J. Indian Inst. Sci., Jan.–Feb. 1997, **77**, 5–14.

Optimization of Membership Functions of Fuzzy Logic Controller for Controlling Liquid Level Using Genetic Algorithm

B. SUBUDHI AND A. K. SWAIN
Department of Electrical Engineering, Indira Gandhi Institute of Technology, Sarang, Dhenkanal-759 146, Orissa.

Abstract

Selection of membership functions (MF) for a fuzzy logic controller (FLC) is an iterative and a time consuming task. This paper describes the use of genetic algorithm (GA) search strategy for optimizing MFs of antecedent and consequent variables of an FLC that has been used for controlling the level of water on real time basis in a laboratory scale pilot plant. For the simulation of GA integral time square error (ITSE), has been used as the fitness function.

Keywords: Membership Function, Genetic Algorithm, Fuzzy Logic Controller, Integral Time Square Error,

1. Introduction

Fuzzy logic is a technique incorporates heuristics developed by practicing engineers and operators into automatic control. It is used to control highly nonlinear, complex systems or systems whose mathematical model is not known. Also in the situations where classical control methods are available, fuzzy logic is introduced to improve the controller performance and in some cases to simplify the control algorithm. It is verified experimentally that the fuzzy controllers perform better than or as good as a PID controller[1–8]. Fuzzy logic technique has also been applied to adapt the current Neural Network configuration to one which is close to the optimal configuration[9]. Several applications of Fuzzy logic have been reported through recent literatures[10–14]. For designing an FLC the MFs should be selected in such a manner that it exhibits desired control behaviour. Hence to MFs tuned to achieve more desired control behaviour. However, iterative approach[15] for selecting MF is a very much time consuming task. Because for change in MF parameters i.e for different shape of the membership functions the performance of FLC also changes.

This work uses GA search strategy based on natural selection[10] and genetics to optimize MFs. A GA requires only information concerning the quality of the solution produced by each parameter set (objective function value information). This characteristic differs from other optimization methods that require derivative information[16]. Particularly it is very difficult to produce derivative information for different fuzzy rules and MF definitions in an FLC. However, GA does not require any derivative information. GA search is basically a computer simulated evolution, which is used in this work to alter the MFs of conventional FLC from one generation to next one, while optimizing ITSE.

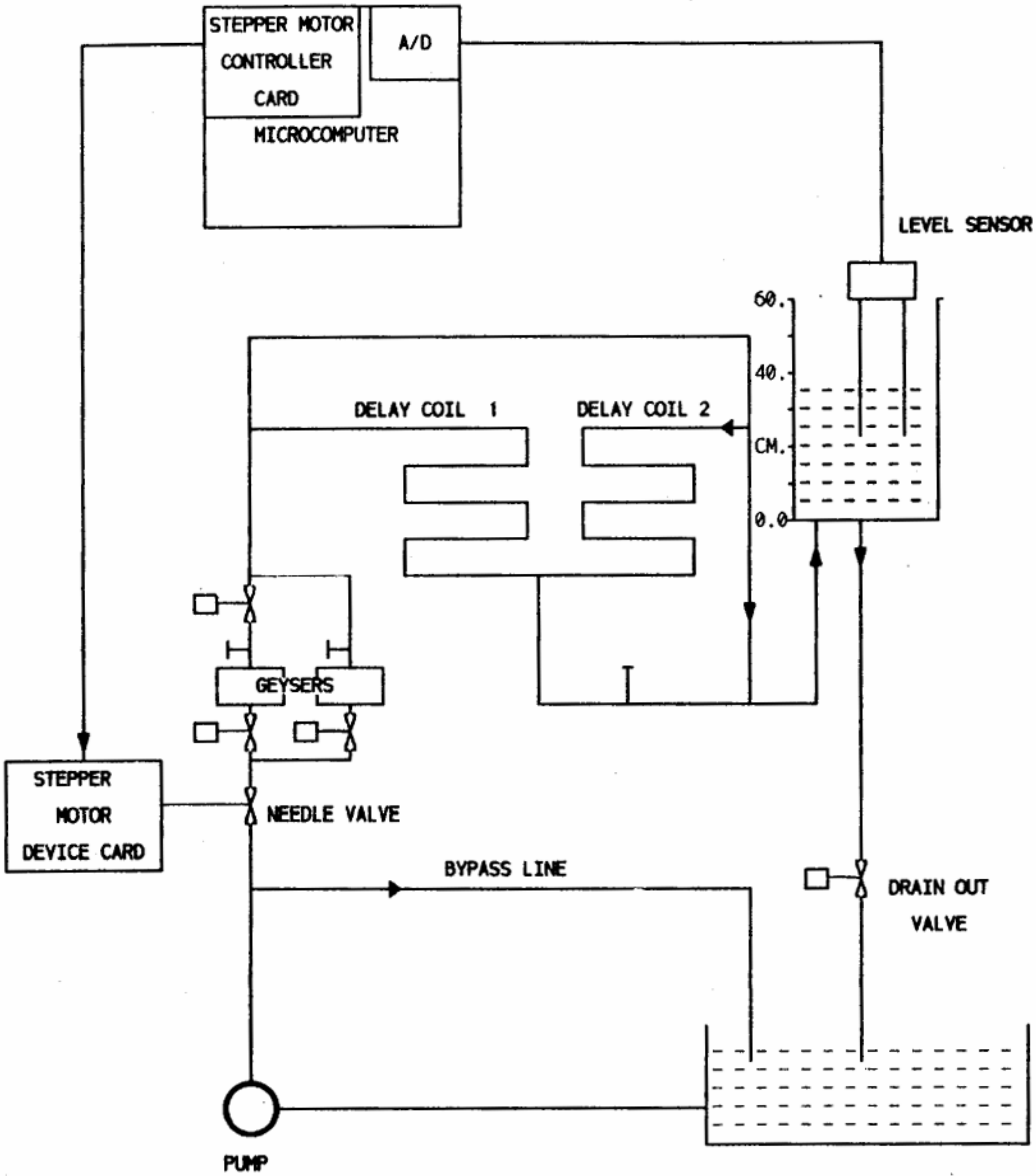

FIG. 1. Schematic diagram of pilot plant.

2. Process Description

Fig. 1 gives the schematics of the pilot plant considered for this work. A capacitance type level sensor is submerged down to the bottom of the tank. Water is being circulated through a centrifugal pump kept at constant speed. Water flow in the pipe is controlled by a stepper motor driven needle valve and manual valves. Needle valves are coupled with a separate stepper motor which is controlled by a stepper controller card. The level sensor and stepper motor are interfaced with an IBM compatible PC/XT. There is a bypass outlet between exit of the pump and needle valve position. Delay coil is an extra arrangement for introducing additional transportation lag in the system. Needle valve is the actuator for the level control system. The transfer function model of this process has been derived using two point, method[17,18]

$$G(s) = \frac{4.2e^{-20s}}{1+210s} \tag{1}$$

where $G(s)$ is the open loop transfer function of the tank: process gain = 4.2: time delay = 20 seconds; and time constant = 210 seconds.

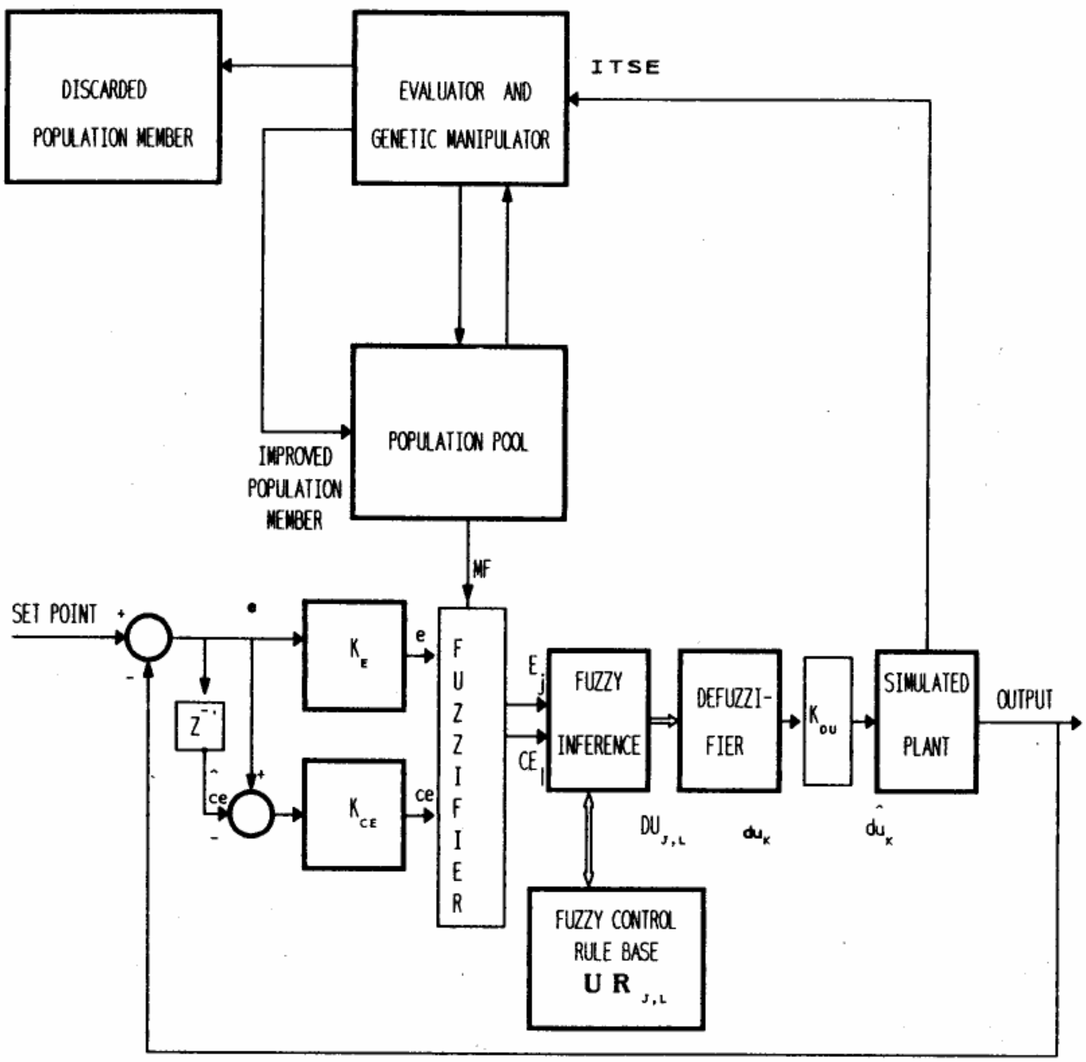

FIG. 2. Block Diagram of GA-FLC.

3. Optimization of membership functions

3.1. *GA-FLC design*

Fig. 2 shows the block diagram of a genetic algorithm based fuzzy logic controller (GA-FLC based FLC). Scale factors K_E for error, K_{CE} for change in error and K_{DU} for change in control action, transform the plant variables error (e), change in error (ce) and change in control action (*du*) into respective computational universes of discourse (UoDs) for simplified calculation. In the initial prototype-FLC design the MFs for the fuzzy sets N, Z and P are selected heuristically as shown in Fig. 3. At each sampling instant error and change in error are fuzzified using MFs as selected by GA.

To initiate the simulated evolution the solutions to the problem are encoded on chromosomes. The coding technique adopted in this work is concatenated unsigned integer coding of points locating the bases of fuzzy triangles N, Z and P to take care of multiple parameter coding[19] GA is used to either expand or shrink the base widths of fuzzy triangles. The FLC is designed with 3 term sets N (negative), Z (zero) and P (positive). The fuzzy triangles for N, Z and P sets used for both process input and output variables are shown in Fig. 3. In this work the parameters used for the MFs simulation for the FLC are the base points L_{ij}, C_{ij} and R_{ij} (left, center and right points that are locating the fuzzy triangles N, Z and P respectively). Let the universes of discourse for error *e*, change in error ce and change in control action du are E, CE and DU respectively. The universes are discretized in to certain number of segments or quantization levels. In Fig. 3, a common UoD is taken to represent UoDs E, CE and DU and the variable '*x*' represents '*e*', '*ce*' or '*du*' . Here the membership function of 'x' takes different values for different fuzzy sets (i = 1,..3 for three fuzzy sets N, Z and P) and quantization levels

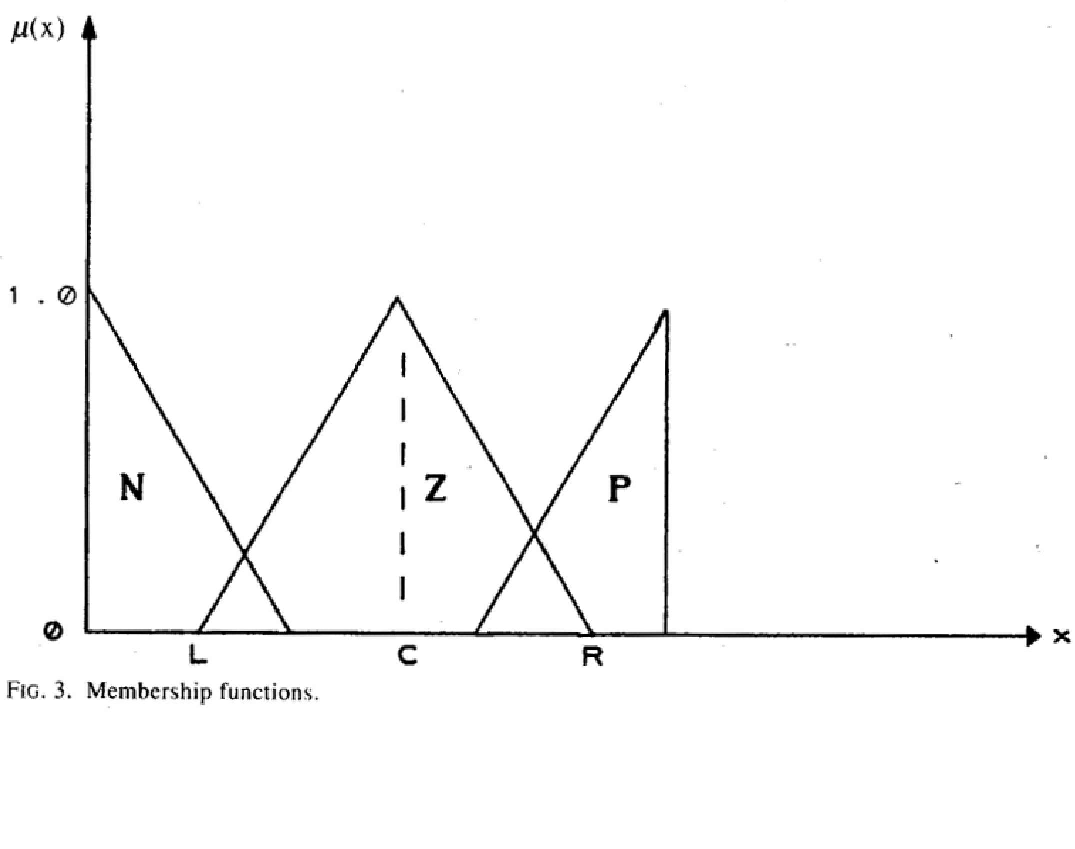

FIG. 3. Membership functions.

(j = 1,....,49). Each chromosome, i.e, one set of nine points is decoded to give membership functions according to a particular input X using equation (2)[17].

$$\mu(X_{ij}) = 1 - M_{L_{ij}}(c_{ij} - x_{ij}) \text{ if } L_{ij} \le x_{ij} \le c_{ij} \tag{2}$$

$$= 1 - M_{R_{ij}}(x_{ij} - c_{ij}) \text{ if } c_{ij} \le x_{ij} \le R_{ij}$$

$$= 0 \qquad \text{if } X_{ij} \le L_{ij} \text{ or } X_{ij} \le R_{ij}$$

where i = 1, ...,3 (for three fuzzy sets),
j = 1, ..., 49 (for quantization levels of a variable),

X_{ij}, an FLC condition or action variable of i-th fuzzy set and j-th quantization levels; M, Slope of MF to the left of the center point; and M , Slope of MF to the right of the center point.

This work uses tournament selection method[3] for selecting efficient chromosomes into the next generation.

3.2. *Fuzzy reasoning*

The fuzzy control rule base consists of a set of statements, relating input variables and the corresponding output variables, to control the process. These statements are in the form of production rules, and let the i-th rule is R which can be expressed as,

R_i : If error (e) is E_i and change in error (ce) is CE_i then change in control action (du) is DU_i (3)

Where E_i, CE_i and DU_i are the fuzzy sets for e, ce and du, respectively.

R_i can be expressed as a fuzzy implication given by ,

$$R_i : E_i \rightarrow CE_i \rightarrow DU_i \tag{4}$$

Let the universes of discourse for error, change in error and change in control action are E, CE and DU, respectively, R_i can be expressed as a fuzzy relation on E, CE, and DU spaces, and is given by the cartesian product

$$R_i = E_i \times CE_i \times DU_i \tag{5}$$

The MF of R_i, i.e., μ_{R_i} is defined as ,

$$\mu_{R_i} = \min(\mu_{E_i}(e), \mu_{CE_i}(ce), \mu_{DU_i}(du)) \tag{6}$$

Using compositional rule of inference for fuzzy reasoning the consequent of each rule for measured values E' and CE' is inferred by using the equation.

$$DU_i' = (E' \times CE') \, o \, R_i \tag{7}$$

where o is composition operator.

The fuzzy control algorithm contains 'r' numbers of fuzzy rules. Therefore, the overall relation matrix R, can be obtained ORing the individual rules, and has been given in equation (8).

$$
\begin{aligned}
R &= R_i \cup R_2 \cup R_r \\
&= \cup R_i = \cup(E_i \times CE_i \times DU_i) \\
&\text{where } i = 1, \ldots, r.
\end{aligned}
\tag{8}
$$

Hence from equations (6) and (8) it can be noted that R can be represented as a matrix of membership functions. Therefore, the fuzzy rule base has been stored in a two dimensional array as shown in Fig. 4. Consequent DU_i' for i-th rule R_i can be accessed using the row index j for error and column index 1 for change in error of the rule base matrix. Hence, rule R_i, consequent DU_i', error E_i, and change in error CE can be redefined as $R_{j,1}$, $DU_{j,1}'$, E_j and CE_1, respectively. Hence equation (7) can be rewritten as,

$$DU_{j,1}' = \left(E_j' \times CE_1'\right) o R_{E_j \to CE_1 \to DU_{j,1}} \tag{9}$$

Where $R_{j,1}$ is redefined as $R_{E_j \to CE_1 \to DU_{j,1}}$

If Sup-min operation is used for composition, the above expression reduces to,

$$\mu_{DU_{j,1}'}(du) = \sup_{e,ce} - \min\left(\min\left(u_{E},(e_k), \mu_{CE'}(ce_k)\right), \mu_{R_{j,1}}(e, ce, du)\right) \tag{10},$$

Equation (10) can be further simplified, for deriving the control action, to,

$$\mu_{DU_{j,1}'}(du) = \min\left[\min\left\{u_{E_j}(e_k), \mu_{CE_1}(ce_k)\right\}, \mu_{DU_{j,1}'}(du)\right] \tag{11},$$

$E_j \downarrow$ / $CE_1 \rightarrow$	N	Z	P
N	N	N	Z
Z	N	Z	P
P	Z	P	P

FIG. 4. Fuzzy control rule base.

The consequent for the complete set of rules is given by,

$$\mu_{DU'} = \max\left(\mu_{DU'_{i,1}}\right) \tag{12}$$

The crisp control action for use in simulated plant is computed using center of area method as ,

$$du_k = \frac{\sum_{i=1}^{n} \mu_{DU'}(du_i) du_i}{\sum_{i=1}^{n} \mu_{DU'}(du_i)} \tag{13}$$

Where n is number of quantization levels,

The transfer function of the liquid level system has been converted into the following discrete equation to get the simulation model,

$$y(k) = d_o \times y(k-1) + c_o \times u(k-s) + c_1 \times u(k-s-1) \tag{14}$$

Where $y(k)$ and $u(k)$ are process output and control input at k-th sampling instant, and d_o, c_o and c_1 are constants, depend upon the sampling time, process gain and time delay t_d and s is the maximum possible integer period of sampling interval in t_d.

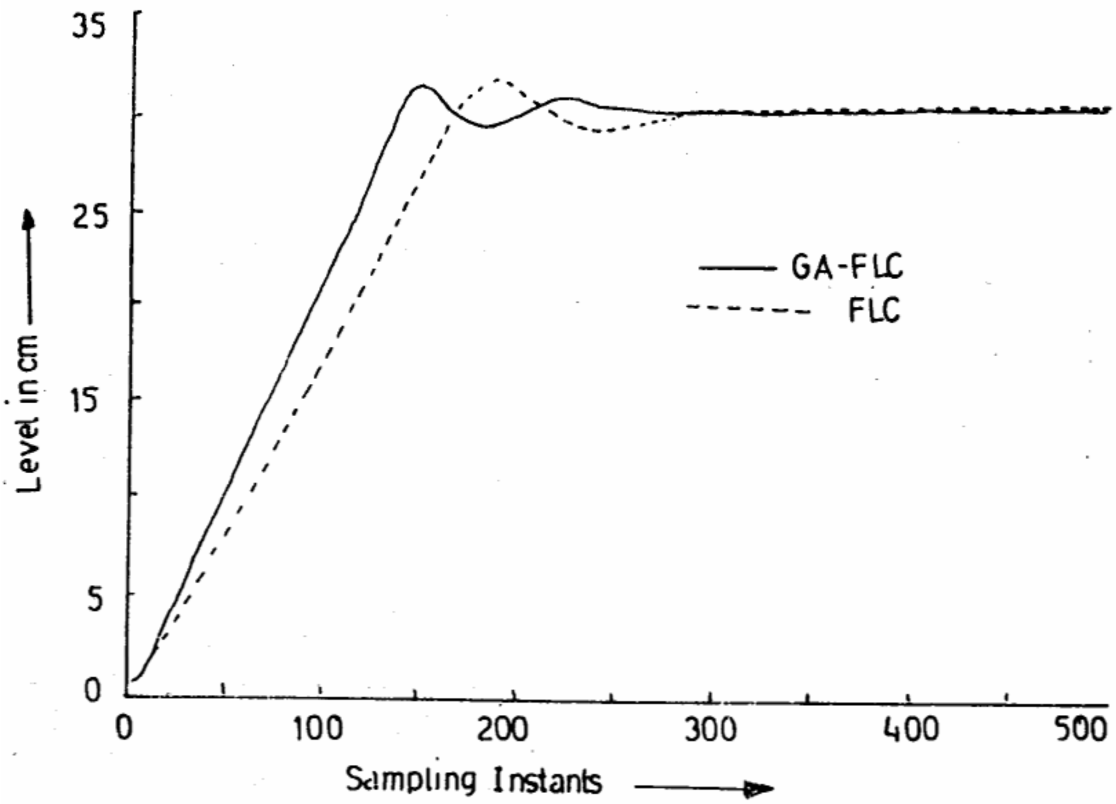

FIG. 5. Comparison of responses of FLC and GA-FLC for $K_E = 12$, $K_{CE} = 50$ and $K_{DU} = 0.01$.

4. Simulation results and discussion

For simulations of the genetic algorithm, the following parameters have been used:

Population size	20
Probability of crossover	0.6
Probability of mutation	0.003

In Fig. 5 the performance of GA-FLC and conventional FLC have been compared for the values of $K_E = 12$, $K_{CE} = 50$, $K_{DU} = 0.01$ and set point = 30 cm. Fig. 6 gives the responses of FLC and *GA-FLC* for $K_E = 14$, $K_{CE} = 21$ and $K_{DU} = 0.01$. It is clear from figures 5 and 6 that GA-FLC performs better than FLC.

5. Conclusions

The proposed approach significantly reduces the time and effort to select MFs for achieving better control behaviour. It also gives a clear understanding of the effect of MFs on the controller performance, and effect of plant parameter variations in terms of MFs. This work describes the superiority of genetic algorithm to a liquid level control with optimal selection of membership function of an FLC. Hence thereby it provides a way to use genetic algorithm in the existing plants controlled by fuzzy logic for better performance. One such case, is the the control of a cement kiln. Since a GA does not

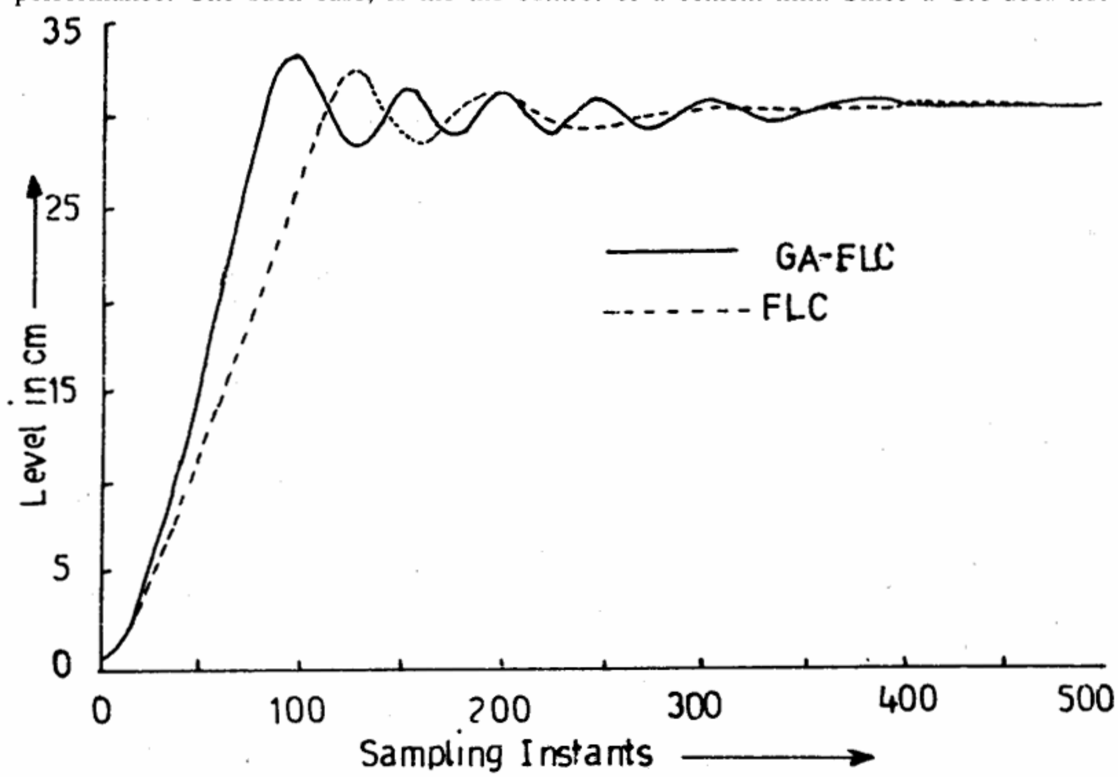

FIG. 6. Comparison of responses of FLC and GA-FLC for $K_E = 14.0$, $K_{CE} = 21.0$ and $K_{DU} = 0.01$.

require any specific information, it is more flexible than any other numerical optimization technique.

Acknowledgement

The first author acknowledges with thanks to the coordinator MAP laboratory, Dept. of Electrical Engg., IIT Delhi, for extending the computational and laboratory facilities for carrying out this research work. Also, the authors are thankful to the reviewers for their careful reading and helpful comments.,

References

1. MAMDANI, E. H. Application of fuzzy algorithm of a simple dynamic plant, *Proc. IEEE*, 1974, Vol.121, pp. 1585–1588.
2. MAMDANI, E. H. AND ASSILIAN, S. A Fuzzy Logic Controller for a Dynamic Plant, *Intl. j. Man-Machine Stud.*, 1975, Vol.7, pp. 1–13.
3. LEMKE, H. R.VAN NAUTA AND KICKERT, W. J. The application of fuzzy set theory to control a warm water plant, J. AI, 1976, Vol. A17, pp. 8–18.
4. KICKERT, W. J. AND LEMKE, H. R. VAN NAUTA. Application of a fuzzy controller in a warm water plant, Automatica, 1976, Vol. 12, pp. 301–308.
5. KING, P. J. AND MAMDANI, E. H. The application of fuzzy control systems to industrial processes, Automatica, 1976, Vol. 13, pp.235–242.
6. RUTHERFORD, D AND CARTER, G. Z. A heuristic adaptive controller for a cinter plant, *Proc. 2nd IFAC Symp Johannesburg*, 1976.
7. ØSTERGARD, J. J., Fuzzy Logic Control of a Heat Exchange Process, *Internal Report 7601*, Electrical Power Engineering Dept., Tech. Univ.of Denmark, 1976.
8. KICKET, W. J., Further Analysis and Application Fuzzy logic Control, *Internal Report F/WK 2/75, Queen Mary's College London*, 1976.
9. SHARP, N. ROBERT, CHOW, M., BRIGGS STEVE, LARRY AND WINDING, L., Windingland., A Methodology using Fuzzy Logic to optimise Feed forward ANN configuration, IEEE Trans. on Systems, Man Cybernetics, May, 1994, Vol. 24.
10. GOLDBERG, D. E. Genetic and Evolutionary Algorithms come of age, Communication of the ACM, March.94, 37:3, pp.113–119.
11. BUCKLEY, J. J. AND HAYASHI, Y., Fuzzy genetic algorithms and applications, Fuzzy sets and systems, Jan., 1994, 61:2, pp. 129–136.
12. ISHIBUCHI, H., NOZAKI, K., YAMAMOTO, N. AND TANAKA, H. Construction of Fuzzy Classifier Systems with rectangular Fuzzy rules using GA, Fuzzy Sets and System, Aug., 1994, pp.237–253.
13. HAYASHI, Y. .Fuzzy neural networks: A survey, Fuzzy Sets and Systems, Aug. 25, 1994, 66:1, pp.1–13.
14. BOUSLAMA, FAOUZI, AND ICHIKAWA, AKIRA, Application of Neural Networks to Fuzzy Control, Neural Networks, 1993, Vol. 6, pp. 791–799.

15. PROCYK, T. J AND MAMDANI, E. H. A Linguistic self-organizing process controller, *Automatica*, 1979, Vol. 15, pp. 15–30.

16. KARR, C. Genetic Algorithms for fuzzy controllers, AI Expert. Feb, 1991, pp. 26–33.

17. SUBUDHI, B., A Genetic Algorithm Based Adaptive Fuzzy Logic Controller for Process Control Applications, M.Tech. Thesis, Department of Electrical Engg., Indian Institute of Technology, Delhi, 1993.

18. CORRIPIO, A. B. Tuning of industrial control systems, Instrument Society of America, North California, 1990.

19. KARR, C. L. AND GENTRY, E. J. Fuzzy control of PH using Genetic Algorithm, *IEEE Trans. on fuzzy systems*, Feb. 1993, Vol. 1, pp. 46–53.

20. AHMAD, N. Real time intelligent distributed control systems, Ph. D. Thesis, Department of Electrical Engg., Indian Institute Technology, Delhi, India, 1992.

21. MAMDANI, E. H. Aplication of fuzzy Logic to Approximate Reasoning using linguistic synthesis, 3IEEE Trans. on Computers, 1977, Vol. C-26, No.12, pp. 1182–1191.

22. SWAIN, A. K. AND SUBUDHI, B. Fuzzy-PID Hybrid control scheme for cement kiln control, Proc. IEEE Singapore Internationål Conference on Intelligent Control & Instrumentation1, 3-7, July' 1995, pp. 103–108.