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DEVELOPMENT OF AN EFFICIENT LINEAR MODEL FOR THE ANALYSIS OF MULTIPLE EFFECT EVAPORATOR SYSTEM

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ABSTRACT

Based on the principles of Process Integration a simplified model has been developed for the analysis of multiple effect evaporator (MEE) systems. It uses the new concepts of stream analysis, temperature path and internal heat exchange for the formulation of the model equations. In addition to it, the above model also takes into account variation in physico-thermal properties as well as boiling point rise while simulating the MEE system. The present model is scalable and is used to simulate three case studies, taken from open literature. The model, consists of linear equations, which are automatically generated through the computer program and are solved using Gaussian Elimination method with partial pivoting. The results of the present model compare well with published models.

Keywords: Mathematical Modeling, Multiple Effect Evaporator, Temperature Path, Internal Heat Exchange.

1. INTRODUCTION

Evaporation is an energy intensive process. This fact has provided necessary motivation for the development of different mathematical models, which could analyze the complex phenomena of evaporation in multiple effect evaporator (MEE) system.

Over last seven decades, mathematical models of MEE system, have been developed by a number of investigators. Some of these are Holland (1975), Lambert et al. (1987), Mathur (1992), Zain and Kumar (1996) and El-Dessouky et al. (2000). These models are generally based on a set of linear and non-linear equations. Further, it has also been observed that model equations developed for a given operating configuration does not work for other operating configurations and in such cases the whole set of governing equations of the model needs to be changed to address the new operating configuration (Mathur, 1992). This further complicates handling of all operating configurations through a single model. To overcome this limitation Stewart & Beveridge (1977) and Ayangbile et al. (1984) developed cascade algorithm in which the model of an effect is solved repeatedly to address the different operating configurations of a MEE system. Further, Bhargava (2004) developed an improved model based on the work of Ayangbile et al. (1984). With the development of Process Integration (PI) principles and stream analysis methods investigators also tried to analyze the complex MEE system with the help of these. Westerberg and Hillenbrand (1988) used the concepts of PI in MEE system to optimize operating temperature of an evaporator and to select optimal feed flow sequence (OFFS). However, they did not demonstrate it with the help of an example of complex MEE system.

Mathematical Models, which describe the complete process, are complex in nature. These complex models use special techniques (Boolean algebra) to address the complexity of the problem such as feed sequencing, splitting and

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bleeding etc. and also pose stability problems during solution. Moreover, many a models are highly sensitive to initial guess of the unknown parameters. On the other hand, though the PI based methods provide useful insights to the problem and suggest remedial measures to improve it, have not been developed to full extent so that these could be used to tackle all complexities of a MEE system.

Thus, under the above backdrop it appears that there is a motivation for the development of a simple PI based model which can be used for the analysis of a MEE system. The present paper uses three primary concepts of Westerberg and Hillenbrand (1988) to develop the model. These are multiple hypothetical streams in a feed, *temperature path* and *internal heat exchange*. Further, these concepts are modified to suit the evaporation process and to include the effect of boiling point rise (BPR). The present model is scaleable and developed for MEE system with n effects and then scale down to solve a number of case studies.

2. MODEL DEVELOPMENT

Generally, to model the complex process of evaporation temperature dependent physico-thermal properties of liquor/fluids are used. Thus, for the present investigation, correlations for heat of vaporization and enthalpy of condensate are developed based on data are taken from standard text of Smith et al. (2001). However, the thermo-physical properties of the feed and liquor are considered based on the case studies selected for comparison.

2.1 The MEE System

The MEE system, consisting of n effects, is selected for model development and shown schematically in Figure 1 in which feed first enters into effect no. n and then follows the backward sequence. In the present model it is assumed that the feed stream is composed of a number of individual streams such as condensate streams which subsequently come out from different effects (except first as it utilizes live steam) and a product stream. These streams are assumed to move through the MEE system as if these are physically separate from each other. For the MEE system with n effects the feed stream, in hypothetical sense, is composed of (n+1) number of streams namely: one product stream, P, and n condensate streams designated as, C_i, (where, i=1 to n).

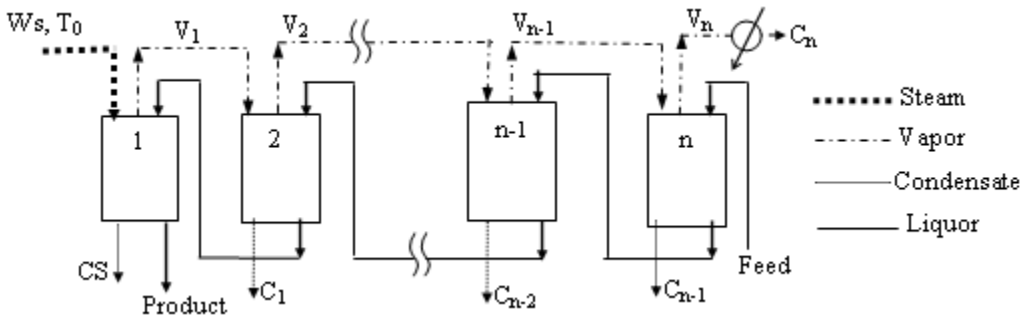

Figure 1: The schematic diagram of a MEE system with 'n' effects.

A stream of feed, while traversing from entry effect to exit effect, passes through different levels of temperatures. This fact is demonstrated using the concept of *temperature path* defined as a "Path followed by the temperatures of a stream when it passes through an effect or a network of it". It plays a vital role in the development of model equations. The *temperature paths* of all (n+1) constituent streams of feed are shown in Figure 2 in which T_1 to T_n are vapor body temperatures of 1^{st} to n^{th} effects. The *temperature paths* are plotted to demonstrate another important concept called "*internal heat exchange*". This concept works on one of the basic principles of PI, called "maximum energy recovery". It allows different streams or their parts to exchange heat with each other in order to facilitate maximum amount of heat to be exchanged through internal exchange and thus provides a mean to minimize the amount of live steam required by the system.

To demonstrate the concept of "*internal heat exchange*" the *temperature path* of P, shown in Figure 2, is considered. It first moves downward in temperature from point "a" to "c" through point "b" and thus behaves as a hot stream from point "a" to "c". However, the same stream from point "d" to "g" through "e" & "f" works as a cold stream as its temperature rises in this section of *temperature path*. The hot stream part of P can exchange heat with

its cold stream part subjected to the driving force constraint, which is in this case $\Delta T_{min.}$ Thus, the hot stream part of product "P" first cools down from point "a" to "b" and the cold stream part rises from point "d" to "e" under this exchange only. Thus, stream "P" enters n^{th} effect at point "b" and is cooled down to T_n+BPR_n by flashing inside the effect. For the product stream, P, maximum feasible *internal heat exchange* is equal to $[P*C_{PP}*(T_F-T_n-BPR_n-\Delta T_{min})]$ kW. Further, the cold part of product stream "P" enters the $(n-1)^{th}$ effect at point "e" (which is $T_{n-1}+BPR_{n-1}$) and also exits it at same temperature corresponding to point "e". The stream "P" then enters effect no. (n-2) at the "e". In this effect, it is heated up to point "f" through sensible heat exchange inside the effect provided by vapor V_{n-3}. Similarly, the heating of stream "P" from "f" to "g" also takes place.

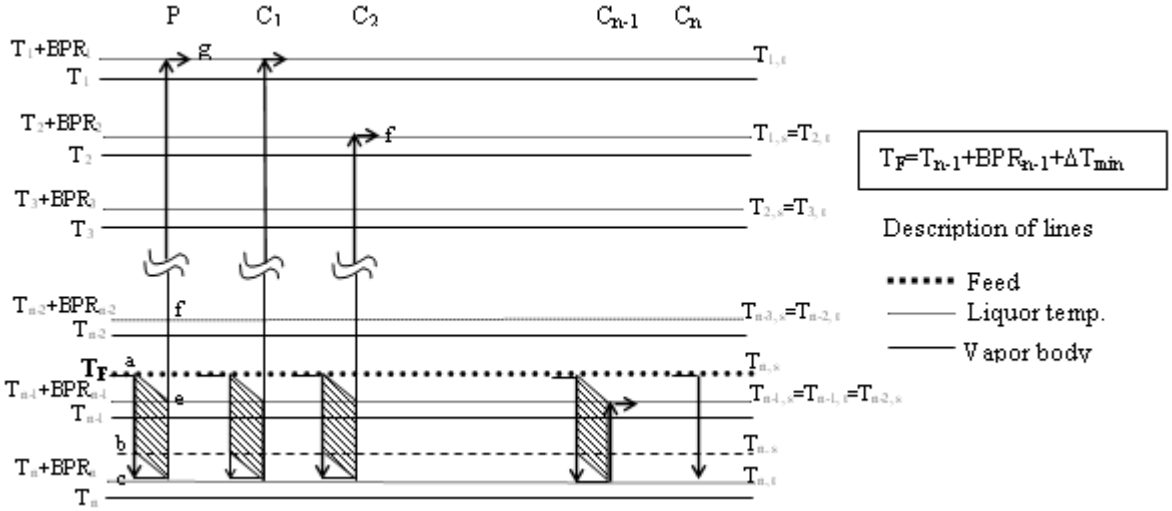

Figure 2: Temperature path s of different streams of feed for a MEE system.

2.2 Formulation of the Model

To develop the model equation of an effect the temperatures at which a stream enters (supply temperature, T_s) into it and exits (target temperature, T_t) from it should be known. These temperatures can be obtained from the *temperature paths* shown in Figure 2. For example, to develop equation for the first effect T_s and T_t of streams, P and C_1 are taken as (T_2+BPR_2) and (T_1+BPR_1) respectively from Figure 2. Similarly, T_s and T_t of different streams that enter into second to n^{th} effect can be extracted from Figure 2.

Table 1: Supply and target temperatures of different streams entering to different effects

S. No.	Effect	$T_{k,s}$	$T_{k,t}$	Streams enter to the effect
1	First	T_2+BPR_2	T_1+BPR_1	P, C_1
2	Second	T_3+BPR_3	T_2+BPR_2	P, C_1, C_2
⋮	⋮	⋮	⋮	⋮
n-2	n-2	$T_{n-1}+BPR_{n-1}$	$T_{n-2}+BPR_{n-2}$	P, C_i (i=1 to n-2)
n-1	n-1	$T_{n-1}+BPR_{n-1}$	$T_{n-1}+BPR_{n-1}$	P, C_i (i=1 to n-1)
n	n	$T_n+BPR_n+\Delta T_{min}$	T_n+BPR_n	P, C_i (i=1 to n-1)
		T_F	T_n+BPR_n	C_n

From Figure 2 and Table 1, it can be evident that due to internal heat exchange streams P & C_i (i=1 to n-1) enter into n^{th} effect at point "b" (equal to $T_n+BPR_n+\Delta T_{min}$) whereas stream, C_n, enters into it at point a (equal to T_F) as internal heat exchange is not possible for this stream. This fact is justified as for the purpose of computation n+1 streams, which comprise feed, move separately through the system without any interference from each other

streams. Thus, hypothetically it can be considered that different streams can enter into an effect at different T_s. In this manner internal heat exchange in temperature paths are utilized in the proposed model. Model equations for the first effect are derived using data shown in S. No. 1 of Table 1. Thus, energy balance around 1st effect at steady state is:

Latent heat supplied by live steam at T_0	+	Sensible heat of entering streams, P & C_1 at $T_{1,s}$	=	Latent heat available with vapor stream, C_1 at $T_{1,t}$ + Sensible heat of exiting streams, P & C_1 at $T_{1,t}$

(1)

Latent heat supplied by live steam at T_0= Ws * λ_s (2)

Sensible heat of streams, P and C_1, which enters the first effect at $T_{1,s}$ is:

$$P * C_{PP} * (T_{1,s} - T_r) + C_1 * [T_{1,s} * C_{PW}(T_{1,s}) - T_r * C_{PW}(T_r)] \quad (3)$$

Latent heat available with the exit vapor stream of amount C_1 at $T_{1,t}$= C_1 * λ_1 (4)

Sensible heat of exiting streams from first effect at $T_{1,t}$ is:

$$P * C_{PP} * (T_{1,t} - T_r) + C_1 * [T_{1,t} * C_{PW}(T_{1,t}) - T_r * C_{PW}(T_r)] \quad (5)$$

Substituting Eqs. 2, 3, 4 and 5 in Eq. 1 and putting the values of $T_{1,s}$ and $T_{1,t}$ from Table 1, following equations for first effect is obtained:

$$Ws * \left[\frac{\lambda_s}{\lambda_1}\right] + C_1 * \left[\frac{b_1}{\lambda_1} - 1\right] + \frac{P * C_{PP} * (T_2 + BPR_2 - T_1 - BPR_1)}{\lambda_1} = 0 \quad (6)$$

Where, $b_1 = [(T_2 + BPR_2) * C_{PW}(T_2 + BPR_2) - (T_1 + BPR_1) * C_{PW}(T_1 + BPR_1)]$

Similarly, equation for k[th] effect can be derived using Figure 3. For this purpose, supply and target temperatures for this effect are $T_{k,s}$ and $T_{k,t}$ as shown in Figure 3. Streams denoted by P, C_1, C_2….C_k are presented in k[th] effect.

$$A + C_{k-1} * \left[\frac{\lambda_{k-1} + b_k}{\lambda_k}\right] + C_k * \left[\frac{b_k}{\lambda_k} - 1\right] + \frac{P * C_{PP} * (T_{k,s} - T_{k,t})}{\lambda_k} = 0 \quad (7)$$

where, A = 0 for the value of k = 2

$A = C_{k-2} * \left[\frac{b_k}{\lambda_k}\right]$ for the value of k = 3

$A = \sum_{i=1}^{k-2} C_i * \left[\frac{b_k}{\lambda_k}\right]$ for the values of k equal to 4 to n

$b_k = T_{k,s} * C_{PW}(T_{k,s}) - T_{k,t} * C_{PW}(T_{k,t})$

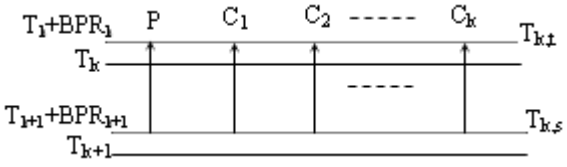

Figure 3: Temperature paths of k+1 number of streams moving from (k+1)[th] effect to k[th] effect.

The above equation, Eq. 7, can generate (n-1) different equations based on values of k equal to 2 to n.

Overall mass balance around the MEE system provides:

$$\sum_{i=1}^{n} C_i = F - P \quad (8)$$

Thus, simplified model for MEE system consists of (n+1) number of linear equations.

3. SOLUTION OF THE MODEL

A set of (n+1) linear algebraic equations is obtained for the present model. It is not out of place to mention that similar non PI based models contain at least 3n number of equations for the MEE system shown in Figure 1. The set

of linear equations is solved simultaneously using Gaussian Elimination Method with partial pivoting. For the solution of the present model a computer program in C++ has been developed. This program plots the *temperature paths* of different streams of feed and also automatically generates the set of governing equations of the model, once the input data to it are provided.

4. RESULTS AND DISCUSSION

The present linear model (Model-SM) is used to simulate three case studies, taken from Holland (1975), Zain and Kumar (1996) and Mathur (1992) and the results are compared with that of above investigators. These investigators have employed different approaches to compute the steam consumption (SC) for the MEE systems. Therefore, Model-SM, which has been developed for a MEE system with n effects and backward feed flow sequence, has been re-configured as per the operating conditions of the above investigations shown in Table 2. For this purpose only input data files need to be changed and not the complete program.

4.1 Case Study 1

Holland (1975) investigated a triple effect evaporator (TEE) system used for the concentration of NaOH and computed the live steam consumption for the operating conditions given in column 2 of the Table 2. The model was based on mass and energy balance equations which incorporated the effect of BPR through a correlation. Further, the author has considered three different values of overall heat transfer coefficient (OHTC) for the three different effects. It was observed that for the TEE system Model-SM provided 2.3 % less SC than the model of Holland (1975). The above departure in the value of SC is primarily due to the fact that the model of Holland (1975), considered different values of OHTC for each effect which gives rise to different driving forces (ΔT) values in different effects (as high as 11.7 % w.r.t. average ΔT). This is in contrast to the assumption of constant ΔT in each effect taken by the Model-SM.

Table 2: Operating parameters for the MEE systems

Parameter(s)	*Value(s) of parameter(s)*		
	Holland (1975)	*Zain & Kumar (1996)*	*Mathur (1992)*
No. of effects	3	3	6
Feed flow sequence	1→2→3	1→2→3	3→4→5→6→2→1
Type of liquor	NaOH	NaOH	Black liquor
T_0, °C	176.7	165.015	138
T_L, °C	76.7	33.54	51
T_F, °C	93.3	110	70
F, kg/h	18144	25000	65318.4
x_F	0.20	0.08	0.15
x_P	0.50	0.35	0.45
Type of model	Non-linear	Nonlinear	Non-linear
Equations used by other authors	12	12	18
Equations used by Model-SM	4	4	7
SC predicted by other authors, kg/h	5477.7	7152	16473.8
SC predicted from Model-SM, kg/h	5354.5	6824.4	17196

4.2 Case Study 2

Zain and Kumar (1996) investigated a TEE system, given in column 3 of Table 2, for the concentration of NaOH. The authors employed empirical models for the computation of OHTC and BPR for all the three effects. A comparison shows that Model-SM predicted 4.8 % less value of SC than model of Zain and Kumar (1996). The departure in the predicted values of SC can be attributed to the same fact as described for the case study-1.

4.3 Case Study 3

Third case study is taken from Mathur (1992), shown in column 4 of Table 2. He investigated a MEE system with six effects which was used to concentrate black liquor in a Pulp and Paper Mill. For above system Mathur (1992) has developed a rigorous model by taking in to account the variation in physico-thermal properties, BPR and OHTC. Gudmundson (1972) model was used for the computation of OHTC for each effect. It was observed that Model-SM predicts 4.2 % more steam consumption. This departure is primarily due to the Gudmundson (1972) model which provides low values of OHTC in first effect which generally contributes maximum towards the steam consumption. Due to highly uneven distribution of OHTC in different effects the driving force distribution is also highly uneven. The present departure is attributed due to this phenomenon.

From the above results it can be concluded that the present scalable model, Model-SM, predicts the values of SC with considerably less effort in comparison to other models. It can also address the real world complexities of MEE system.

5. CONCLUSIONS

1. The present scalable model, Model-SM, though based on simplified assumptions can be used as an analysis tool if an average error of 4.2 % can be tolerated for the prediction of the value of SC. It does the prediction of SC with less effort as compared to other models. Model-SM can address real world complexities of a MEE system such as variation in physical properties and BPR.
2. The solution of the Model-SM is easy and does not pose any instability or oscillation problems as generally observed in other simulation models based on sets of non-linear equations.

NOMENCLATURE

CS, C_1 to C_n	Flow rate of condensate streams, kg/h
C_{PP}	Specific heat capacity of product, kJ/kg/K
C_{PW}	Specific heat capacity of condensate, kJ/kg/°C
F	Feed flow rate, kg/h
P	Flow rate of product stream, kg/h
T	Vapor body temperature, °C
V	Vapor flow rate, kg/h
x	Mass fraction

Subscripts

0	Live steam entering into first effect
1,2,…., n, k	Effect
F	Feed
P	Product
r	Reference
s	Supply
t	Target

Greek letters

Δ	Difference in values of a parameter at two points
λ	Heat of vaporization, kJ/kg

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