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An Overview and Comparison of Four Sequence Generating Methods for Robotic Assembly

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Abstract:
The work presents an appropriate methodology for generation of assembly sequences. Several existing methods are studied and applied on randomly chosen products, which are then used as building blocks for development of a simplified and appropriate methodology for generation of robotic assembly sequences. The developed methodologies are validated logically. The suitability of these methods with respect to various aspects of robotic assembly is examined and the appropriate one is selected for use. The outcome of the present work is poised to make the robotic assembly system more efficient and flexible.

1. Introduction

It is known that, on the average, assembly cost accounts for 10 – 30% of the total cost of most industrial products [1], and thus reducing the assembly costs may significantly reduce total cost of a product. Consequently, much research effort has been made in enhancing assembly-system performance, either by investigating Design for assembly (DFA) [2] or through in-depth study of the assembly sequence itself [3]. Robotic assembly system is programmable and hence provides a cost effective solution for the assembly tasks. However, a product is assumed to be suitable for robotic assembly, when it is composed of rigid parts interconnected with each other in mutually orthogonal directions. An assembly sequence needs to be carefully determined in order to produce the best output in terms of throughput, feasibility and convenience in the context of robotic assembly. Essentially, generation of robotic assembly sequence involves two major issues; i) determination of assembly directions to avoid geometric interference and physical instability during part connections, and ii) determination of an assembly sequence satisfying the extracted assembly directions while minimizing assembly cost and time for a given assembly task. The present work addresses different methodologies for determination of assembly sequences and then these methods are critically studied on some example products.

Several methods have been developed for generating assembly/disassembly sequences by a number of researchers. Most of these methods follow certain distinct steps to find out the correct assembly sequences. Kuo [4] determined the disassembly sequences that can be advantageously used for obtaining the assembly sequence. The determination of disassembly sequences in the said method involves representation of relationship of components of the products by a precedence matrix, application of graph theoretic approach to split the complete product to multiple subassemblies, evaluation of the disassembly sequences and finally construction of a disassembly tree. In order to identify the precedence relation, several methods depending on the query-and-answer [5] about the precedence relations have been reported. The precedence relationship is used for the generation of assembly sequences. An approach has been suggested by Park and Chung [6] that determines parallel assembly sequences by separating mating parts/subassemblies from a target product without violating the constraints. Santochi and Dini [7] suggested the method for detection of possible subassembly in a product by following certain rules for the generation of sequences for each subgroup and for the whole product from a proper analysis of the connections among the elements. In another approach [8], the product is described through three matrices representing the interference, the contact and the connection amongst mating parts. These matrices are then used for the determination of disassembly sequences. Shpitaini, Elber et al. [9] presented a method in which the target product is represented by connectivity graph (CG) for the generation of disassembly sequences. The CG shows the precedence relationship amongst the parts. The node (part) in the connectivity graph having no precedence is called sink or free node. The decision regarding the sequence and direction are then made based upon traversing the connectivity graph for removal of sink nodes. Once a free node is removed the connectivity graph is updated. The same procedure is repeated till the connectivity graph becomes an empty. Genetic algorithm (GA) has also been used for directly generating the assembly sequence of a product [10] using the biological principles of selection and inheritance to converge towards a complete structure from the information on the components. The use of GA gives rise to optimal or suboptimal solution to the problem.

2. Assembly sequence generating methods

An assembly task is defined as an action, which joins one subassembly (or part) to another subassembly (or part) to produce a larger subassembly or the final product. The problem of sequencing has a primary role in the development of computer-aided assembly planning system. Several methods [3,5,6,9,11] have been developed and tested to generate feasible sequences. Many of the methods use soft computing tools for generating assembly sequences that give optimal sequences. However these methods are based on large number of assumptions or trivial data during the process of generation. The present work aims at finding feasible and stable sequences that can be accomplished by a

robot. Once assembly constraints have been inferred, assembly sequences satisfying the assembly constraints can be generated. Such assembly sequences are called the feasible assembly sequences. The feasible assembly sequences, however, do not always guarantee the parts to fix onto an in-process subassembly, parts may be loosely connected, and come apart when the subassembly is turned or moved. Such assembly sequences that keep the stability of in-process subassembly movement are called the stable sequences, by which the parts can be successfully assembled to form an end product.
The methods under study in the present work are: i) Constraint method, ii) Connectivity Graph method, iii) Liaison method, and iv) Matrix method. These methods are common, conventional and easy to use. Since these methods use different principles for generating assembly/disassembly sequences their applicability and capability are different.

2.1 Constraint method

The method uses two assembly constraints, viz. 'G' constraint and 'C' constraint. Once constraints for each part of the product are acquired, assembly sequences for the product are generated by recording all assembly tasks that do not violate the assembly constraints. The procedure followed in this method for the generation of assembly sequences is summarized as follows:

Step 1: Study the product.
Step 2: List all the individual parts of the product and put them in a single set called master set.
Step 3: Construct the liaison diagram.
Step 4: Determine the 'G' constraints for each part in the product.

'G' constraint: The assembly constraints caused by the geometry of parts are called 'G' constraint, i.e. if a part blocks path in any direction during its removal, then the part has got 'G' constraint.

Step 5: Determine the 'C' constraint for each part.

'C' constraint: The assembly constraint caused by the contact coherence is called 'C' constraint. 'C' constraint is determined from the liaison diagram of the product. A part is said to have 'C' constraint if during the removal of part, its neighboring parts get disconnected from the liaison diagram.

Step 6: Remove a component/subassembly(C/S) from the master set which is not having any constraints. The C/S which is removed is placed at the beginning of the disassembly set in ordered manner. In case, multiple C/S are free from constraints at same point of time, the C/S are removed in parallel and are arranged in separate disassembly set.
Step 7: Update the master set.
Step 8: Repeat step 6 and 7 until the successive master set is empty.

The whole disassembling process is represented as a directed graph, which can be edited and used for finding only the feasible and stable assembly sequences.

2.2 Connectivity Graph (CG) method

Connectivity graph is a directed graph representing the connection relationship of all the parts of the product. The free part is a node with only incoming arrows but no outgoing arrows. This node has no constraint and also its removal will not cause instability. However, the constrained part is a node with either only outgoing arrows or both incoming and outgoing arrows. The method is described in the following steps:

Step 1: Study the product
Step 2: Construct of CG in +Z direction. It is a directed graph. The preferred direction for disassembling is +Z axis since the robotic assembly is facilitated by vertical assembly operation and hence the directions of +X, -X,+Y and –Y are of lower priorities. This graph shows the removal of parts in +Z direction in sequential order.

Direction of arrows in the CG: If the removal of any part(s) is obstructed by the presence of any other part(s) the arrow is directed towards the later part(s) and if there is no obstruction faced by the part for its removal, arrow is directed towards the former part.

Step 3: Remove free nodes sequentially from +Z axis connectivity graph. Once the parts are removed, the CG is updated after removing the free nodes.
Step 4: If none of the parts are further disassembled along +Z axis, the method looks into the CG for the remaining parts along +X-axis. Positive axis graph shows the removal of parts in +X direction in sequential order.
Step 5: If, some parts are left to be removed, then look into the CG drawn for the remaining parts along -X-axis. This graph shows the removal of parts in -X direction in sequential order.
Step 6: If, however, some parts are still left to be removed, repeat step 3 through step 5 until the CG is completely empty.
Step 7: Store the parts removed sequentially to generate disassembly sequence

The reverse of the disassembly sequence is known as assembly sequence.

2.3. Liaison method

Liaison sequence analysis is a systematic way to generate all the feasible assembly sequences for a product. Steps involved in this method are as follows:

Step 1: List all the individual parts of the product and put them in a single set called master set.

Step 2: Draw the liaison diagram. This shows connections between the parts. The connecting lines between the parts are known as the liaisons. List all the liaisons and number them.

Step 3: Find out the precedence relationship amongst parts. This is obtained by answering the following questions:

(a) What liaison must be done before doing L_i, the i^{th} Liaison?

(b) What liaison must be done after doing L_i?

Step 4: Write precedence relations from the answers.

Step 5: Use the relations to generate a network of feasible sequences.

Step 6: Edit the resulting network and discard those sequences which are difficult to assemble and are unstable in order to obtain a few good sequences.

The procedure adopted for representing the liaison sequences in a graphically can be detailed as;

1. Generation of sequences starts with the 0^{th} rank where no liaisons have been established, the establishment of 1^{st} rank starts with the independent liaison i.e. the one having no precedence.
2. For each possible first liaison, one has to explore all possible subsequent states by scanning the liaison list, the precedence relations and other constraints imposed on the assembly. In this way 2^{nd} rank is established.
3. There will be as many ranks as parts.
4. No two same states are allowed in a state transition.
5. A single liaison is established per state transition in case of L = (n-1) and in case of L > (n-1), specific state transitions involves establishing two or more liaisons, where L= number of liaisons and n = number of parts.
6. If addition of one part forms one loop, two liaisons are added and if addition of one part forms two loops, three liaisons are added in the state transition and so on.

2.4 Matrix method

The method uses three matrices viz. interference matrix, connection matrix, and contact matrix for modeling the product. These matrices are then utilized as inputs for the generation of assembly sequences. The steps involved in this method are summarized as follows:

Step 1: The matrices viz. connection matrix, interference matrix, and contact matrices are constructed by analyzing the product.

Interference matrix: It is that square matrix of order 'n' where a_{ij} = 1, If the element e_i interferes with the element e_j during the translation along the direction +k, otherwise a_{ij} = 0. As a convention a_{ii} is always equal to zero.

Connection matrix: The connection matrix C_k of a product formed by 'n' elements $e_1, e_2, \ldots, e_n$, in that square matrix of order 'n' where each c_{ij} assumes a numerical code, function of the kind of connection existing between the elements e_i and e_j along the direction 'K' (Table-1). For example: If the connection between e_i and e_j is a threaded one and the component e_i is removed from the element e_j along 'k' direction, then a numerical value is assigned to C_{ij} as +1. If element e_i cannot be removed from the element e_j which is connected through thread, then $C_{ij} = -1$.

Table1: Values of C_{ij} for various connections

Connection between elements i and j	Dis-assemblablity of element i	C_{ij}
Threaded connection	yes	1
Threaded connection	no	-1
Drive fit	yes	2
Drive fit	no	-2
Elastic ring	yes	3
Elastic ring	no	-3
O – ring	yes	4
O – ring	no	-4
Absence of the previous connection	yes	0

Contact matrix: The contact matrix, B_k, of a product formed by 'n' elements e_1, e_2, $e_3, \ldots, e_n$, is that square matrix of order 'n' where $b_{ij} = 1$, if the element e_i is in contact with the element e_j along the direction +k, otherwise $b_{ij} = 0$. As a convention, b_{ii} is always equal to zero. A component is in contact with other elements along a direction, means that the elements that physically avoid the displacement of a component along the mentioned direction.

Step 2: If the product contains subassembly, then another matrix called contracted matrix is drawn.

Contracted matrix: Let A_k be the interference matrix of a product of element $e_{1,}$, $e_2, \ldots, e_n$ and let 'h' be a set of element $S_{1,}$, $S_2, \ldots, S_{m.}$ Contracted matrix $A^{\bullet}_k$ is defined as the order (n-m+1) obtained from A_k considering the previous set as a single element that cannot be disassembled.

Step 3: Construct the interference/contracted matrix and connection matrix in +Z axis. Remove a part/subassembly whose elements contains all zero in the interference/contracted matrix and none of the elements is negative corresponding to selected part in the connection matrix.

Step 4: Update the interference/contracted matrix as well as connection matrix.

Step 5: Repeat step 3 to 4 until the interference/contracted matrix is completely empty or further disassembly is not possible along +Z-axis.

Step 6: Repeat step 3 to 4 in +X-axis direction for the remaining parts.

Step 7: Stop, if the interference/contracted matrix is completely empty or further disassembly is not possible along +X-axis direction.

Step 8: Repeat step 3 to 4 in -X-axis direction for the remaining parts.

Step 9: Stop, if the interference/contracted matrix is completely empty or further disassembly is not possible along -X-axis direction.

Step 10: If, however, some parts are still left to be removed, then repeat step 2 through step 9 until the interference/contracted matrix is completely empty.

3. Case Study

The four methods for generating assembly sequences are studied by applying them on four example products. The products under consideration are; a) air craft engine component (product 1), b) grinder sub-assembly(product 2), c) automobile engine component (product 3), and d) ball point pen(product 4). The assembly sequences for these example products are generated applying the methodologies described in the previous section.

3.1. The constraint method

The method is applied to four different products. The first product considered here is an air craft engine component(product 1) {Figure 1(a)} consists of four parts viz. A, B_1, B_2, and C. On examination of the product and its components, it is observed that parts A, B_1, and B_2 have 'G' constraint and none of the parts has 'C' constraint.

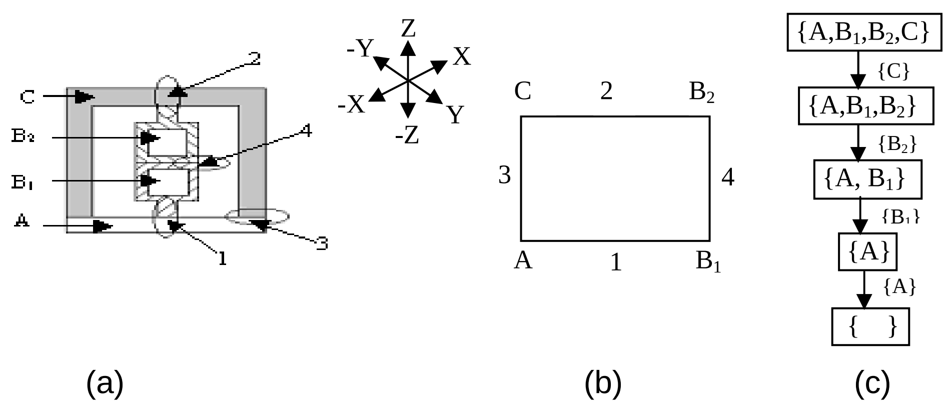


Figure1 (a).Geometrical model of product 1, (b) Liaison diagram of the product, and (c) Disassembly diagram of the product.

Based on the constraints generated and following the laid out procedure, the disassembly sequence is found to be: C – B_2 – B_1 – A as evident from Figure 1(c). Therefore, the assembly sequence is: A – B_1 – B_2 – C.

The second product considered here is a grinder subassembly(product 2) {Figure 2(a)}.

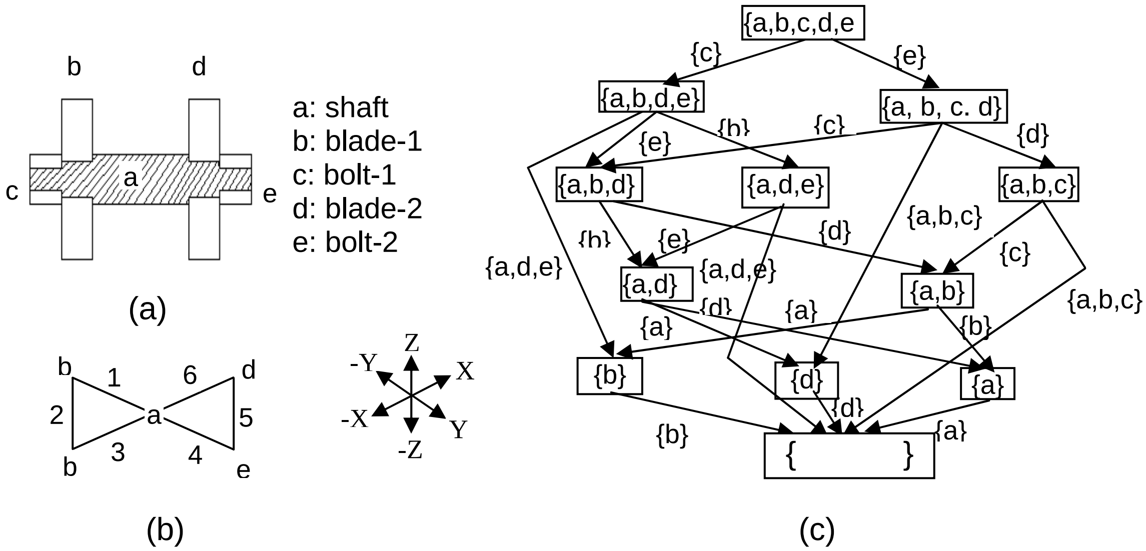


Figure 2(a). Model of product 2, (b) Liaison diagram of the product , and (c) Disassembly diagram of the product.

On inspection and verification of the drawing of the product, it is found that only part ‘b’ and part ‘d’ possess ‘G’ constraint and the part ‘a’ possesses ‘C’ constraint. The product is disassembled by taking out the component or group of components behaving as a single component (subassembly), which does not have any constraints either ‘C’ or ‘G’. This process of identifying the constraints and their disassembling is continued with all possible alternatives till every component is disassembled. The process is represented by Figure 2(c).

(a)	(b)
1) c – {a ,d ,e} – b	1) b – {a, d, e} – c
2) c – e – b – a – d	2) d – a – b – e – c
3) c – e – b – d –a	3) a – d – b – e – c
4) c – e –d –a – b	4) b – a – d – e – c
5) c – e – d – b – a	5) a – b – d – e – c
6) c – b – e – a – d	6) d – a – e – b – c
7) c – b – e – d – a	7) a – d – e – b – c
8) c – b – {a , d, e}	8) {a, d ,e} – b – c
9) e – c – b – a – d	09) d – a – b – c – e
10) e – c – b – d – a	10) a – d – b – c – e
11) e – c – d – a – b	11) b – a – d – c – e
12) e – c – d – b – a	12) a – b – d – c – e
13) e – { a , b, c } –d	13) d – {a, b, c} – e
14) e – d – c – a – b	14) b – a – c – d – e
15) e – d – c – b – a	15) a – b – c – d – e
16) e – d – { a , b, c }	16) {a, b, c } – d – e

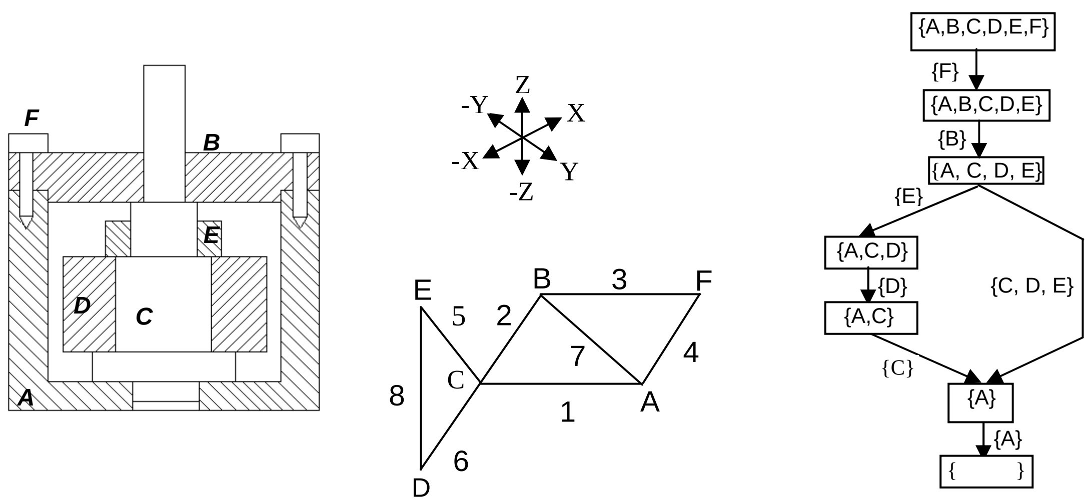
Figure 3(a). Disassembly sequences and (b) Assembly sequences of product 2.

The assembly process should start with a base part and others are to be mated in predefined manner to build the complete assembly. However, the assembly sequences mentioned in Sl. no. 1, 2, 4, 6, 9, 11, 13 and 14 are not convenient, as they do not start with base part ‘a’. (The base part is chosen using the criteria of maximum number of mating links, maximum volume, maximum weight etc.). Therefore, rejecting the sequences in the aforementioned sequences, the convenient sequences in the context of robotic assembly are shown in Fig.3(c)

i.	a – d – b – e – c
ii.	a – b – d – e – c
iii.	a – d – e – b – c
iv.	{a, d ,e} – b – c
v.	a – d – b – c – e
vi.	a – b – d – c – e
vii.	a – b – c – d – e
viii.	{a, b, c } – d – e

Fig.3(c) Feasible and stable assembly sequences for product 2.

The third product considered here is an automobile engine component (product 3) {Figure 4(a)}, and it consists of six parts viz. A, B, C, D, E and F.



(a) (b) (c)

Figure 4 (a). Geometric model, (b) Liaison diagram, and (c) The disassembly diagram of product 3.

On application of procedure on the entire product for acquiring the ‘G’ and ‘C’ constraints, it is observed that parts A, B, C, D, and E have ‘G’ constraint and ‘B’ and ‘C’ have ‘C’ constraint. The associated disassembly diagram is shown in Figure 4(c). The feasible disassembly sequences are:

1) F – B – E – D – C – A
2) F – B – {C, D, E} – A

Hence, the corresponding assembly sequences are:

1) A – C – D – E – B – F
2) A – {C, D, E} – B –F

The fourth product considered is a ballpoint pen(product 4) consisting of six parts {Figure 5(c)}.

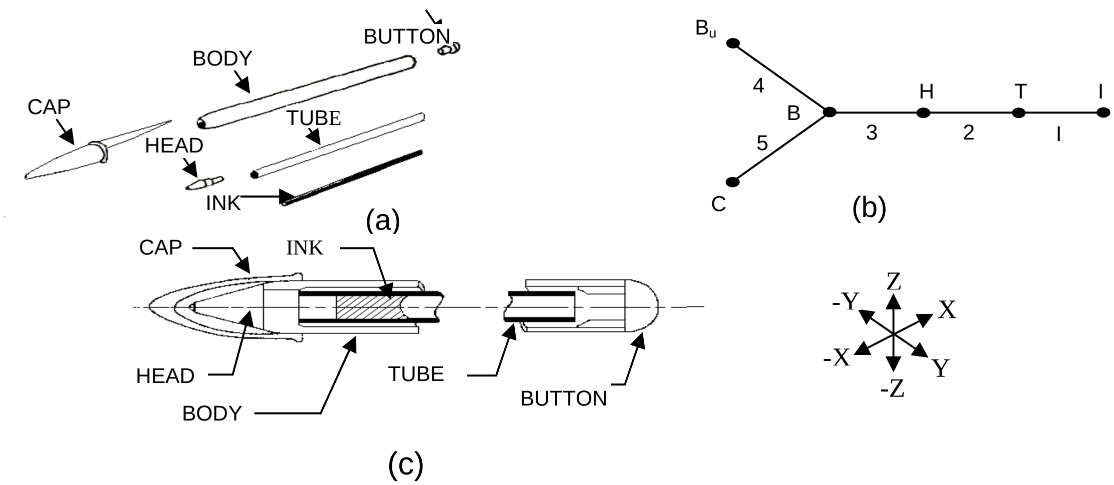


Figure 5(a).Components of the product 4, (b) Liaison diagram the product, and (c) Complete assembled product.

On inspection and verification of the drawing of the product, it is found that parts H, B, T and I have ‘G’ constraint and Parts H, B, and T have ‘C’ constraint. The disassembly structure of the product is shown in Figure 6.

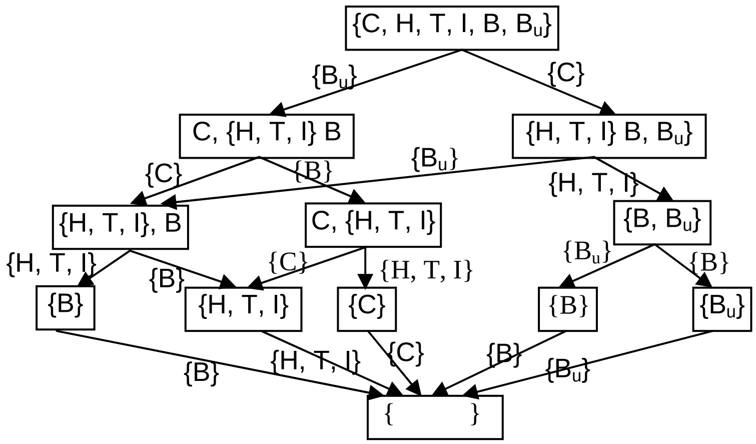


Figure 6. The disassembly structure product 4.

The disassembly sequences are found by traversing the disassembly diagram and are presented in the Figure 7(a) and the corresponding assembly sequences are given in the Figure 7(b).

1. B_u – C – {H, T, I} – B
2. B_u – C – B – {H, T, I}
3. B_u – B – {H, T, I} – C
4. B_u – B – C – {H, T, I}
5. C – {H, T, I} – B_u – B
6. C – {H, T, I} – B – B_u
7. C – B_u – B – {H, T, I}
8. C – B_u –{H, T, I} – B

(a)

1. B – {H, T, I} – C – B_u
2. {H, T, I} – B – C – B_u
3. C – {H, T, I} – B – B_u
4. {H, T, I} – C – B – B_u
5. B – B_u – {H, T, I}– C
6. B_u – B – {H, T, I}– C
7. {H, T, I} – B – B_u– C
8. B– {H, T, I} – B_u –C

(b)

Figure 7(a).Disassembly sequences of product 4, (b) The corresponding assembly sequence.

3.2 Connectivity graph (CG) method.

The method is applied to the same set of products for generating the necessary assembly sequences.Considering the product 1 in Figure 1, the +Z connectivity graph is drawn on priority basis as shown in Figure 8.It is verified from this figure that ‘C’ is the sink node as there are no outgoing arrows from it [8]. Therefore, part ‘C’ is removed first. Further removal of parts are carried out in the order B_2, B_1, and A successively. Hence, the disassembly sequence is found to be C – B_2 – B_1 – A and therefore, assembly sequence is: A – B_1 – B_2 – C.

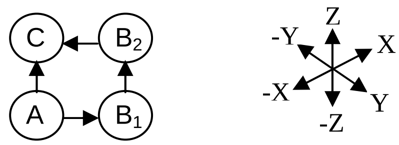
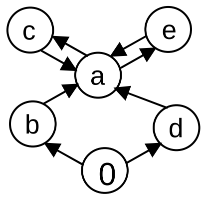
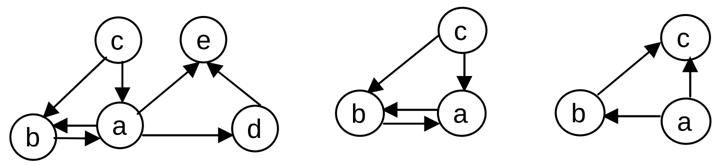


Figure 8. Z-axis CG product 1.

Considering product 2 in Figure 2(a), the Z-connectivity graph is drawn as in Figure 9. Since none of the parts in +Z-axis CG is a sink node, the disassembly is not possible in the + Z direction.





(a) (b) (c)

Figure 9.+Z CG of product 2.

Figure10 (a). +X-axis CG, (b) +X-axis CG after the removal of parts ‘d’ and ‘e’ and (c) -X axis CG of the parts ‘a’, ‘b’, and ‘c’.

Therefore, connectivity graph in the positive direction of X axis is drawn on next priority basis as shown in Figure 10(a). Initially, part ‘a’ and then part ‘e’ are removed and the modified graph is drawn {Figure 10(b)}. Absence of any sink node in Figure 10(b) restricts further removal of parts in the positive direction of X-axis. Therefore, connectivity graph is drawn in the negative direction of X-axis on the basis of next priority as shown in Figure 10(c). It is evident from the Figure 10 (c) that part removal can be made in the order c, b, and a successively. Therefore the disassembly sequence is: e – d – c – b – a, and the assembly sequence is found to be a –b – c – d – e.

Considering the product 3 in Figure 4 (a), the Z-connectivity graph is drawn on priority basis (Figure 11).

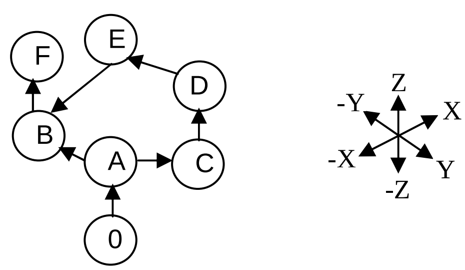
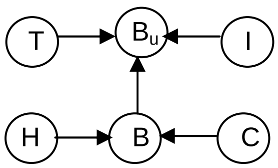
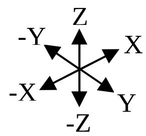


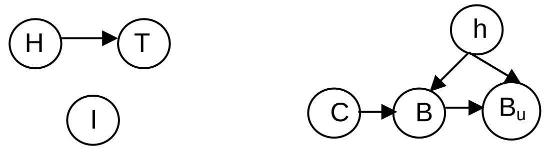
Figure 11. +Z axis CG of product 3.

Sink node ‘F’ is removed first and this causes part ‘B’ to become sink node, which is removed next. Likewise, other parts are removed successively in the order ‘E’, ‘D’, ‘C’, and ‘A’. The disassembly sequence is found to be F – B –E – D – C – A and therefore, the assembly sequence is: A – C – D – E – B – F.

Considering product 4 in Figure 5(c), it is observed that disassembly is not possible in + Z-axis. Therefore, +X- axis connectivity graph is drawn on next priority (Fig. 12)





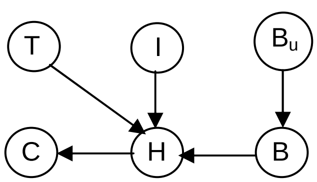


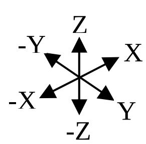
(a) (b)

Figure12.+X axis CG of product 4.

Figure 13 (a). +X axis CG of subassembly {H,T, I}, and (b) +X- axis CG of subassembly ‘h’ and remaining parts of the product.

The connectivity graph of the head, tube and the ink, which forms a stable subassembly, is shown in the positive direction of X-axis in Figure 13(a). The disassembly sequence for the above subassembly is found to be I –T – H and the corresponding assembly sequence is H –T – I which is found to be suitable. Considering the subassembly {H, T, I} to be ‘h’, the CG of the subassembly ‘h’ and the remaining parts of the product is drawn in the positive direction of X-axis as shown in Figure 13 (b). On analysis of the above diagram, the disassembly sequences are determined to be i) B_u – B – C – h, and ii) B_u – B – h – C and the assembly sequences are; i) h – C – B –B_u, and ii) C – h – B –B_u. Practically, none of the above sequences are suitable. Since, once the part C is engaged with the subassembly ‘h’, it is not possible for the subassembly ‘h’ to be mated with body ‘B’. In order to overcome this difficulty, the -X connectivity graph of the product is drawn on the next priority basis (Figure 14)





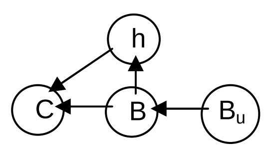


Figure 14.-X axis CG of the product 4.

Figure15. –X axis CG of the Subassembly ‘h’ with the rest of the parts of the product.

The connectivity graph of the subassembly ‘h’ with the rest of the parts is drawn in the negative direction of X-axis as shown in Figure 15. On verification the above diagram, the disassembly sequence is C –h – B – B_u and the assembly sequence is: B_u– B – h – C = B_u– B – {H– T – I} – C

3.3 Liaison method

On analysis of product 1 and its liaison diagram in Figure 1(a) and 1(b) respectively, it is found that liaison to be done before L_i are:

$i = 3 \Rightarrow 1$ and 4 or 2 and 4 or 1 and 2 $\rightarrow$ 3 (The symbol '$\rightarrow$' is read "must precede") and liaison to be done after L_i are:

$i = 1 \Rightarrow 1 \rightarrow 3$; $i = 2 \Rightarrow 2 \rightarrow 3$; $i = 4 \Rightarrow 4 \rightarrow 3$

The precedence relations are summarized as follows:

1 and 4 or 2 and 4 or 1 and 2 $\rightarrow$ 3, Independent liaisons are 1, 2 and 4

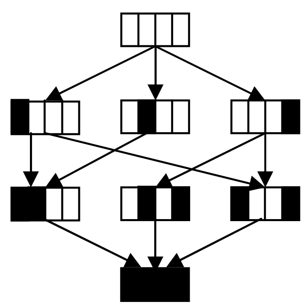


Figure 16. Graphical representation of liaison sequences for product 1.

The sequences of liaison obtained from this are:

1) 1 – 2 – 3 – 4 2) 1 – 2 – 4 – 3 3) 1 – 4 – 2 – 3 4) 1 – 4 – 3 – 2
5) 2 – 1 – 3 – 4 6) 2 – 1 – 4 – 3 7) 4 – 1 – 2 – 3 8) 4 – 1 – 3 – 2
9) 4 – 2 – 1 – 3 10) 4 – 2 – 3 – 1

However, on verification of the sequences of liaison obtained Vis-à-vis the part connectivity of the product, it is observed that:

i) it is not suitable to move forward with the process of assembly after part A is mated with part C (liaison 3)
ii) it is not suitable to proceed further with the assembly process after combination of liaison 1and liaison 2

(Liaison 1 signifies, mating of part A with part B_1, and liaison 2 signifies mating of part B_2 with part C).Therefore, the feasible and convenient liaisons and the corresponding sequences of assembly of the parts are:

i) 1 – 4 – 2 – 3 $\Rightarrow$A – B_1 – B_2 – C; ii) 2 – 1 – 4 – 3$\Rightarrow$C – B_2 –B_1 –A

On analysis of the drawing of Product-2 in Figure 2(a) and its liaison diagram in Figure 2(b), it is observed that the precedence of liaisons are:

$i = 2 \Rightarrow 1 \rightarrow 2$

$i = 3 \Rightarrow 1 \rightarrow 3$

$i = 4 \Rightarrow 6 \rightarrow 4$

$i = 5 \Rightarrow 6 \rightarrow 5$

and the succeeding liaisons are:
i = 1 ⇒ 1 → 2
i = 4 ⇒ 4 → 5
Liaisons 1and 6 are Independent liaisons

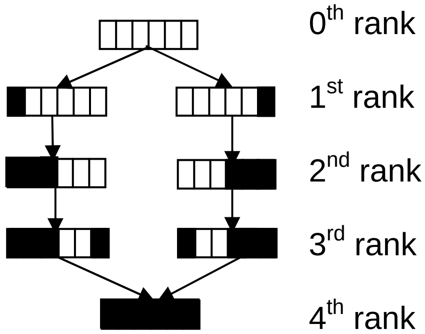


Figure 17. Graphical representation of all valid liaison sequences for product 2.

Table 2: Additional constraint resulting in assembly without plurality of unconnected sub- assembly

Established Liaison	Prospective next Liaison
1	2,3,4,6
2	1,3
3	1,2,4
4	1,3,5,6
5	4,6
6	1,3,4,5

It is evident from the diagram that there are 6^{th} paths from 0^{th} rank to 4^{th} rank. The liaison sequences and the corresponding sequences of assembly of parts are as follows:
1) 1 – 2 – 3 – 6 – 5 – 4 ⇒ a – b – c – d – e
2) 1 – 2 – 3 – 6 – 4 – 5 ⇒ a – b – c – d – e
3) 6 – 5 – 4 – 1 – 2 – 3 ⇒ a – d – e – b – c
4) 6 – 5 – 4 – 1 – 3 – 2 ⇒ a – d – e – b – c
5) 6 – 4 – 5 – 1 – 2 – 3 ⇒ a – d – e – b – c
6) 6 – 4 – 5 – 1 – 3 – 2 ⇒ a – d – e – b – c

Ignoring the repetitions, the sequences for assembly are:

i) a – b – c – d – e, and ii) a – d – e – b – c.

Applying this method for determination of assembly sequences for product 3 (Figure 4), it is observed that precedence of liaisons is:

i = 2 ⇒1 and 5 and 6 → 2
i = 3 ⇒1or 2 → 3
i = 4 ⇒3 and 7 → 4
i = 5 ⇒ 6 → 5
i = 7 ⇒1 and 2 and 5 and 6 → 7
i = 8 ⇒ 5 and 6→8
and the succeeding liaisons are :
i = 1 ⇒ 1 → 7
i = 2 ⇒ 2 → 3
i = 5 ⇒ 5 → 2

i = 6 ⇒ 6 → 5
i = 7 ⇒ 7 → 4
i = 8 ⇒ 8 → 7
Summary of the precedence relationship:
1 and 5 and 6 → 2
7 → 4
6 → 5
1 and 2 and 5 and 6 → 7
2 → 3, and liaisons 1 and 6 are independent.

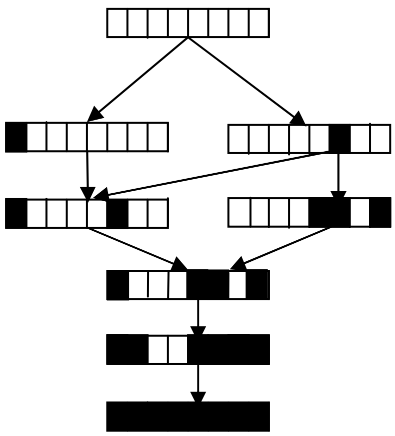


Figure 18. Graphical representation of liaison sequences for product 3.

Table 3: Additional constraint resulting in assembly without plurality of unconnected sub- assembly

Established liaison	Prospective Next liaison
1	2,4,5,6,7
2	1,3,5,6,7
3	2,4,7
4	1,3,7
5	1,2,6,8
6	1,2,5,8
7	1,2,3,4
8	5,6

On verification from Figure18, the liaison sequences and the corresponding sequences of assembly of parts are as follows:

1) 1 – 6 – 5 – 8 – 2 – 7 - 3 – 4 2) 6 – 1 – 5 – 8 –2 – 7 – 3 – 4

3) 6 – 5 – 1- 8– 2 – 7 – 3 – 4 4) 6 – 8 - 5 – 1 – 2 – 7 – 3 – 4

5) 6 – 5 – 8 -1 – 2 – 7 – 3 – 4 6) 6 – 8 - 5 – 1 – 2 – 7 – 3 – 4

Ignoring the repetitions, the sequences for assembly are found as follows:

i) 1 – 6 – 5 – 8 – 2 – 7 - 3 – 4 ⇒ A – C – D – E – B – F

ii) 6 – 1 – 5 – 8 –2 – 7 – 3 – 4 ⇒ C – D – A – E – B – F

iii) 6 – 5 – 8 -1 – 2 – 7 – 3 – 4 ⇒ C – D – E – A – B – F

Considering the product 4 in Figure 5 (c) and its liaison diagram in Figure 5(b), it is observed that the precedence of liaison are:

i = 1 ⇒ 2 → 1

i = 5 ⇒ 3 → 5

and the succeeding liaisons are:

i = 1 ⇒ 1 → 4 and 5

i = 2 ⇒ 2 → 4 and 5 i.e. 1 or 2 → 4 and 5

i = 3 ⇒ 3 → 5

Precedence relationships are summarized as follows: 2 → 1, 3 – 5, 1 or 2 → 4 and 5, and liaisons 2, 3, 4 are independent liaisons

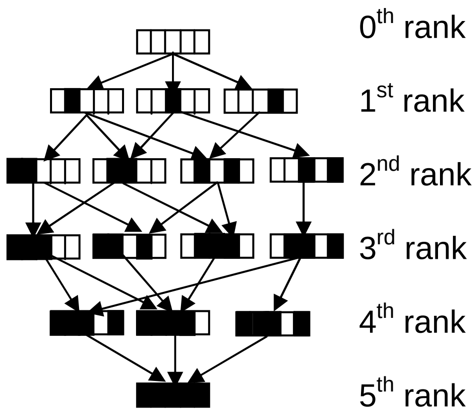


Table 4: Additional constraint resulting in assembly without plurality of unconnected sub- assembly

Established Liaison	Prospective Next Liaison
1	2
2	1,3
3	2,4,5
4	3,5
5	3,4

Figure 19. Graphical representation for product 4.

On verification the diagram in Figure19, the following liaison sequences are obtained.

1) 2 – 1 – 3 – 5 – 4; 2) 2 – 1 – 3 – 4 – 5; 3) 2 – 1 – 4 – 3 – 5;
4) 2 – 3 – 1 – 5 – 4; 5) 2 – 3 – 1 – 4 – 5; 6) 2 – 3 – 4 – 1 – 5;
7) 2 – 3 – 4 – 5 – 1; 8) 2 – 4 – 3 – 1 – 5; 9) 2 – 4 – 3 – 5 – 1;
10) 2 – 4 – 1 – 3 – 5; 11) 3 – 2 – 1 – 5 – 4 12) 3 – 2 – 4 – 1 – 5;
13) 3 – 2 – 4 – 5 – 1; 14) 3 – 5 – 2 – 1 – 4; 15) 3 – 5 – 2 – 4 – 1;
16) 4 – 2 – 1 – 3 – 5; 17) 4 – 2 – 3 – 1 – 5;

However, on verification of the sequences of liaison obtained and the part connectivity of the product, it is observed that the assembly sequences mentioned in Sl. no.4 through Sl no.10 and Sl.no.15 are not convenient since it is difficult to put ink after the tube is mated with the head while head is inside the body. Further, the assembly sequences mentioned in Sl. no. 11 – 14 are not accepted since it is difficult to assemble the tube with the head while the later part is already mated with the body. The assembly sequence in Sl.16 is also not considered, as ink cannot be put in the tube unless head is fitted to tube.

Rejecting all these infeasible sequences, the feasible liaison sequences and the corresponding sequences of assembly of parts are as follows:

i) 2 – 1 – 3 – 5 – 4 ⇒ T – H – I – B – C – B_u
ii) 2 – 1 – 3 – 4 – 5 ⇒ B_u – B – {H - T – I} – C
iii) 2 – 1 – 4 – 3 – 5 ⇒ T – H – I – B – Bu – C
iv) 4 – 2 – 3 – 1 – 5 ⇒ Bu – B – T – H – I – C

3.4 Matrix method.

The matrix method [7] is applied to the same set of products to find out the assembly sequences. Considering product 1, the interference matrix for the product in the positive direction of Z-axis is presented in Figure 20

		A	B_1	B_2	C
	A	0	1	1	1
A_Z =	B_1	0	0	1	1
	B_2	0	0	0	1
	C	0	0	0	0

Figure 20. Interference matrix for product 1.

From the corresponding interference matrix, it is evident that part ‘C’ can be removed first.Further removal of parts take place in the order B_2 , $B_{1,}$ and A succcessively.The disassembly sequence is determined to be C-B_2-B_1-A. Therefore, the assembly sequence is: A-B_1-B_2-C.

Considering the product 2, it is observed that the product cannot be disassembled in +Z direction. Therefore, the connection matrix and the interference matrix for the product are drawn in the positive direction of X-axis as shown in Figure 21 and 22 respectively.

		a	b	c	d	e
	a	0	-2	-1	-2	-1
	b	2	0	0	0	0
$C_{(X)}$ =	c	1	0	0	0	0
	d	2	0	0	0	0
	e	1	0	0	0	0

Figure 21. Connecion matrix.

		a	b	c	d	e
	a	0	0	0	1	1
	b	1	0	0	1	1
$A_{(+x)}$ =	c	1	1	0	1	1
	d	0	0	0	0	1
	e	0	0	0	0	0

Figure 22. Interference matrix.

The connection matrix shown in Figure 21 allows the part ‘e’ to be disconnected as none of the elements corresponding to part ‘e’ are negative and it is observed from the interference matrix shown in (Figure 22) that part ‘e’ does not interfere with any parts as the row corresponding to part ‘e’ contains all zeros. Therefore, the part ‘e’ can be removed from the interference matrix by deleting row and column corresponding to part ‘e’. The removal of part ‘e’ makes the part ‘d’ free. The part ‘d’ is then removed. The matrix is then updated and shown in Figure 23(a). Although the row corresponding to part ‘a’ in the matrix shown in Figure 23(a) contains all zeros, part ‘a’ cannot be removed in the positive direction of X-axis since the second condition of disassemblablity is not satisfied (i.e. all the elements in the row corresponding to the part ‘a’ is negative as shown in the connection matrix in Figure 21). Considering the generation of sequences for the subassembly from the negative direction of X-axis shown in Figure 23(b), part ‘c’ can be removed since both the conditions of part disassemblablity are satisfied as evident from its interference matrix and the connection matrix. Further, disassembling of parts takes place in the order b, a successively. The disassembly sequence is e – d – c – b – a and the assembly sequence is: a – b – c – d – e.

	a	b	c
a	0	1	1
b	0	0	1
c	0	0	0

(a)

	a	b	c
a	0	0	0
b	1	0	0
c	1	1	0

(b)

Figure 23 (a). Updated matrix after the removal of parts‘d’ and ‘e’, (b) Interference matrix of subassembly {a b c} in -X direction.

Considering product 3, the interference matrix for the product is prepared in +Z-axis and presented in Figure 24.

		A	B	C	D	E	F
	A	0	1	1	1	1	1
	B	0	0	0	0	0	1
$A_{(+x)}$ =	C	0	1	0	1	1	0
	D	0	1	0	0	1	0
	E	0	1	0	0	0	0
	F	0	0	0	0	0	0

Figure 24. Interference matrix in +Z axis.

It is evident from the interference matrix that part ‘F’ can be removed first. Then the interference matrix is modified by deleting row and column corresponding to part ‘F’ .Then part ‘B’ is removed. Again the matrix gets updated. This leads to removal of another part. In this process the entire product gets disassembled. The disassembly sequence is F – B – E – D – C – A and the assembly sequence for the product is: A – C – D – E – B – F.

Considering product 4, the interference matrix is given in Figure 25.

		I	T	H	B	B_u	C
	I	0	0	1	0	0	1
	T	0	0	1	0	0	1
$A_{(-X)}$ =	H	0	0	0	0	0	1
	B	0	0	1	0	0	1
	B_u	1	1	1	1	0	1
	C	0	0	0	0	0	0

Figure 25.Interference matrix of product 4.

		I	T	H
	I	0	0	0
$A_{h(X)}$ =	T	0	0	0
	H	1	1	0

Figure 26.Iinterference matrix of subassembly ‘h’ of product 4.

Considering the head, tube and ink form a subassembly ‘h’, its interference matrix is drawn in the +X-axis (Figure 26). The disassembly sequences of the subassembly ‘h’ are found to be; 1) I-T-H and 2) T-I-H. Therefore, assembly sequences are; 1) H-T-I and 2) H-I-T. The assembly sequence, H-T-I, is found to be most appropriate since the other assembly sequence can not be accepted as mating of head with the ink is not feasible. The contracted matrix is constructed along the +X-axis considering the subassembly {H-T-I} to be ‘h’ (Figure 28).

		h	B	B_u	C
	h	0	1	1	0
$A_{(+X)}$ =	B	0	0	1	0
	B_u	0	0	0	0
	C	1	1	1	1

Figure 27.Contracted matrix along +X-axis.

		h	B	B_u	C
	h	0	0	0	1
$A_{(-X)}$ =	B	1	0	0	1
	B_u	1	1	0	1
	C	0	0	0	0

Figure 28.Contracted matrix along –X-axis.

On verification of the contracted matrix (Figure 28), the parts are removed in the order B_u – B – h – C and corresponding assembly sequence is: C – h – B – B_u. The generated sequence is not suitable, since once the part C is engaged with the subassembly ‘h’, it is not possible for the subassembly ‘h’ to be mated with the body ‘B’. Therefore, the contracted matrix is constructed along the –x-axis on next priority, and is presented in Figure 29. On verification of the contracted matrix, parts are removed in the order C, h, B, and B_u successively. The disassembly sequence is: C – h – B – B_u. Therefore, the assembly sequence is B_u – B – h – C is: B_u – B – {H – T – I} – C which is suitable.

4. Results and discussion

The results obtained by using the four methods for the four products under consideration are presented in Table 5. The results clearly indicate that both Constraint and Liaison method provide multiple solutions. However, the outcome of these methods is quite dependent on type of product under consideration. The detailed comparisons amongst the methods are discussed in the next section.

Table 5: Assembly sequences of the products by the selected methods

Method	Product-1	Product-2	Product-3	Product-4
Constraint	A- B_1- B_2- C	1) a – d – b – e – c 2) a – b – d – e – c 3) a – d – e – b – c 4) a – b – c – d – e 5) a – d – b – c – e 6) a – b – d – c – e 7) {a,d,e} – b – c 8) {a,b,c} – d – e	1) A – C – D – E – B – F 2) A – {C, D, E} – B –F	1) B – {H, T, I} – C - B_u 2) {H, T, I} – B – C – B_u 3) C – {H, T, I} – B – B_u 4) B – B_u – {H, T, I} –C 5) B_u – B – {H, T, I} –C
CG	A- B_1- B_2-C	a – b – c – d – e	A – C – D – E – B – F	B_u – B – {H, T, I} – C
Liaison	1) A-B_1-B_2-C 2) C-B_2-B_1-A	1) a – b – c – d – e 2) a – d – e – b – c	1) A – C – D – E – B – F 2) C – D – A – E – B – F 3) C – D – E – A – B – F	1) T – H – I – B – C – B_u 2) B_u – B – {H, T, I} – C 3) T – H – I – B – B_u – C 4) B_u – B – T – H – I – C
Matrix	A-B_1-B_2-C	a – b – c – d – e	A – C – D – E – B – F	B_u – B – {H, T, I} – C

4.1 Comparison of the methods

The present work considered four different methods for generating assembly sequences in four different products. Although the methods have their own advantages and limitations, a few interesting observations are made with regard to their suitability so far as robotic assembly is concerned.

Constraint method: The constraint method needs to be carefully handled for selection of the constraints and requires a number of iterations for determining the constraints depending upon the number of parts/components in the product assembly. It uses only two parameters and does not depend upon the directions. However, the method has its limitation for being suitable to products with fewer components as it gets choked with large number of sequences in the disassembly diagram. The method has multiple solutions, which further need to be converged to a single solution with additional considerations. The suitability of this method for robotic assembly generation is not clear as it does not indicate the direction of assembling the components.

Matrix method: This method has the capability of being utilized for a large number of applications. Particularly for assembly sequence generation, this is a very useful tool and can cater with products with large number of components. This method is also easier for integration to automation processes and can be built into the robot motion control program. It has good converging characteristics. However, formation of matrices is to be done carefully to get the correct solution.

Liaison method: It uses a logical method through a set of questions that resulted in the desired precedence relationship among the parts. The precedence relationships are used for the generation of assembly sequences. The success of this method depends upon the answers resulting from a pair of queries made on each liaison. The suitability of this method is associated with the products of fewer parts/components. The method gives out multiple solutions.

Connectivity graph (CG) method: It is the simplest of the four to build up the process. The method uses mainly one parameter, sink node, which is easily found in the CG. The process is suitable to products with any number of components and provides likely one optimal solution. The method can be conveniently used for robotic assembly.

5. Conclusion

In designing robotic assembly system, the generation of assembly sequence is a fundamental task because of the fact that the sequence crucially affects the system layout and efficiency. This paper presents our research efforts in developing an appropriate methodology for the generation of robotic assembly sequences. In our approach, four different sequence generation techniques are applied on different product types. The robotic assembly system is a programmable one. It is appropriate to develop method(s) which can be incorporated with the robot motion program thereby improving the efficiency and effectiveness of the system. The present exercise has given out two different methods of generating assembly sequences which can be advantageously used for a robotic assembly system. The methods are;1) Connectivity method, and 2) Matrix method. In connectivity method, the graphs can be represented in matrix form and the matrices so obtained can be used directly as the input for the robot program. In the matrix method the matrices developed for the product in question can be used for giving sequencing information to the robot. Both Connectivity graph method and Matrix method generate very few sequences, and sometimes only one which may be considered to be near optimal and further editing of sequences is hardly necessary. The outcome of the analysis clearly indicates that both Connectivity graph method and Matrix method are suitable for the goal set. However, the draw back in the Connectivity method is that the method is suitable only for the products where no threaded connection amongst the parts exists. On the other hand, Matrix method is a generic method and more appropriate as it can be easily automated and computerized for quick and efficient results.

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