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# Optimal Area Targeting of Heat Exchanger Network Using Nonlinear Programming

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## Abstract

This paper presents a mathematical modeling approach based on Non-Linear Programming (NLP) formulation to compute area targeting of heat exchanger network (HEN). The heat integration is represented by stage wise Superstructure which depends on number of hot and cold streams present in the problem.

The present HEN problem is taken from open literature. The problem involves two hot and two cold streams along with hot and cold utilities. For a specified minimum temperature difference ($\Delta T_{min}$) equal to 20 °C, the hot and cold utility requirements are 605 and 525 kW respectively. For simplicity heat transfer coefficients for all streams are considered to be same in the initial stage of the problem. Then it is varied to see its effect on area targeting. Since there are two hot and two cold streams, the proposed representation for heat integration includes two stage-wise Superstructure. The NLP formulation has six linear constraints in which five are equality and one is inequality constraint. The objective function of area targeting is non-linear. For the present problem, Chen's approximation is used to compute log mean temperature difference.

The NLP formulation is solved using mathematical modeling and optimization software, GAMS, with MINOS solver. It takes 0.063 seconds on an AMD Athlon processor to complete the simulation. The solution indicates an area target equal to 1326.97 $m^2$, which is very close to the value given in the literature. The present model can account for streams with different values of heat transfer coefficients thus can accommodate non-vertical heat transfer.

**Key words:** Area targeting, Heat Exchange Network, Nonlinear Programming, Superstructure.

## Nomenclature

| | |
|---|---|
| C | Cooler |
| H | Heater |
| HX | Heat exchanger |
| $MC_p$ | Heat capacity flow rate, kW/ °C |
| NC | Number of cold streams |
| NH | Number of hot streams |
| *NOK* | Number of stages in superstructure |
| Q | Minimum heat load |
| T | Temperature, °C |
| Superscripts | |
| s | Supply |
| t | Target |
| ‘ | Two stage superstructure |
| ‘’ | Three stage superstructure |
| ‘’’ | Four stage superstructure |
| Subscript | |
| m | 1, 2, 3, 4 and represents stages |

## 1. Introduction

Over the last two decades, Heat Exchanger Network (HEN) synthesis has progressed extensively, particularly in the field of utilization of energy, area and units targets for HEN [1]. The area targeting of HEN was first demonstrated by Linnhoff & Townsend [2] with the application of Pinch Technology [3]. Pinch Technology was a graphical approach for area targeting but with strict restriction of vertical heat transfer. Therefore, Pinch Technology did not consider streams with different values of heat transfer coefficients for feasible match. Gundersen and Grossmann [4] proposed an optimization strategy for automated HEN synthesis, based on strict vertical heat transfer. The methods, discussed above [2], [3] & [4], do not account for different design constraints that can accommodate non-vertical heat transfer. Colberg and Morari [5] presented a NLP transshipment model to target required area of HEN and considered process streams with unequal heat transfer coefficients but used fixed temperature or enthalpy intervals which do not provide flexibility for the design of HEN.

This limitation can be overcome with the help of mathematical modeling approach. For optimal area targeting, a Nonlinear Programming (NLP) model [6] is proposed which can easily relax the assumption of constant heat transfer coefficients and allows for the possibility of non-vertical heat transfer. The mathematical modeling uses stage wise superstructure to target optimal area that is applicable for HEN with streams having fixed flow rates, fixed target & supply temperatures and fixed hot & cold utilities requirements. This model gives exact and accurate solution for the problems and requires very reasonable computation time. The model can also account for different design constraints.

## 2. Problem Statement

A typical problem [7] from open literature has been considered for targeting the optimal area for HEN. The present problem is a practical example involving a portion of petrochemical process that has two hot and two cold streams. The process involves an exothermic reaction, followed by separation using distillation. The process flow sheet is shown in Fig. 1. The stream data for the present HEN problem is given in Table 1.

The minimum temperature difference ($\Delta T_{min}$) responsible for heat transfer between hot stream and cold stream is taken as 20 °C for the present problem. The fixed values of hot and cold utilities are 605 kW and 525 kW. For the present problem, steam and cold water are proposed as hot and cold utilities, respectively.

Table 1: The Stream data for the present problem

| **Stream (s)** | **$T^s$ (°C)** | **$T^t$ (°C)** | **$MC_p$ (kW/ °C)** | **Heat transfer coefficient ($kW/m^2$ °C)** |
|---|---|---|---|---|
| HOT1 | 175 | 45 | 10 | 0.2 |
| COLD1 | 20 | 155 | 20 | 0.2 |
| HOT2 | 125 | 65 | 40 | 0.2 |
| COLD2 | 40 | 112 | 15 | 0.2 |
| Steam | 180 | 179 | - | 0.2 |
| Cold Water | 15 | 25 | - | 0.2 |

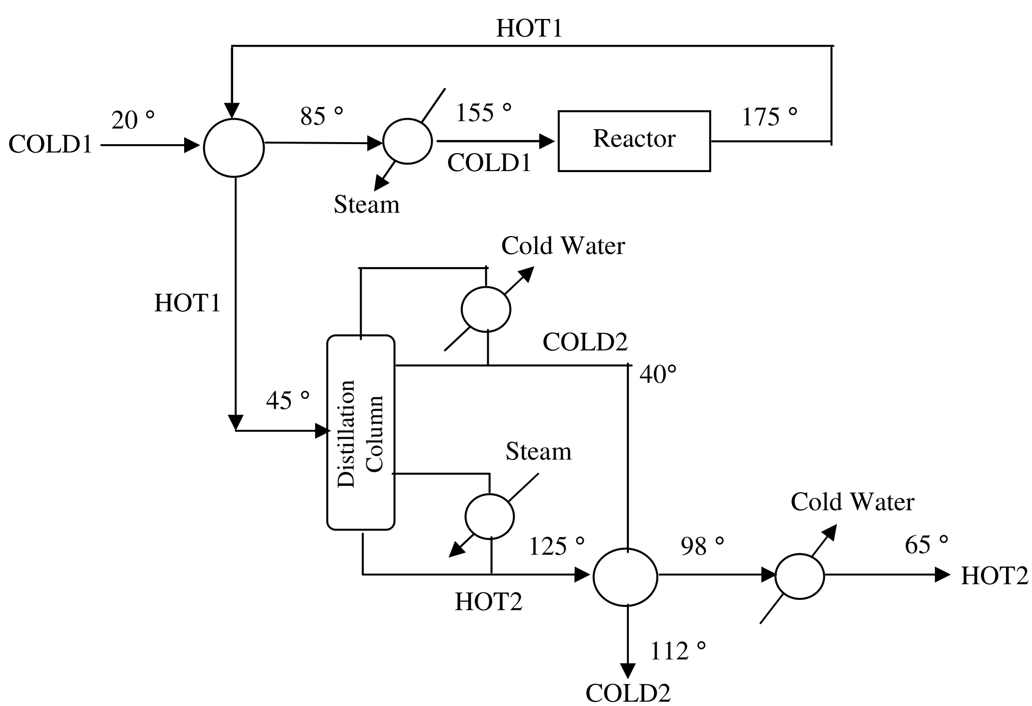


Fig. 1. Process flow sheet for the present problem

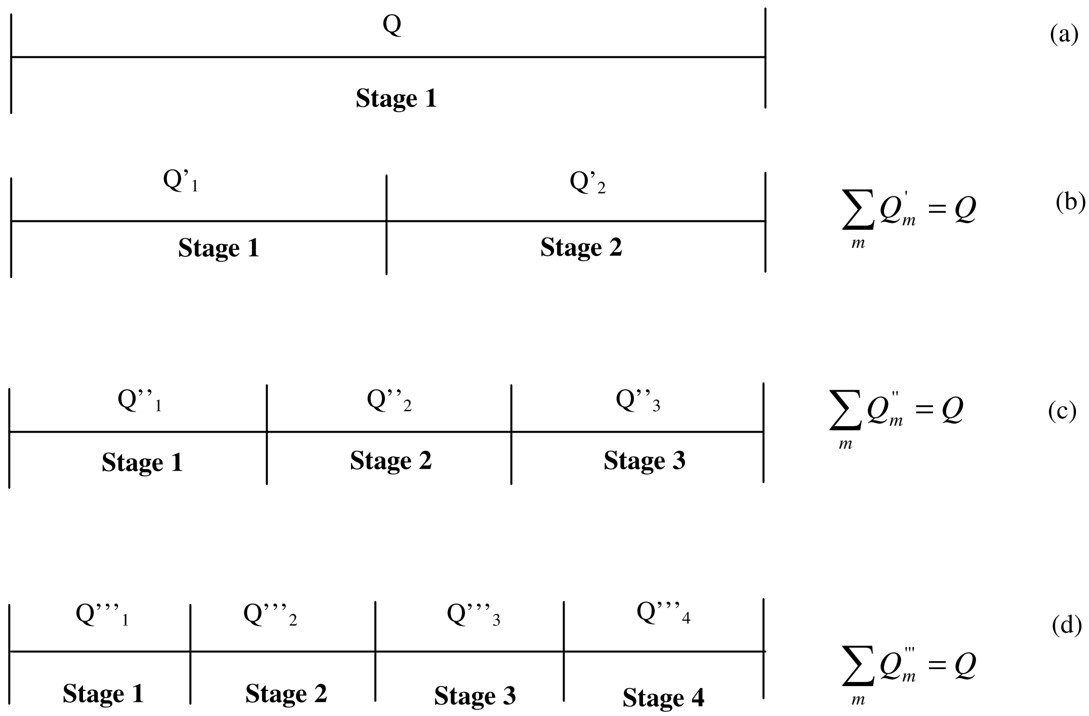


Fig. 2. Single stage and multiple stage superstructure representation of the HEN problem

### 2.1. Superstructure Representation of HEN Problem

The NLP model, presented in this work, uses stage wise superstructure to represent HEN problem. The main advantage of superstructure representation is that it allows for different possibilities for matching streams. The superstructure for any HEN problem can be proposed as follows:

1) The number of stages of a superstructure, to represent HEN, depends on a stream with minimum heat load amongst all streams present in the problem. This stream can be a hot stream or a cold stream. The HEN for a process, having a stream with minimum heat load, Q, can be expressed as a superstructure with different stages as shown below:
   (a) single stage superstructure, as shown in Fig. 2a.
   (b) two stage superstructure, as shown in Fig. 2b.
   (c) three stage superstructure, as shown in Fig. 2c.
   (d) four stage superstructure, as shown in Fig. 2d.

   While formulating superstructure with different stages, the heat load, Q, is distributed amongst the stages depending upon feasible matches between hot and cold stream as per Eq. 9e. For the present problem, the stream COLD2 has minimum heat load, Q, equal to 1080 kW.
   In general, the number of stages required to model the HEN would seldom be greater than either the number of hot streams or the number of cold streams. This is due to the fact that an optimal design usually does not require a large number of exchangers.
2) At each stage of superstructure, the corresponding hot and cold streams entering into that stage are split and directed to a heat exchanger for a feasible match between hot and cold stream. The split streams are mixed together at the outlet of exchanger and is directed to the next stage of superstructure. The heat of mixing of streams in the proposed superstructure is assumed to be zero. By assuming negligible heat of mixing, the nonlinear heat balance around each exchanger and the heat of mixing equations can be eliminated. This assumption significantly simplifies the model as for each stream; only an overall heat balance must be performed within each stage.
3) The temperatures corresponding to each stage are treated as variables, which accommodates the possibility of criss-cross heat exchange between hot and cold streams having different values of heat transfer coefficients.
4) For simplicity, the hot and cold utilities are placed at the outlet of the superstructure.

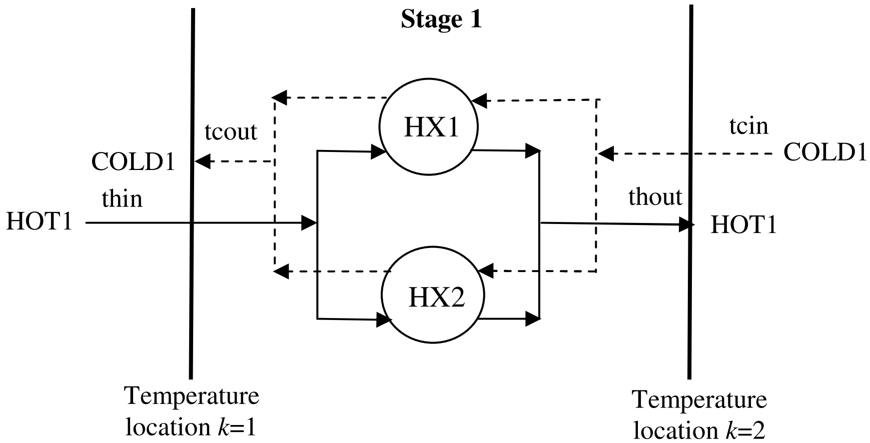


Fig. 3. One-stage superstructure

To give better understanding of the superstructure, Fig. 3 is drawn. It shows a one stage superstructure which involves one hot and one cold stream and is represented by two heat exchangers with two feasible matches.

## 3. Formulation of Area Targeting Model of HEN

To simplify the model formulation, following assumptions are to be considered that can easily be relaxed:

1) Heat capacities of streams are not functions of temperature.
2) Constant heat transfer coefficients for all streams.
3) Countercurrent heat exchange between hot and cold streams.
4) Only one type of hot and one type of cold utilities are proposed.

### 3.1. NLP Model

The necessary definitions, used in present model, are as follows:

Indices:

*i* = number of hot streams

*j* = number of cold streams

*k* = index for stages 1…*NOK* and temperature location 1…*NOK*+1

Sets:

*HP* = {*i*|*i* is a hot process stream}

*HU* = hot utility

*CP* = {*j*|*j* is a cold process stream}

*CU* = cold utility

*ST* = { *k*|*k* is a stage in the superstructure, *k* = 1,…*NOK*}

The above statement shows that in the superstructure, utilities streams are treated as process streams with unknown flow rates.

Parameters:

*thin(i)* = supply temperature of hot stream, *i*

*thout(i)* = target temperature of hot stream, *i*

*tcin(j)* = supply temperature of cold stream, *j*

*tcout(j)* = target temperature of cold stream, *j*

*fh(i)* = heat capacity flow rate of hot stream, *i*

*fc(j)* = heat capacity flow rate of cold stream, *j*

*hh(i)* = heat transfer coefficient of hot stream, *i*

*hc(j)* = heat transfer coefficient of cold stream, *j*

*hhu* = heat transfer coefficient of hot utility

*hcu* = heat transfer coefficient of cold utility

*hch(i)*=heat load of hot stream, *i*

*hcc(j)*=heat load of cold stream, *j*

*QCU* = total cold utiltiy usage

*QHU* = total hot utility usage

*thuin* = inlet temperature of hot utility

*thuout* = outlet temperature of hot utility

*tcuin* = inlet temperature of cold utility

*tcuout* = outlet temperature of cold utility

*mtd* = minimum temperature difference between hot and cold streams

*NOK* = total number of stages

Variables:

*q(i,j,k)* = heat load between hot process stream *i* and cold process stream *j* at stage *k*

*qcu(i)* = heat load between hot stream, *i*, and cold utility

*qhu(j)* = heat load between cold stream, *j*, and hot utility

*th(i,k)* = temperature of hot stream, *i*, at hot end of stage *k*

*tc(j,k)* = temperature of cold stream, *j*, at hot end of stage *k*

*dt(i,j,k)* = temperature approach between hot stream, *i*, and cold stream, *j*, at location *k*

*dthu(j)* = temperature approach between cold stream, *j*, and the hot utility

*dtcu(i)* = temperature approach between hot stream, *i*, and cold utility;

Constraints:

The constrains responsible for targeting optimal area of HEN are given below:

1) Overall heat balance for each stream

The overall heat balance is required to estimate the total heat availability of each process stream. The overall heat balance of each stream must equal to the sum of the heat it exchanges with other process streams at each stage and with utility stream.

$$(thin(i)-thout(i))\times fh(i)=\sum_{k\in ST}\sum_{j\in CP}q(i,j,k)+qcu(i)$$

$$i\in HP \qquad (1a)$$

$$(tcout(j)-tcin(j))\times fc(j)=\sum_{k\in ST}\sum_{i\in HP}q(i,j,k)+qhu(j)$$

$$j\in CP \qquad (1b)$$

2) Heat balance for each stage

To determine the temperature of stages of superstructure, heat balance at each stage is required. For a superstructure with *NOK* stage, *NOK*+1 temperature locations must be defined. These constraints consider the fact that for two adjacent stages, the outlet temperature of the first stage corresponds to the inlet temperature of the second stage. To define the stages and temperature locations, index *k* is used. For stage or temperature location, *k*=1, involves the highest temperature. The heat balances at each stage are given as follows:

$$(th(i,k)-th(i,k+1))\times fh(i)=\sum_{j\in CP}q(i,j,k)$$

$$k\in ST, i\in HP \qquad (2a)$$

$$(tc(j,k)-tc(j,k+1))\times fc(j)=\sum_{i\in HP}q(i,j,k)$$

$$k\in ST,\ j\in CP \qquad (2b)$$

3) Assignment of superstructure inlet temperature

Inlet temperatures of process streams are assigned as the inlet temperatures to the superstructure. The present model considers the countercurrent heat exchange and thus for hot streams, the superstructure inlet corresponds to temperature location *k*=1, while for cold streams, the inlet corresponds to temperature location *k* = *NOK*+1.

$$thin(i)=th(i,k) \qquad i\in HP, k=1 \qquad (3a)$$

$$tcin(j)=tc(j,k) \qquad j\in CP, k=NOK+1 \qquad (3b)$$

4) Feasibility of temperatures

These constraints specify the decrease of temperature at each successive stage k. The outlet temperature of each stream at its last stage does not necessarily correspond to the stream target temperature since utility exchangers, heaters and coolers, can occur at the outlet of the superstructure.

$$th(i,k)\ge th(i,k+1) \qquad k\in ST, i\in HP \qquad (4a)$$

$$tc(j,k)\ge tc(j,k+1) \qquad k\in ST,\ j\in CP \qquad (4b)$$

$$thout(i)\le th(i,k) \qquad i\in HP, k=NOK+1 \qquad (4c)$$

$$tcout(j)\ge tc(j,k) \qquad j\in CP, k=1 \qquad (4d)$$

5) Hot and Cold utility load

The difference between outlet temperature in last stage and target temperature of each process streams determines the hot and cold utility requirement that can be defined as:

$$(th(i,k)-thout(i))\times fh(i)=qcu(i)$$
$$i\in HP, k=NOK+1 \qquad (5a)$$

$$(tcout(j)-tc(j,k))\times fc(j)=qhu(j)$$
$$j\in CP, k=1 \qquad (5b)$$

6) Utility requirements for HEN

These constraints can be expressed as the sum of the utility required by each process stream. The hot and cold utilities required by the present problem are fixed.

$$\sum_{i\in HP}qcu(i)=QCU, \qquad (6a)$$

$$\sum_{j\in CP}qhu(j)=QHU. \qquad (6b)$$

Objective function:

The objective function of area targeting for fixed hot and cold utility requirement is given by:

$$\sum_{i\in HP}\sum_{j\in CP}\sum_{k\in ST}(q(i,j,k)\times((1/hh(i))+(1/hc(j)))/LMTD(i,j,k))-$$
$$\sum_{i\in HP}(qcu(i)\times((1/hh(i))+1/hcu)/LMTD(i,CU))+$$
$$\sum_{j\in CP}(qhu(j)\times((1/hc(j))+1/hhu)/LMTD(HU,j))$$

(7)

The Chen's approximation [8] is proposed for computing log mean temperature difference (*LMTD*).

*LMTD* between hot stream, *i*, and cold stream, *j*, at each stage *k* is:

$$LMTD(i,j,k)=(dt(i,j,k)\times dt(i,j,k+1)\times$$
$$\left[\frac{dt(i,j,k)+dt(i,j,k+1)}{2}\right]+10^{-6})^{1/3} \qquad (8a)$$

Where,

$$dt(i,j,k)=th(i,k)-tc(j,k)$$

$$dt(i,j,k+1)=th(i,k+1)-tc(j,k+1)$$

*LMTD* between hot stream, *i*, and cold utility is expressed as:

$$LMTD(i,CU)=((thout(i)-tcuin)\times dtcu(i)\times$$
$$\left[\frac{(thout(i)-tcuin)+dtcu(i)}{2}\right]+10^{-6})^{1/3} \qquad (8b)$$

Where,

$$dtcu(i) = th(i,k) - tcuout \qquad k = NOK + 1$$

*LMTD* between hot utility and cold stream, *j*, is:

$$LMTD(HU, j) = ((thuin - tcout(j)) \times dthu(j) \times \left[\frac{(thuin - tcout(j)) + dthu(j)}{2}\right] + 10^{-6})^{1/3} \quad (8c)$$

Where,

$$dthu(j) = thuout - tc(j,k) \qquad k = 1$$

Though, Chen's approximation slightly underestimates the *LMTD* and thus overestimates the HEN area but it avoids numerical difficulties and inaccuracy. The approach temperatures of both sides of heat exchanger are equal or when either of them equals to zero, which will lead the objective function to be divided by zero. A very small positive tolerance, $10^{-6}$, is added to the *LMTD* term in Eqs. 8a, 8b & 8c, to prevent this infeasibility.

The proposed model has 6 linear constraints in which 5 are equality and one is inequality constraint. These constraints decide the boundary of the feasible region for the present area targeting model. The objective function for optimal area targeting is nonlinear. The nonlinearity is due to *LMTD* term. The proposed model has no fixed heat recovery approach temperatures since these are treated as variables for optimization.

Initializations:

To increase the possibility of obtaining the globally optimal solution for the present NLP formulation, a good initial guess is required.

The temperature variables are bounded and initialized to provide a high driving force for the heat exchangers.

$$thout(i) \le th(i,k) \le thin(i) \quad i \in HP, k \in ST \quad (9a)$$

$$tcin(j) \le tc(j,k) \le tcout(j)\ , \ j \in CP, k \in ST \quad (9b)$$

$$th(i,k).I \le thin(i) \qquad i \in HP, k \in ST \quad (9c)$$

$$th(j,k).I \le tcin(j) \qquad j \in CP, k \in ST \quad (9d)$$

Where *.I* represents the initialization for the particular variable.

The heat load for each heat exchanger can be initialized to a value that distributes the heat load of stream amongst *NOK* stages. The maximum heat transfer for a match of hot & cold stream is the minimum of either of hot or cold stream heat loads as expressed below.

$$q(i, j, k).I = \min[hch(i), hcc(j)] / NOK$$

$$i \in HP, j \in CP, k \in ST \quad (9e)$$

Where,

$$hch(i) = fh(i) \times (thin(i) - thout(i))\text{, and}$$

$$hcc(j) = fc(j) \times (tcout(j) - tcin(j))$$

Though the utility loads are fixed, the variables for hot and cold utilities can be initialized to evenly distribute the required utility heat loads amongst the streams.

## 4. Results and Discussions

The NLP formulation for targeting optimal area of HEN involved 52 constrains and 45 variables. This model is solved using mathematical modeling and optimization software, GAMS 2.50, with an NLP solver, MINOS 5.4. The constraints of NLP formulation are linear; hence MINOS 5.4 uses reduced gradient method [9] to converge the solution of the present model.

Since the problem involves two hot and two cold streams, a two stage superstructure for NLP formulation is solved. It took 0.063 seconds on an AMD Athlon processor to execute the simulation. The solution of NLP formulation determines the optimal area target as 1326.97 m$^2$ for the HEN with specified hot and cold utilities requirements. The targeted optimal value of area appears to be very satisfactory and is very close to the value given in literature [7]. The value obtained from this model is 1.1 % more than that is given in literature. The difference can be explained primarily as follows: in the present model, the Chen's approximation [8] is used to compute *LMTD* term whereas, in literature, following *LMTD* term has been used:

$$LMTD(i, j, k) = \left[\frac{dt(i, j, k) - dt(i, j, k+1)}{\ln\left[\dfrac{dt(i, j, k)}{dt(i, j, k+1)}\right]}\right]$$

$$k \in ST \quad (10)$$

Table 2. The heat load between hot stream, *i*, and cold stream, *j*, at each stage of two stage superstructure

| Feasible match between Hot stream & Cold stream (i.j) | Stage 1 Heat load (kW) | Stage 2 Heat load (kW) |
|---|---|---|
| 1.1 | 374.5 | 607.6 |
| 1.2 | 114.5 | |
| 2.1 | | 1112.83 |
| 2.2 | | 965.5 |

Table 3. The Heat Exchangers used in a two stage superstructure with their heat loads and areas

| Heat exchangers | Heat load (kW) | Area ($m^2$) |
|---|---|---|
| HX1 | 374.5 | 113.9 |
| HX2 | 114.5 | 29.2 |
| HX3 | 607.6 | 196.3 |
| HX4 | 1112.83 | 336.99 |
| HX5 | 965.5 | 367.14 |
| H (Heater) | 605 | 159.9 |
| C1 (Cooler) | 203.3 | 57.99 |
| C2 (Cooler) | 321.7 | 65.4 |

### 4.1. HEN Configuration

The solution of the present model (Eq. 1 to 9) also can generate the HEN configurations. Table 2 provides the required heat load for the feasible match between hot stream, *i*, and cold stream, *j*, at each stage when HEN is represented by a two stage superstructure. The minimum heat load, Q, of COLD2 has been distributed in two stages with heat loads of 114.5 and 965.5 kW as per Eq. 9e. These matches are restricted to 6 linear constraints, given by Eqs. 1, 2, 3, 4, 5 & 6.

Therefore, the feasible HEN meets above targeted area (1326.97 $m^2$) with 5 heat exchangers, one heater and two coolers. The heat exchangers with different heat loads and corresponding optimal areas are presented in Table 3.

The two stage superstructure for HEN of the present problem, which achieves the above targets, is shown in Fig. 4.

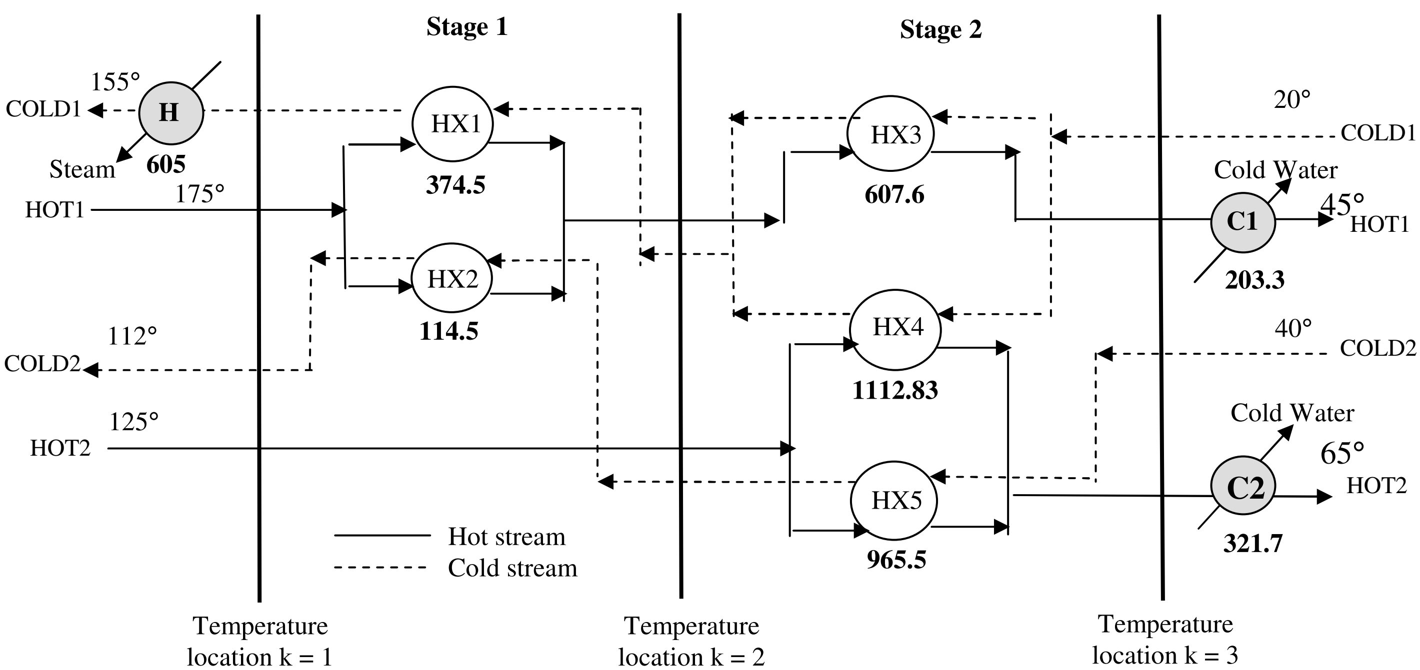


Fig. 4 Feasible network achieving the area target

Table 4. Heat load between hot stream, *i*, and cold stream, *j*, at each stage of a three stage superstructure

| Feasible match between Hot stream & Cold stream (i.j) | Stage 1 Heat load (kW) | Stage 2 Heat load (kW) | Stage 3 Heat load (kW) |
|---|---|---|---|
| 1.1 | 402.8 | | |
| 1.2 | 84.6 | 454.0 | 32.7 |
| 2.1 | | 1217.9 | 474.2 |
| 2.2 | | 508.7 | |

Table 5. Targeted area for HEN, represented by one stage, two stage, three stage and four stage superstructures

| Number of stages | Targeted area ($m^2$) | Difference from literature value (%) | Number of heat exchangers |
|---|---|---|---|
| One | 2143.7 | 63.3 (more) | 5 |
| Two | 1326.97 | 1.1 (more) | 8 |
| Three | 1315.39 | 0.2 (more) | 10 |
| Four | 1313.9 | 0.1 (more) | 12 |

There is a flexibility in the present model for selecting number of stages in the superstructure. If higher number of stages is selected than max(NH, NC), it increases solution time but simultaneously decreases required area of HEN. The rate of reduction however decreases when one moves to higher number of stages. This effect can be shown using three stage model for the present problem. In this case, the number of constraints and variables are 71 & 57 respectively. The solution time increases up to 0.125 seconds, which is higher than two stage model. The targeted optimal area in this case comes out to be 1315.39 $m^2$ that is 0.72 % less than that for two stage model and 0.2% more than that given in literature. For three stage superstructure, the heat loads for different matches are shown in Table 4. It needs 7 heat exchangers, one heater and two coolers.

Table 5 shows the optimal area and number of heat exchangers of HEN, when HEN is represented by one stage, two stage, three stage and four stage superstructures. Table 5 shows that the area computed with one stage superstructure is 63.3 % more than the literature value. The area obtained using three stage and four stages superstructures is very close to the value given in literature and that can be achieved with 10 and 12 heat exchangers respectively, whereas in literature the targeted area, 1312.57 $m^2$, can be achieved using 8 heat exchangers. It is clearly concluded from Table 5 that as stages of superstructure increases, number of heat exchangers increase which leads to a high value of capital cost even if the total heat transfer area of HEN remains almost constant. Therefore, one can account two stage superstructure to represent HEN for the present problem.

The present model can also account for the streams with different heat transfer coefficients. If heat transfer coefficients for streams HOT1, HOT2, COLD1 and COLD2 are taken as 0.8, 0.02, 0.8 & 0.02 kW/$m^2$ °C, respectively, the targeted area for HEN comes out to be 5155.6 $m^2$.

## 5. Concluding Remarks

1) The present model does not require the temperature or enthalpy interval and gives the HEN configuration based on the constraints. In other words the model gives the targeted area as well as heat load for each possible matches simultaneously.
2) This NLP formulation depends on the *LMTD* approximation and thus estimates the targeted area of HEN accordingly.
3) The model is not restricted to number of stages in superstructure. However, there is a trade-off between number of stages of superstructure and number of heat exchangers required to achieve the targeted area of HEN.
4) The streams with different heat transfer coefficients can also be accounted in this model.

## 6. References


[1]. Zhu, X. X., B. K. O'Neill, J. R. Roach & R. M. Wood, "A new method for heat exchanger network synthesis using area targeting procedures", Comp. Chem. Engg., Vol 19, No. 2, pp 197-222, 1995.

[2]. Linnhoff, B. and Townsend, D. W., Surface area targets for heat exchanger networks, IchemE Annual Res. Mtg, Bath (1984)

[3]. Linnhoff, B., Townsend, D. W., Boland, D., Hewitt, G. F., Thomas, B. E. A., Guy, A. R. and Marsland, R. H. (1982), User guide on process

integration for the efficient use of energy, The Institution of Chemical Engineers, Rugby, U.K.; available in the U.S. through Pergamon Press, Inc. Elmsord, N.Y.

[4]. Gundersen T. and I. E. Grossmann, "Improved optimization strategies for automated heat exchanger network synthesis through physical insights" Presented at the *Annual AIChE Meeting,* Washington, D.C. (1988).

[5]. Colberg R. D and M. Morari, "Area and capital cost targets for heat exchanger network synthesis with constrained matches and unequal heat transfer coefficients", Comp. Chem. Engg., Vol 14, pp 1-22, 1990.

[6]. Yee T. F., I. E. Grossmann and Z. Kravanja, "Simultaneous Optimization models for heat integration-I. Area and Energy targeting and modeling of multi-stream exchangers', Comp. Chem. Engng., Vol. 14, No. 10, pp. 1151-1164, 1990.

[7]. Shenoy, U. V. *Heat Exchange Network Synthesis: Process Optimization by Energy and Resource Analysis*; Gulf Publishing Company: Tokyo, 1995.

[8]. Chen J. J. J., Letter to the Editors: Comments on improvement on a replacement for the logarithmic mean, Chem. Engg. Sci., **42**, 2488-2489 (1987).

[9]. Murtagh B.A. and M.A. Saunders, MINOS 5.0 User's guide. Tech. Report SOL 83-20, Systems Optimization Laboratory, department of Operation research, Stanford University (1985).