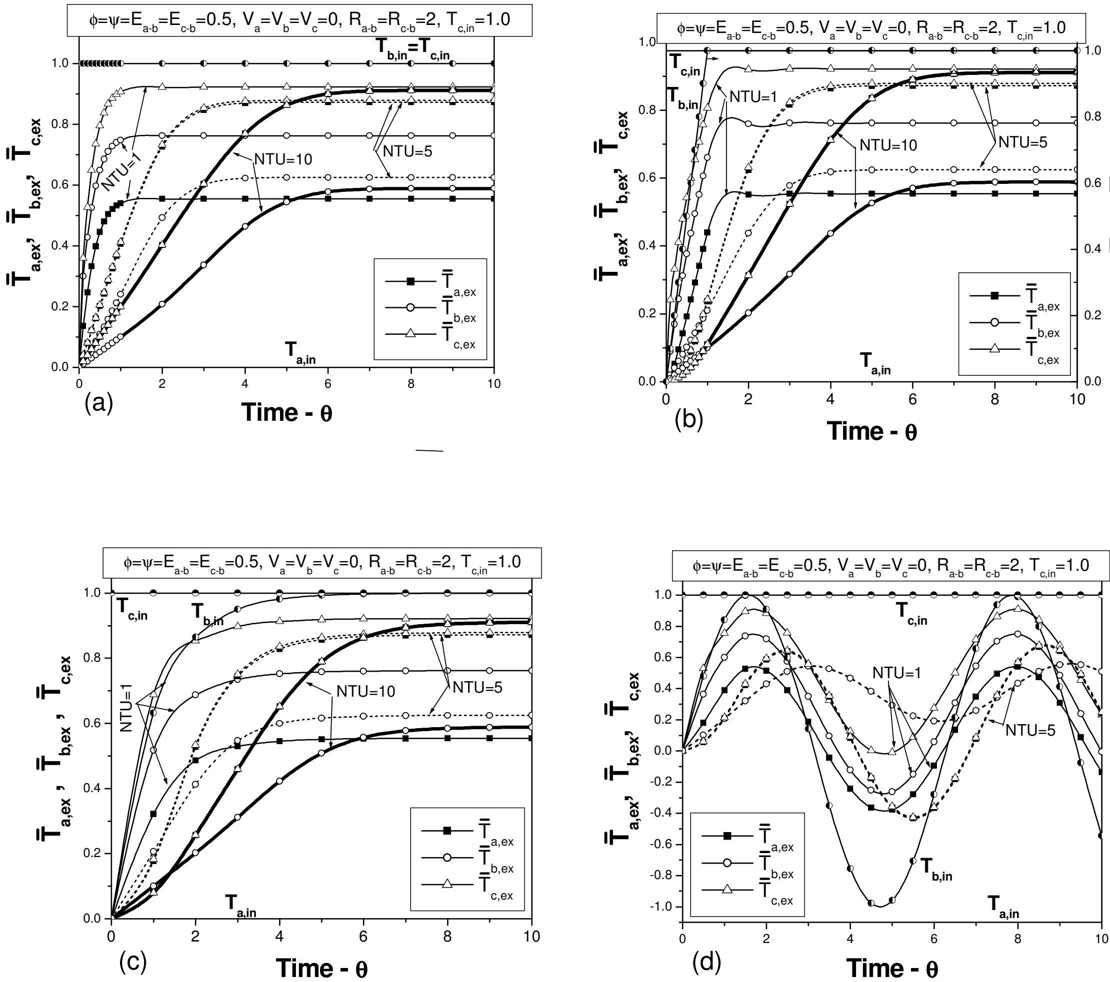


Fig.4, Mishra, JHT

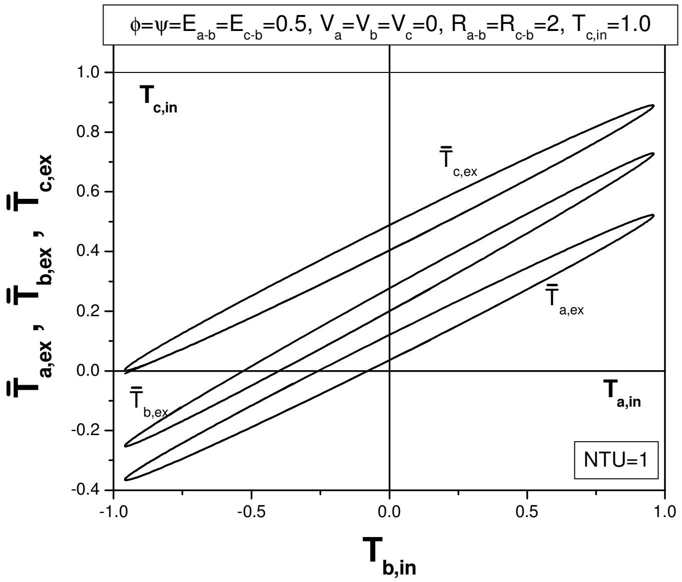


Fig.5, Mishra, JHT

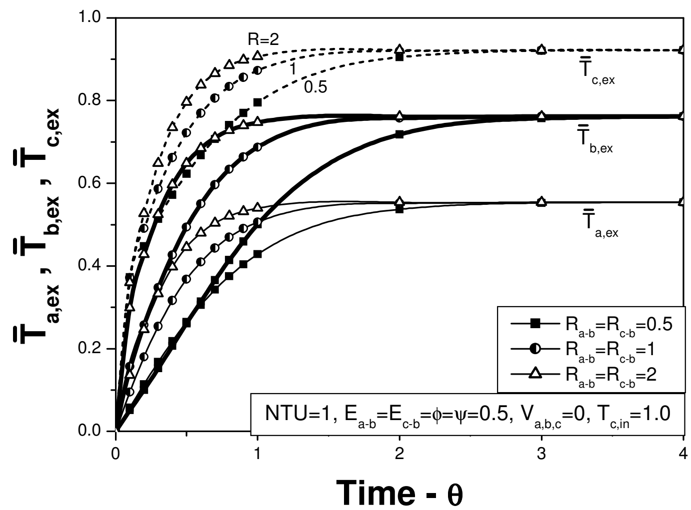


Fig.6, Mishra, JHT

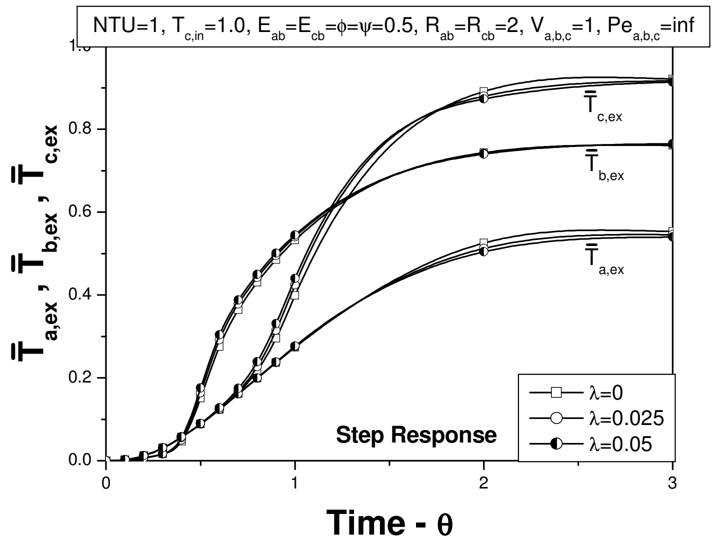


Fig.7, Mishra, JHT

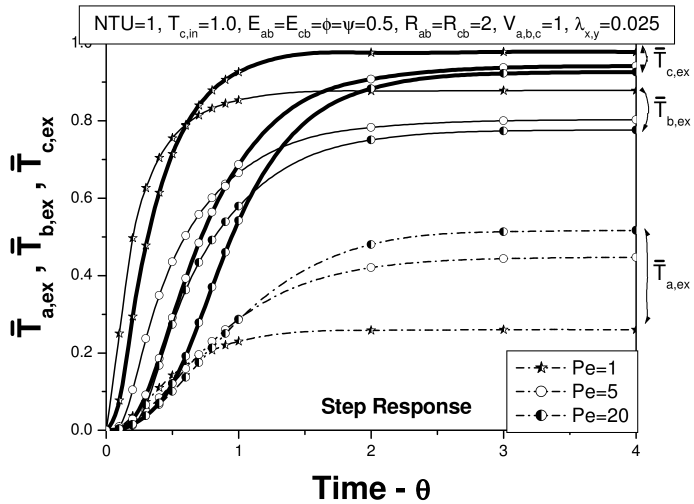
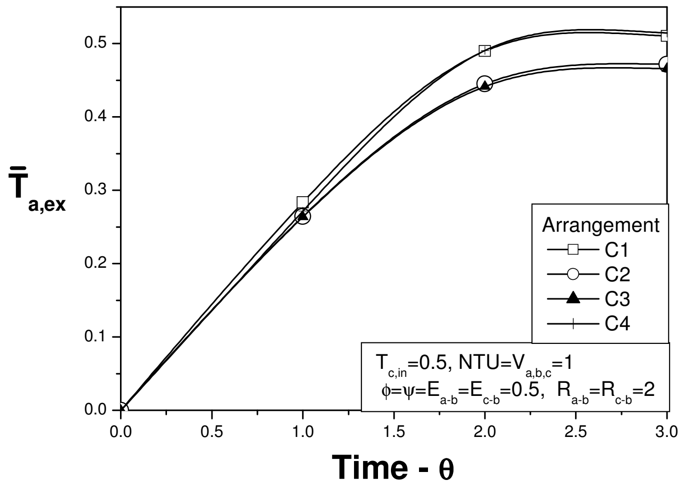
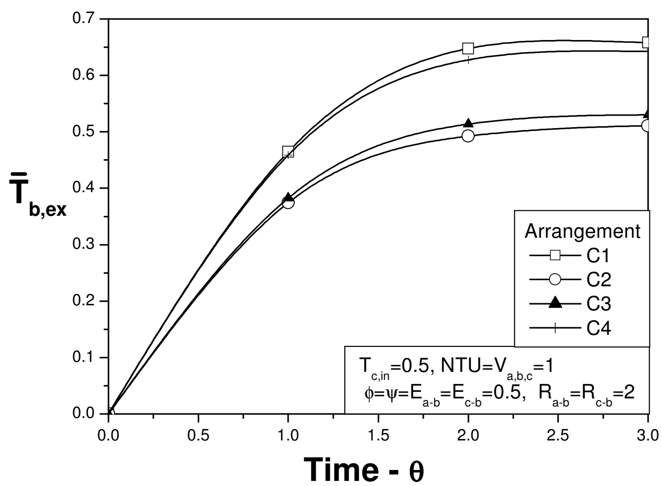


Fig.8, Mishra, JHT

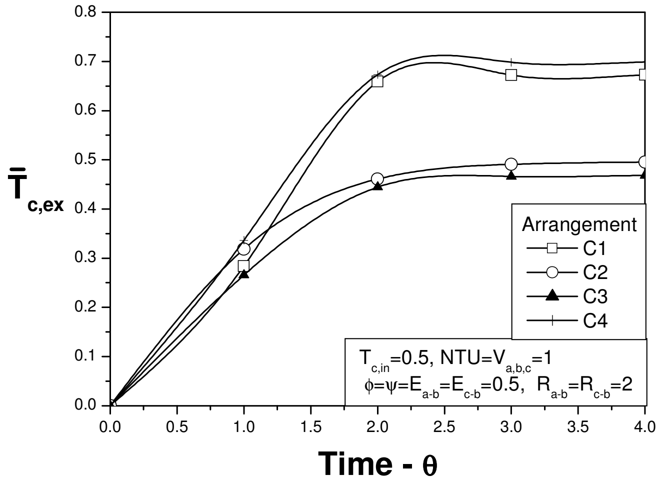


(a)



(b)

Fig.9, Mishra, JHT



(c)

Fig.9, Mishra, JHT