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Flux and speed estimation in decoupled induction motor drive using Kalman Filter

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ABSTRACT

This paper presents the rotor flux and speed estimation of a decoupled induction motor drive using Kalman filter. The induction motor model in rotor reference frame is considered, with its torque and flux decoupled. Kalman algorithm is first used to estimate the rotor flux. Then speed is estimated by Kalman algorithm using the estimated flux. Controllers are used for sensorless speed control of the drive. Studies show that the estimation algorithm works well and the sensorless speed control scheme can achieve fast transient response as good as that of drives with sensors, and at the same time maintain a wide speed control range.

KEYWORDS: Induction motor, decoupling control, sensorless control, flux estimation, speed estimation, Kalman algorithm

1. INTRODUCTION

Induction motors are increasingly used in variable speed drive applications with the development of vector control technology[1, 2]. There are two forms of vector or field oriented control: *direct* field orientation, which relies on direct measurement or estimation of the rotor flux, and *indirect* field orientation, which utilizes an inherent slip relation. Though indirect field orientation essentially uses the command (reference) rotor flux, some recent works using the actual rotor flux are reported to achieve perfect decoupling.

The implementation of direct field orientation via airgap flux measurement has typically been plagued by the complexities and lack of mechanical robustness associated with intrusive sensors located within machine airgap. Furthermore, a correction is required for the rotor leakage flux if rotor flux field orientation is to be achieved. Estimation rather than measurement of the rotor flux is an alternative approach for both direct and indirect field orientation that has received considerable attention[3-8]. In many popular implementations of field oriented induction machine drives, rotor flux is estimated from the terminal variables such as stator voltage and current, and rotor speed. The task of rotor flux estimation may also be expected to arise in other approaches to control and monitoring of induction machines.

In many applications it is neither possible nor desirable to install speed sensors from the standpoints of cost, size, noise immunity and reliability of the induction motor drive. So, the development of shaft sensorless adjustable speed drive has become an important research topic[9, 10]. There are two major concerns in the sensorless speed control of induction motor drive. One is the control scheme, and other one is the estimation algorithm. Both are highly dependent on the motor parameters. Accurate estimation of flux and speed in the presence of measurement and system noise, and parameter variations is a challenging task. Kalman filter named after Rudolph E. Kalman[11] is one of the most well known and often used tools for stochastic estimation. An extensive literature on Kalman filter and its applications is also available[12]. The Kalman filter is essentially a set of mathematical equations that implement a predictor-corrector type estimator that is optimal in the sense that it minimizes the estimated error covariance – when some presumed conditions are met. For the flux and speed estimation problem of induction motor, where parameter variation and measurement noise is present, Kalman filter is the ideal one.

In the present paper, induction motor model is reviewed in section 2. Input-output linearization and decoupling scheme is also discussed. In section 3, the Kalman filter for flux and speed estimation is presented. Section 4 details the sensorless control scheme. Results are discussed in section 5.

2. INDUCTION MOTOR MODEL

From the voltage equations of the induction motor in the arbitrary rotating d-q reference frame, the state space model with stator current and rotor flux components as state variables is[8] :

$$\frac{d}{dt}\begin{bmatrix} i_s \\ \psi_r \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}\begin{bmatrix} i_s \\ \psi_r \end{bmatrix} + \begin{bmatrix} B_1 \\ 0 \end{bmatrix} v_s \tag{1}$$

where,

$i_s = y = [i_{ds} \quad i_{qs}]^T$, $\psi_r = [\psi_{dr} \quad \psi_{qr}]^T$, $v_s = [v_{ds} \quad v_{qs}]^T$,

$A_{11} = -a_1\, I - \omega_e\, J$, $A_{12} = a_2\, I - P\, a_3\, \omega_r\, J$, $A_{21} = a_5\, I$,

$A_{22} = -a_4\, I - (\omega_e - P\,\omega_r)\, J$, $B_1 = c\, I$.

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, and $J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$c = L_r/(L_s L_r - L_m^2)$, $a_1 = c\, R_s + c\, R_r\, L_m^2 / L_r^2$, $a_2 = c\, R_r\, L_m / L_r^2$,

$a_3 = c\, L_m / L_r$, $a_4 = R_r / L_r$, $a_5 = R_r\, L_m / L_r$

The torque developed by the motor is:

$$T_e = K_t\, (\psi_{dr}\, i_{qs} - \psi_{qr}\, i_{ds}) \tag{2}$$

where, torque constant, $K_t = 3\, P\, L_m / 2\, L_r$, P= number of pole pairs.

The speed dynamics of the motor is given as:

$$\dot{\omega}_r = (T_e - T_l - \beta\omega_r)/J \tag{3}$$

Equations (1) and (3) describe the fifth order state model of the induction motor. In the motor model described by eqns. (1-3), nonlinearities and interaction exist. The conditions required for decoupling control of the motor are:

$$\psi_{qr} = 0 \quad \text{and} \quad \dot{\psi}_{qr} = 0 \tag{4}$$

From (1), decoupling is obtained, when

$$\omega_{sl} = \frac{R_r\, L_m}{L_r} \cdot \frac{i_{qs}}{\psi_{dr}} \tag{5}$$

The nonlinearities in the overall system are eliminated by using input-output linearizing control approach[8]. This approach consists of change of coordinates and use of nonlinear inputs to linearize the system equations. Developed torque, T_e is considered as a state variable, replacing i_{qs} to describe the motor dynamics. Nonlinear control inputs u_1 and u_2 are used to linearize the motor equations[8]. The input voltages, v_{ds} and v_{qs} to the motor in terms of u_1 and u_2 are:

$$v_{ds} = \frac{1}{c}(-\omega_e i_{qs} + u_1) \tag{6}$$

$$v_{qs} = \frac{1}{c}\left[P\omega_r(i_{ds} + a_3\psi_{dr}) + \frac{u_2}{K_t\psi_{dr}}\right] \tag{7}$$

The induction motor system with these new inputs, is decoupled into two linear subsystems: electrical, and mechanical. The electrical subsystem is described by eqns. (8-9).

$$\dot{i}_{ds} = -a_1 i_{ds} + a_2\psi_{dr} + u_1 \tag{8}$$

$$\dot{\psi}_{dr} = -a_4\,\psi_{dr} + a_5\,i_{ds} \tag{9}$$

The mechanical subsystem is described by torque and speed dynamic eqns. (10-11).

$$\dot{T}_e = -(a_1 + a_4)T_e + u_2 \tag{10}$$

$$\dot{\omega}_r = (T_e - T_L - \beta\omega_r)/J \tag{11}$$

The state space model of the electrical subsystem is:

$$\dot{\boldsymbol{x}}_1 = \boldsymbol{A}_1\,\boldsymbol{x}_1 + \boldsymbol{B}_1\,\boldsymbol{u}_1 \tag{12}$$

$$\boldsymbol{y}_1 = \boldsymbol{C}_1\,\boldsymbol{x}_1 \tag{13}$$

where, $\boldsymbol{x}_1 = [i_{ds}\quad \psi_{dr}]^T$, $\boldsymbol{y}_1 = i_{ds}$, $\boldsymbol{B}_1 = [1\quad 0]^T$, $\boldsymbol{C}_1 = [1\quad 0]$

The state space model of the mechanical subsystem is:

$$\dot{\boldsymbol{x}}_2 = \boldsymbol{A}_2\,\boldsymbol{x}_2 + \boldsymbol{B}_2\,\boldsymbol{u}_2 + \boldsymbol{D}_2\,T_L \tag{14}$$

$$\boldsymbol{y}_2 = \boldsymbol{C}_2\,\boldsymbol{x}_2 \tag{15}$$

where, $\boldsymbol{x}_2 = [T_e\quad \omega_r]^T$, $\boldsymbol{y}_2 = T_e$, $\boldsymbol{B}_2 = [1\quad 0]^T$, $\boldsymbol{C}_2 = [1\quad 0]$, $\boldsymbol{D}_2 = [0\quad -1/J]^T$

The rotor flux ψ_{dr} is estimated by applying the Kalman Filter to discrete time form of eqns. (12-13). The motor speed ω_r is estimated by applying the same algorithm to discrete time form of eqns. (14-15). The Kalman's algorithm for state estimation in linear systems is explained in the next section.

3. KALMAN FILTER FOR FLUX AND SPEED ESTIMATION

The discrete time model of both electrical subsystem and mechanical subsystem is:

$$\boldsymbol{x}(k+1) = \boldsymbol{F}(k)\,\boldsymbol{x}(k) + \boldsymbol{u}(k) \tag{16}$$

$$\boldsymbol{y}(k) = \boldsymbol{H}(k)\,\boldsymbol{x}(k) + \boldsymbol{w}(k) \tag{17}$$

where, $\boldsymbol{x}(k)$ and $\boldsymbol{y}(k)$ are the state vector and output, respectively at the k-th sampling instant. $\boldsymbol{F}(k)$ is the state transition matrix (2×2). $\boldsymbol{H}(k)$ is the measurement matrix (1×2). $\boldsymbol{u}(k)$ is the random disturbance input. It is the sum of physical input and the system noise. $\boldsymbol{w}(k)$ is the measurement noise. Both $\boldsymbol{u}(k)$ and $\boldsymbol{w}(k)$ are assumed to be white noise with zero mean.

Let, $\hat{\boldsymbol{x}}(k)$ = estimate of $\boldsymbol{x}(k)$ by Kalman's algorithm from the measurement of $\boldsymbol{y}(k)$

$\overline{\boldsymbol{x}}(k)$ = extrapolated value of $\boldsymbol{x}(k)$ from the previous estimate, $\hat{\boldsymbol{x}}(k-1)$

$\hat{\boldsymbol{x}}(0)$ = priori estimate of $\boldsymbol{x}(k)$, or the initial guess of $\hat{\boldsymbol{x}}(k)$

$\boldsymbol{P}(0)$ = Error covariance matrix of initial guess $\hat{\boldsymbol{x}}(0)$

The first step of Kalman's algorithm in estimating $\boldsymbol{x}(1)$ is to determine the extrapolated value as follows:

$\bar{\boldsymbol{x}}(1) = \boldsymbol{F}(0)\ \hat{\boldsymbol{x}}(0)$

For a general notation at any sampling instant, dropping the arguments:

$$\bar{\boldsymbol{x}} = \boldsymbol{F}\ \hat{\boldsymbol{x}} \tag{18}$$

where, $\hat{\boldsymbol{x}}$ is the previous estimate, and

$\bar{\boldsymbol{x}}$ is the present extrapolated value based on previous estimate.

The error covariance matrix of the new $\bar{\boldsymbol{x}}$ is:

$\bar{\boldsymbol{P}}(1) = \boldsymbol{F}\ \boldsymbol{P}(0)\ \boldsymbol{F}^{\mathrm{T}} + \boldsymbol{Q}$

Again dropping arguments for a general notation,

$$\bar{\boldsymbol{P}} = \boldsymbol{F}\ \boldsymbol{P}\ \boldsymbol{F}^{\mathrm{T}} + \boldsymbol{Q} \tag{19}$$

Dr. Kalman says the new optimal estimate is:

$$\hat{\boldsymbol{x}} = \bar{\boldsymbol{x}} + \boldsymbol{K}\ (\ \boldsymbol{y} - \boldsymbol{H}\ \bar{\boldsymbol{x}}\ ) \tag{20}$$

where, $\boldsymbol{K}$ is the Kalman filter gain

The optimal gain of Kalman filter is given by[12] :

$$\boldsymbol{K} = \bar{\boldsymbol{P}}\ \boldsymbol{H}^{\mathrm{T}}\ (\ \boldsymbol{H}\ \bar{\boldsymbol{P}}\ \boldsymbol{H}^{\mathrm{T}} + \boldsymbol{R}\ )^{-1} \tag{21}$$

The new estimate $\hat{\boldsymbol{x}}$ has an error covariance matrix, which is given by

$$\boldsymbol{P} = (\ \boldsymbol{I} - \boldsymbol{K}\ \boldsymbol{H}\ )\ \bar{\boldsymbol{P}}\ (\ \boldsymbol{I} - \boldsymbol{K}\ \boldsymbol{H}\ )^{\mathrm{T}} + \boldsymbol{K}\ \boldsymbol{R}\ \boldsymbol{K}^{\mathrm{T}} \tag{22}$$

The Kalman filter consists of repeated use of eqns.(18-22) for each measurement.

4. SENSORLESS CONTROL SCHEME

The block diagram of the sensorless speed control scheme is shown in Fig. 1. This sensorless speed control system consists of three major parts: P-I controllers for speed and current, flux-weakening controller, flux and speed estimator.

4.1 P-I Controllers for speed and current

One P-I controller is used for the flux, or flux component of current as it is adequate for good dynamic response. One P-I controller is used for the speed control, and another for the torque, or torque component of current. The reason for using two P-I controllers (one for speed and the other for torque) in a nested fashion is the significant difference in the time constants of the speed and current, or the electromagnetic torque. The design procedure for these P-I controllers are detailed[8]. The gains are:

K_{pd} = 151.24, K_{id} = 43640, K_{pw} = 0.26, K_{iw} = 1.98, K_{pq} = 100, K_{iq} = 29877.

4.2 Flux weakening controller

The flux weakening controller is used to regulate the magnitude of rotor flux linkage command ψ^*_{dr} such that the motor will operate in constant torque mode when motor speed is below base speed and in constant power mode when motor speed is above the base speed. The flux weakening control algorithm is as follows.

$$\psi^*_{dr} = \begin{cases} \psi_R & \text{if } \hat{\omega}_r \le \omega_b \\ \psi_R \dfrac{\omega_b}{\hat{\omega}_r} & \text{if } \hat{\omega}_r \ge \omega_b \end{cases} \tag{23}$$

where, ψ_R = rated rotor flux linkage in V· s

ω_b = base speed in rad/s,

$\hat{\omega}_r$: estimated rotor angular (mechanical) speed

The rotor flux command is then converted to an equivalent field current command in the rotating reference frame.

4.3 Flux and Speed Estimator

The flux and speed estimator using Kalman filter is described in section 3. Only two voltage sensors and two current sensors are used. Current measurements are required for both estimation and control purposes. But, voltage measurements are taken only for control purpose. Measured currents are transformed from 3-phase to rotating d-q reference frame components, i_{ds} and i_{qs} , through the flux vector angle, θ_e. Current component, i_{ds} is used to estimate the rotor flux through eqns. (18-22). Then the estimated rotor flux and the current component, i_{qs} are used to determine the developed torque, T_e using eqns. (2) and (4). The speed is estimated by Kalman filter eqns. (18-22) using this developed torque. The estimated speed added with slip speed, given by eqn.(5) is integrated to obtain the flux vector angle, θ_e, which is used in coordinate transformation.

5. RESULTS AND DISCUSSIONS

The simulation study of the drive system has been carried out with an induction motor whose rating and parameters are given in Table 1.

Table – 1 Rating and Parameters of the Induction Motor

Three phase, 50 Hz, 0.75 kW, 220V, 3A, 1440 rpm
Stator and rotor resistances: R_s = 6.37 Ω, R_r = 4.3 Ω
Stator and rotor self inductances: L_s = L_r = 0.26 H
Mutual inductance between stator and rotor: L_m = 0.24 H
Moment of Inertia of motor and load: J = 0.0088 Kg ·m^2
Viscous friction coefficient: β = 0.003 N ·m ·s/rad

The rotor flux is estimated by Kalman's algorithm. Using the estimated rotor flux speed is also estimated by Kalman flter. Then the estimated rotor flux and the estimated speed are used in the input-output decoupling and linearizing control algorithm. The simulation result is presented in Fig. 2, for flux and speed estimation with a step decrease in speed command from 1500 r/min to 1000 r/min. The command flux linkage is 0.45 V·s. The estimated speed is similar to the actual speed response, except the temporary deep of 27r/min. The actual rotor flux, estimated rotor flux and error in estimation of flux and speed are also shown. For a step increase in speed command from 1500 r/min to 1800 r/min with weakening of command flux linkage from 0.45 V·s to 0.375 V·s, the simulation result is presented in Fig. 3. The estimated speed is similar to the actual speed response, except the temporary spike of 18 r/min.

6. CONCLUSIONS

The estimation of rotor flux and speed of induction motor, using Kalman filter is presented. Torque and rotor flux are decoupled and the induction motor model is linearized using input-output linearization approach. Rotor flux and speed are estimated by Kalman filter. Sensorless control of the linearized and decoupled drive using estimated flux and speed, is simulated and results presented. Kalman filter is found to be very good and fast for flux and speed estimation in the presence of system and measurement noise. The dynamic response of the sensorless drive is as fast as that of drives with physical sensors. Sensorless speed control scheme works for a wide speed control range.

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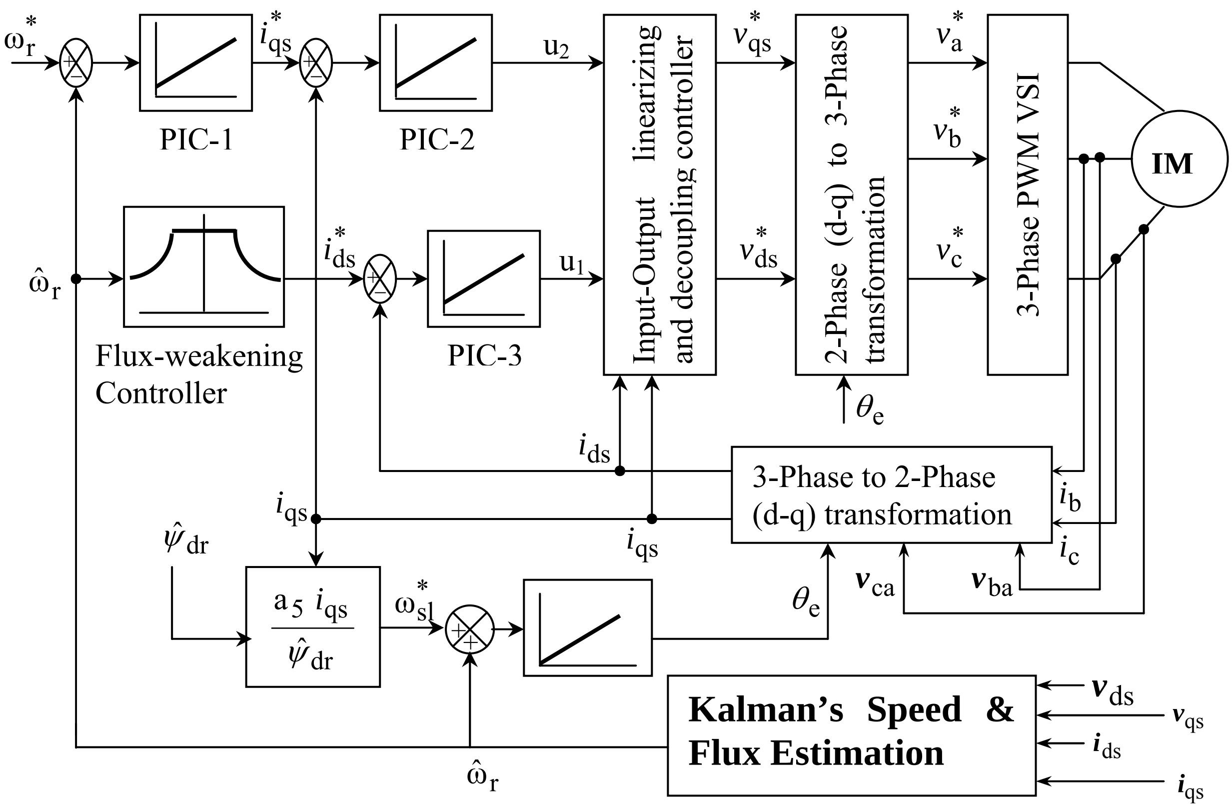

Fig. 1 Block diagram of the sensorless speed control scheme

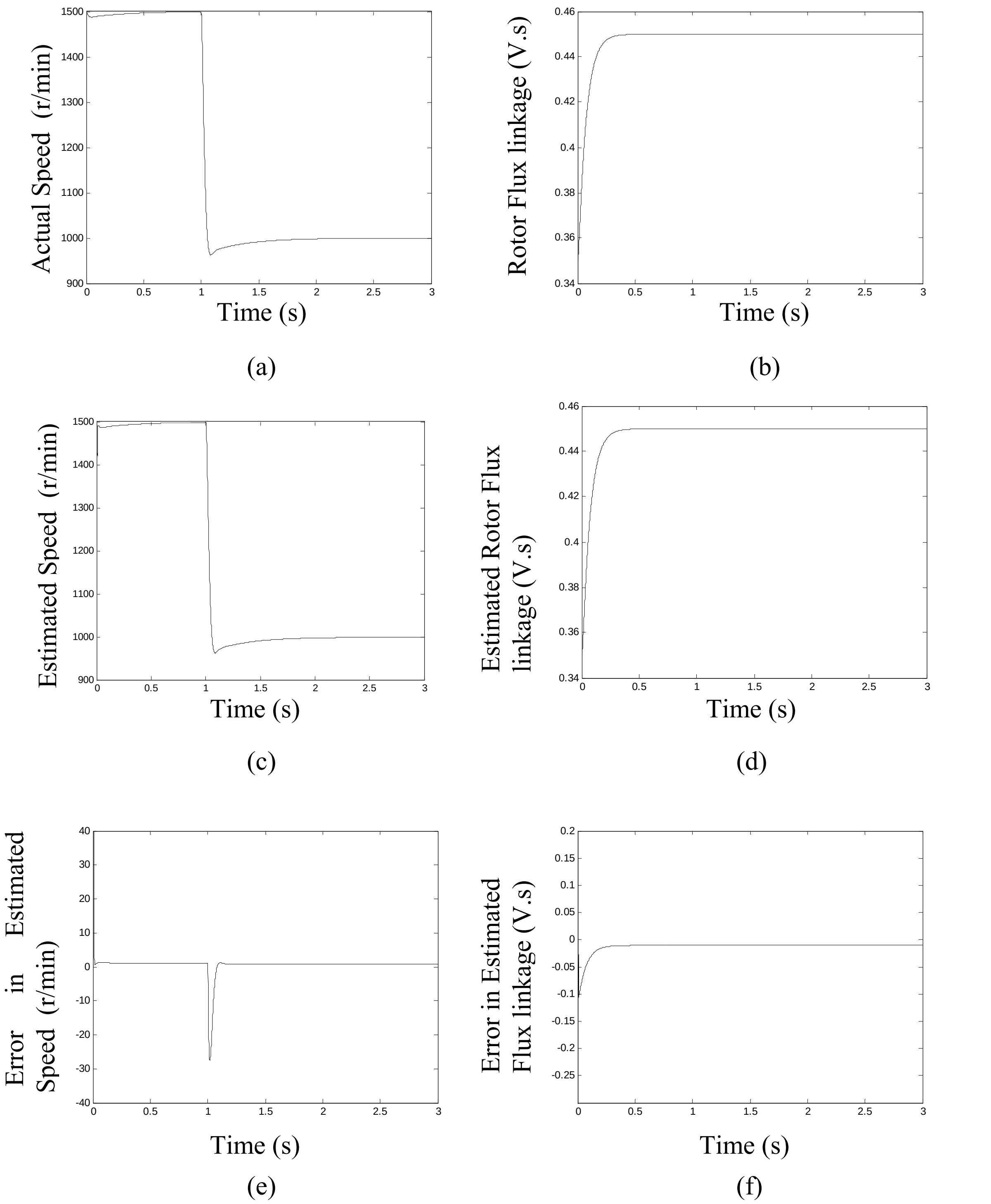

Fig. 2 Simulation response for speed and flux estimation with step change in speed:
(a) Actual Speed, (b) Actual Rotor Flux Linkage, (c) Estimated Speed, (d) Estimated Rotor Flux Linkage, (e) Error in Estimated speed, (f) Error in Estimated Rotor Flux Linkage

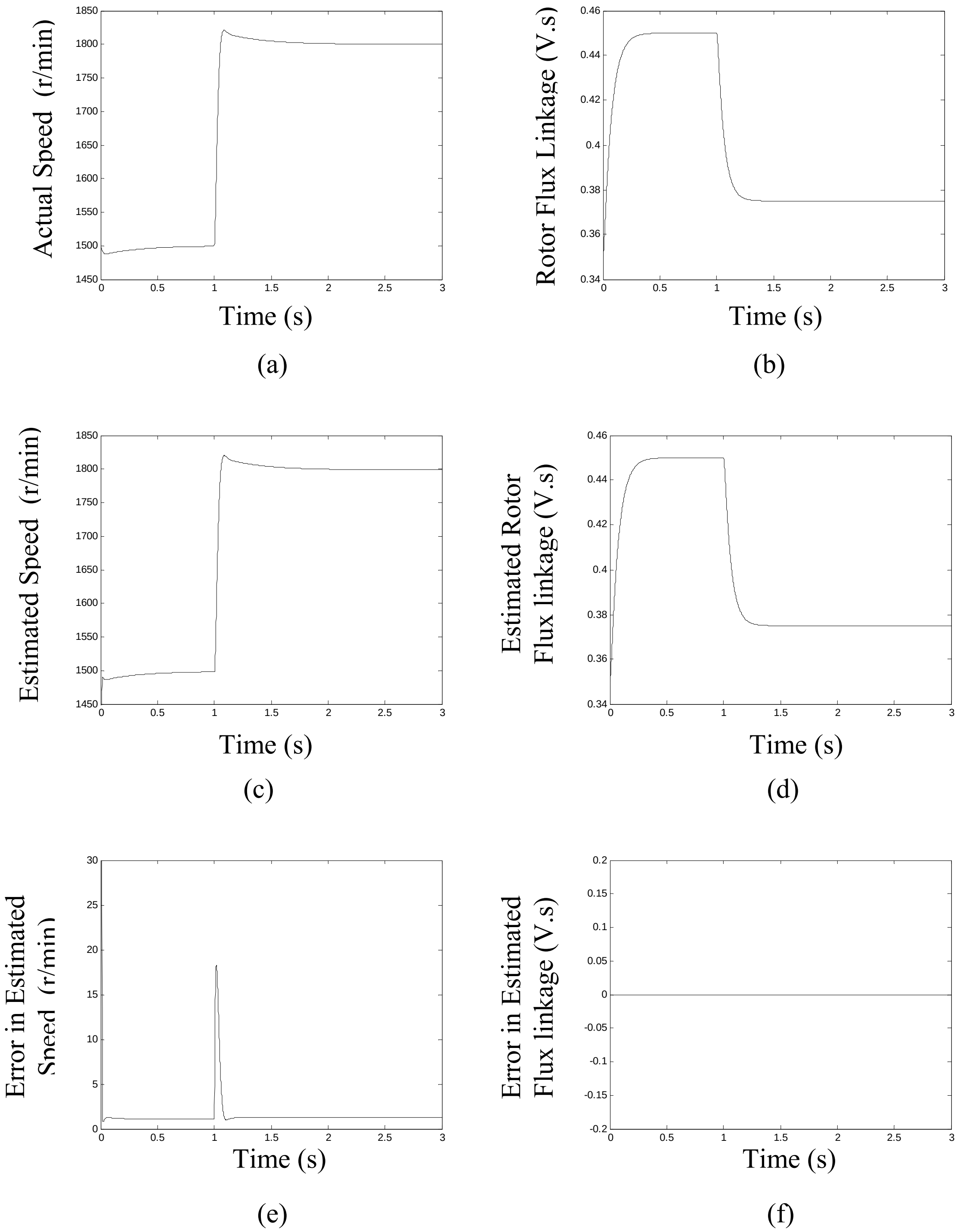

Fig. 3 Simulation response for speed and flux estimation with step increase in speed and flux weakening: (a) Actual Speed, (b) Actual Rotor Flux Linkage, (c) Estimated Speed, (d) Estimated Rotor Flux Linkage, (e) Error in Estimated Speed, (f) Error in Estimated Rotor Flux Linkage