Linearizing Control of an Induction Motor

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ABSTRACT: **An attempt has been made in the present work to improve the dynamic and steady state performances of the vector controlled induction motor drive. Input-output linearization technique is used to linearize the model of drive system and decouple the flux and torque loop. State feedback controllers are designed for the linearized system using pole placement technique to obtain desired dynamic and steady state responses. Test results show that the performance of the drive system with the proposed control technique and designed controller is an improvement over the vector controlled drive and the proposed drive system is more flexible.**

I. INTRODUCTION

Induction motors are widely used in industry, for their simple and robust structure, higher torque-to-weight ratio, higher reliability and ability to operate in hazardous environment. However, unlike DC motor, their dynamic response is sluggish and control is a challenging task, because of the inherent coupling between rotor current and airgap flux, responsible for the torque production. The control of IM in field coordinates using vector control (also known as field oriented control) [1], leads to decoupling between the flux and torque, thus, resulting in improved dynamic torque and speed responses. Significant advances have been made in vector control of induction motors since its inception. A disadvantage of the conventional field-oriented controller is, the method assumes that [1], the magnitude of the rotor flux is regulated to a constant value. Though good dynamic current (or torque) and speed responses are obtained with vector control, the torque is only asymptotically decoupled from the flux, i.e., decoupling is obtained only in steady state, when the flux amplitude is constant. Coupling is still present, when flux is weakened in order to operate the motor at higher speed within the input voltage saturation limits, or when flux is adjusted in order to maximize power efficiency [2]. This has led to further research on application of differential geometry [2-7], to develop the control techniques for linearization and decoupling control. After the theory was proposed [3, 4], it has drawn attention of many researchers for further development and implementation. These techniques have resulted in solutions to several problems, including feedback linearization, input-output linearization and decoupling control. Reference [5] achieved decoupling of torque and flux by a static multivariable state-feedback controller. Decoupling is also obtained in [6] by a static state-feedback controller using the amplitude and frequency of the supply voltage as inputs. A voltage command input-output linearization controller is developed in [7], and a current command input-output linearization controller is reported in [2]. Feedback linearizing control technique is used in [8] to design a controller for switched reluctance motor. A new control scheme is presented in this paper based on [2-11] for linearization and decoupling control of induction motor, and also to improve the dynamic torque and speed responses for high performance motion control applications.

In section II, the induction motor model in synchronously rotating reference frame is reviewed. In section III, the induction motor model is linearized by input-output linearization technique. Linear state feedback controllers are synthesized to obtain good dynamic and steady state response. Test results are discussed in section IV.

II. STATE VARIABLE MODEL

From the voltage equations of the induction motor in the synchronously rotating d-q axes reference frame, the state space model with stator current and rotor flux components as state variables is :

$$\frac{d}{dt}\begin{bmatrix} i_s \\ \psi_r \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}\begin{bmatrix} i_s \\ \psi_r \end{bmatrix} + \begin{bmatrix} B_1 \\ 0 \end{bmatrix} v_s \qquad (1)$$

The stator current which is measurable is taken as the output, which is expressed as

$$i_s = \begin{bmatrix} I & 0 \end{bmatrix}\begin{bmatrix} i_s \\ \psi_r \end{bmatrix} \qquad (2)$$

where, $i_s = y = \begin{bmatrix} i_{ds} & i_{qs} \end{bmatrix}^T$, $\psi_r = \begin{bmatrix} \psi_{dr} & \psi_{qr} \end{bmatrix}^T$, and $v_s = \begin{bmatrix} v_{ds} & v_{qs} \end{bmatrix}^T$.

$A_{11} = -a_1 I - \omega_e J$, $A_{12} = a_2 I - P a_3 \omega_r J$, $A_{21} = a_5 I$,
$A_{22} = -a_4 I - (\omega_e - P\omega_r) J$, $B_1 = c I$.

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, and $J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$.

$c = L_r / (L_s L_r - L_m^2)$,

$a_1 = c R_s + c R_r L_m^2 / L_r^2$, $a_2 = c R_r L_m / L_r^2$,

$a_3 = c L_m / L_r$, $a_4 = R_r / L_r$, $a_5 = R_r L_m / L_r$,

R_s, R_r, L_s, L_r, L_m : Motor parameters (given in appendix),

P : Number of pole pairs,

ω_r : Mechanical rotor angular velocity,

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ω_e : Synchronous electrical angular velocity,
v_{ds} , v_{qs} : d-q axes stator phase voltages,
i_{ds} , i_{qs} : d-q axes stator phase currents,
ψ_{dr} , ψ_{qr} : d-q axes rotor fluxes.

The torque developed by the motor is :

$$T_e = K_t (\psi_{dr} i_{qs} - \psi_{qr} i_{ds}) \tag{3}$$

where, $K_t = 3 P L_m / 2 L_r$.

III. CONTROLLER DESIGN

A. Linearizing Control

The conditions required for vector control [1] are :

$\psi_{qr} = 0$ and $\dot{\Psi}_{qr} = 0$.

From (1),

$$\dot{\psi}_{qr} = a_5 i_{qs} - (\omega_e - P \omega_r) \psi_{dr} - a_4 \psi_{qr} .$$

Indirect vector control is obtained, when

$$a_5 i_{qs} = (\omega_e - P \omega_r) \psi_{dr}$$

$$\text{or, } \omega_e = P \omega_r + a_5 i_{qs} / \psi_{dr} \tag{4}$$

When (4) is satisfied, the dynamic behaviour of the induction motor is :

$$\dot{i}_{ds} = - a_1 i_{ds} + a_2 \psi_{dr} + \omega_e i_{qs} + c v_{ds} \tag{5}$$

$$\dot{i}_{qs} = - \omega_e i_{ds} - a_1 i_{qs} - P a_3 \omega_r \psi_{dr} + c v_{qs} \tag{6}$$

$$\dot{\psi}_{dr} = - a_4 \psi_{dr} + a_5 i_{ds} \tag{7}$$

$$T_e = K_t \psi_{dr} i_{qs} \tag{8}$$

The concept behind field oriented control is that rotor flux can be controlled according to (7), with i_{ds} acting as the control input. The q-axis current component i_{qs} serves as an input in order to control the torque (8) as a product of ψ_{dr} and i_{qs}. Even the field oriented induction motor model described by (5-8) has nonlinearity and interaction. The speed emf term $(\omega_r \psi_{dr})$ appearing in (6) makes the current dynamics nonlinear and speed dependent. Equations (5) and (6) show that interaction between current components exists, in the rotating reference frame. The transition from field oriented voltage components, v_{ds} and v_{qs} to current components as in (5) and (6) involves leakage time constants and interactions. During the flux transient period (7), coupling of flux and torque is apparent from (5) to (8). The interaction between current components and nonlinearity in the overall system is eliminated by using the linearization control approach given in [2-7]. This approach consists of change of coordinates and use of nonlinear inputs to linearize the system equations.

The developed torque, T_e is considered as a state variable, replacing i_{qs} in the induction motor model. Differentiating (8) and simplifying with substitution of (4), (6), (7):

$$\dot{T}_e = - (a_1 + a_4) T_e + K_t \psi_{dr} \left[c v_{qs} - P \omega_r (i_{ds} + a_3 \psi_{dr}) \right] \tag{9}$$

The nonlinearities in (5) and (9) are put together and then replaced by nonlinear functions of the form u_1 and u_2 respectively. With these linearizing inputs u_1 and u_2 , (5) and (9) are then modified to (10) and (12) respectively. The induction motor system is now transformed into two linear and decoupled subsystems: electrical and mechanical.

Electrical subsystem is represented by the state equation:

$$\dot{i}_{ds} = -a_1 i_{ds} + a_2 \psi_{dr} + u_1 \tag{10}$$

$$\dot{\psi}_{dr} = -a_4 \psi_{dr} + a_5 i_{ds} \tag{11}$$

Mechanical subsystem is represented by the state equation:

$$\dot{T}_e = - (a_1 + a_4) T_e + u_2 \tag{12}$$

$$\dot{\omega}_r = (T_e - T_L - \beta \omega_r) / J \tag{13}$$

The stator input voltage components v_{ds} and v_{qs} in terms of u_1 and u_2 are:

$$v_{ds} = (- \omega_e i_{qs} + u_1) / c \tag{14}$$

$$v_{qs} = \frac{1}{c} \left[P \omega_r (i_{ds} + a_3 \psi_{dr}) + \frac{u_2}{K_t \psi_{dr}} \right] \tag{15}$$

The transformed model given above is valid only for $\psi_{dr} \neq 0$. Since the induction motor system described by (10)-(13) is linear and decoupled, the developed torque (or the speed) and the rotor flux are independently controlled. State Feedback Controllers (SFC) are designed to obtain desired transient and steady state performance.

B. Design of State Feedback Controller (SFC)

A general representation of the electrical subsystem (10), (11), and the mechanical subsystem (12), (13) is given by

$$\dot{x} = A x + B u + E d \tag{16}$$

$$y = C x \tag{17}$$

where, for the electrical subsystem: $u = u_1$, $y = i_{ds}$, $d = 0$,

$$x = \begin{bmatrix} i_{ds} \\ \psi_{dr} \end{bmatrix}, \; A = \begin{bmatrix} -a_1 & a_2 \\ a_5 & -a_4 \end{bmatrix}, \; B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \; C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

For the mechanical subsystem: $u = u_2$, $y = \omega_r$, $d = T_L$,

$$A = \begin{bmatrix} -(a_1 + a_4) & 0 \\ \frac{1}{J} & -\frac{\beta}{J} \end{bmatrix}, \; B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \; C = \begin{bmatrix} 0 & 1 \end{bmatrix}, \; E = \begin{bmatrix} 0 \\ -\frac{1}{J} \end{bmatrix}$$

Since the state feedback control is basically a proportional control, the steady-state error may exist. To remove this error, the deviation term between the output y and the reference value y_r is introduced as a new state.

$$\tilde{y} = y - y_r \tag{18}$$

Assuming y_r and d to be constant, differentiating (16) and (18) and arranging, a standard form of state equation is obtained as:

$$\dot{z} = \hat{A} z + \hat{B} v \tag{19}$$

where a new augmented state vector, z and a new control vector, v are defined as

$$z = \begin{bmatrix} \dot{x} \\ \tilde{y} \end{bmatrix} , \quad v = \dot{u} \quad \text{and} \quad \hat{A} = \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix} , \quad \hat{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}$$

A linear state feedback control law of the form:

$$v = K z \tag{20}$$

can be designed for the augmented system of (19) by the pole placement technique [9], where K is the feedback gain matrix. For the closed loop system (19), with the state feedback control (20) to have the desired eigenvalues, controllability conditions should be satisfied as follows [10]:

1. the pair ($\hat{A}$, $\hat{B}$) is controllable
2. the matrix $\begin{bmatrix} A & B \\ C & 0 \end{bmatrix}$ has full rank.

Since these conditions are satisfied for the given system, the control law can be derived by substituting z into (20), and then integrating. The linear state feedback control law for both electrical and mechanical subsystems is

$$u = K_p\ x + K_i \int_0^t (y - y_r) dt \tag{21}$$

The above control law comprises of the feedback of the sates (first term) as well as the integral of the output errors (IOE) (second term) and does not require the knowledge of the disturbance vector. The IOE feedback makes the controller fairly robust by making it insensitive to modeling imperfections and step like disturbances.

For the electrical subsystem, the control law is

$$u_1 = K_{p1}\ i_{ds} + K_{p2}\ \Psi_{dr} + K_{i1} \int_0^t (i_{ds} - i_{ds}^*) dt \tag{22}$$

The control law for the mechanical subsystem is

$$u_2 = K_{p3}\ T_e + K_{p4}\ \omega_r + K_{i2} \int_0^t (\omega_r - \omega_r^*) dt \tag{23}$$

In the above two control laws, the proportional gains K_{p1} ,K_{p2} , K_{p3} , K_{p4} and the integral gains K_{i1} , K_{i2} are determined using the pole placement technique. The eigenvalues of the augmented system matrix of the electrical subsystem are –288.55, –10.22 and 0.0. To place the closed loop poles of the electrical subsystem at –288.55, –20 and –20, the gains of the state feedback controller are, K_{p1} = –29.78 , K_{p2}= –2130.2 and K_{i1} = –28,922. The eigenvalues of the augmented matrix of the mechanical subsystem are –298.77, –0.34 and 0.0. To place the closed loop poles of the mechanical subsystem at –298.77, –10 and –8, the gains of the state feedback controller are, K_{p3} = –17.66, K_{p4} = –47.1 and K_{i2} = –210.33. The block diagram of the electrical subsystem with SFC is shown in Fig. 1. The block diagram of the mechanical subsystem with SFC is shown in Fig. 2. The test results of the closed loop system with these controllers are presented in the next section.

IV. TEST RESULTS AND DISCUSSIONS

The proposed controller has been simulated using MATLAB on an induction motor drive system, whose data are listed in Table- I. Two cases of simulation tests are presented here. First the drive is subjected to a benchmark test, as shown in Fig. 3. The unloaded motor is required to accelerate from standstill condition to 800 rpm in 0.5 sec. Then reference speed is kept constant at 800 rpm from time, t=0.5 sec to 1.5 sec. Rated load torque of 5 N.m is applied from t=0.8 sec to 1.2 sec. Then reference speed is changed from 800 rpm at t=1.5 sec to –800 rpm at t=2.5 sec in no load condition, kept constant at –800 rpm until t=3.5 sec, load torque of –5 N.m is applied from t=2.8 sec to 3.2 sec. After that motor is made to stop from –800 rpm in 0.5 sec. Dynamic response to the benchmark test is shown in Fig. 4. After starting of motor, the rated rotor flux linkage of 0.45 V.s is established in 0.5 sec and remains at the rated value after that, irrespective of speed and load torque changes. The quadrature component of the rotor flux remains zero, through-out, indicating decoupling of flux and torque. The rotor speed tracks the reference speed with a little delay, and reaches 800 rpm at t=0.7 sec without any overshoot. When the rated load torque is applied, there is a temporary dip in speed of 145 rpm. After the load is released, again there is a temporary speed overshoot of 140 rpm. When the reference speed is reduced to zero linearly, rotor speed follows with a delay, and becomes zero at t=2.14 sec. For reverse motoring (second half cycle), the dynamic response is similar to that of forward motoring.

In the second case, the reference speed is changed in steps. First the speed is increased from 1000 rpm to 1500 rpm at rated rotor flux linkage of 0.45 V.s and then further increased to 1800 rpm with flux weakening, and then decreased to 1500 rpm with flux strengthened to rated value. Load torque is kept constant at 1 N.m. The dynamic response is shown in Fig. 5. During flux weakening control above the base speed (1500 rpm to 1800 rpm), the rotor flux linkage reduces from 0.45 V.s to 0.375 V.s in 0.5 sec. The flux linkage also increases from 0.375 V.s to 0.45 V.s in 0.5 sec, when speed is reduced from 1800 rpm to 1500 rpm. During constant torque mode of operation, the rotor flux linkage is constant at 0.45 V.s. The quadrature component of the flux is zero, through-out, indicating decoupling of flux and torque. Speed response does not have any overshoot.

The fact that, speed response does not have any overshoot, is a great advantage of this controller. Speed response also tracks the command value very fast. The responses are better than vector controlled IM drive and this is achieved when the motor is fed from a Voltage Source Inverter (VSI). Fast torque response is another advantage of this system. However, when the model of induction motor is not known exactly, or the parameters used in the model changes due to change in operating conditions, the linearization does not hold good. The decoupling of torque and flux is not obtained, and so also the linearization of the motor model. Because the nonlinearity cancellation is not perfect, under such conditions torque and flux still remains coupled during the transient period, and hence fast transient response can not be obtained. The drive system with the proposed control is now being implemented in the laboratory.

TABLE – I RATING AND PARAMETERS OF THE MOTOR

Three phase, 50 Hz, 0.75 kW, 220V, 3A, 1440 rpm
Stator and rotor resistances: R_s = 6.37 Ω, R_r = 4.3 Ω
Stator and rotor self inductances: L_s = L_r = 0.26 H
Mutual inductance between stator and rotor: L_m = 0.24 H
Moment of Inertia of motor and load: J = 0.0088 Kg ·m^2
Viscous friction coefficient: β = 0.003 N ·m ·s/rad

V. CONCLUSION

A linearization and decoupling control technique for induction motor drive is developed. State feedback controllers are designed using pole placement technique to control the dynamic and steady state responses of the drive. The test results show that the dynamics are controlled in about 0.5 second, fast for such systems, and also the voltage and current variations are within limits. There is no overshoot in speed at all. This is an improvement of conventional vector controlled IM drives. The implementation is underway in the laboratory.

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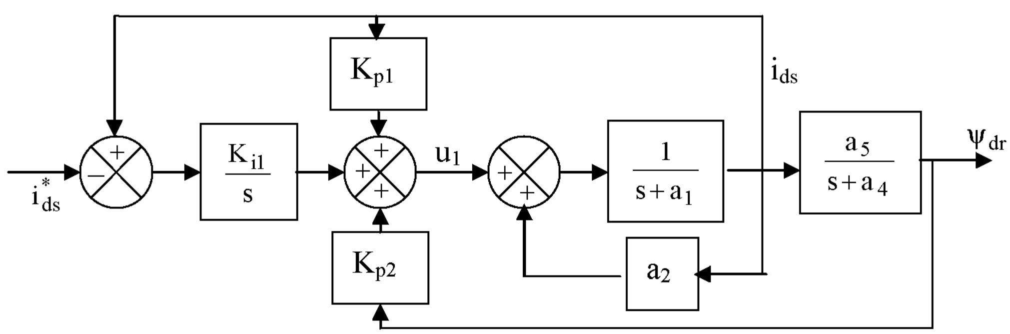

Fig. 1. Block diagram of the electrical subsystem with SFC

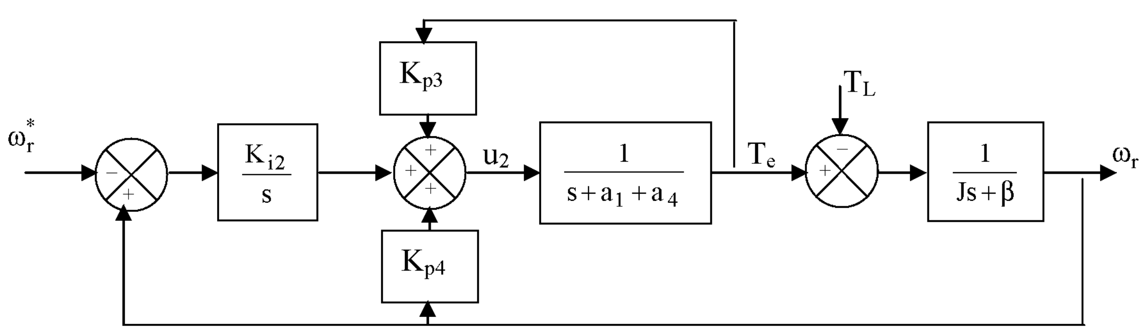

Fig. 2. Block diagram of the mechanical subsystem with SFC

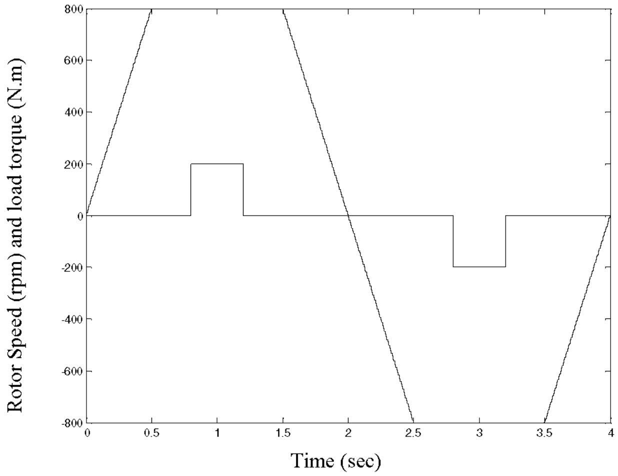

Fig. 3 Benchmark test

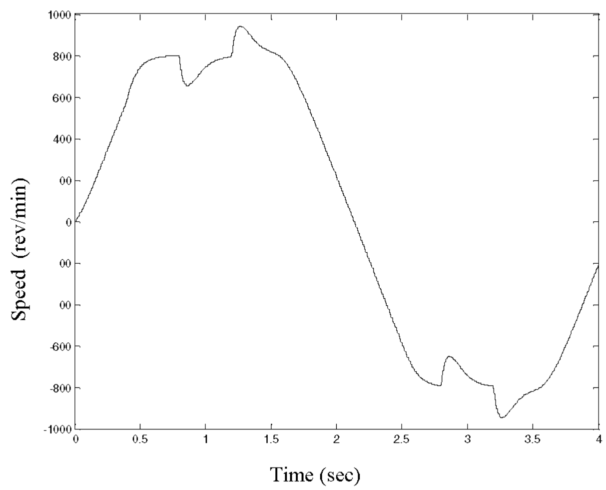

Fig. 4 (a)

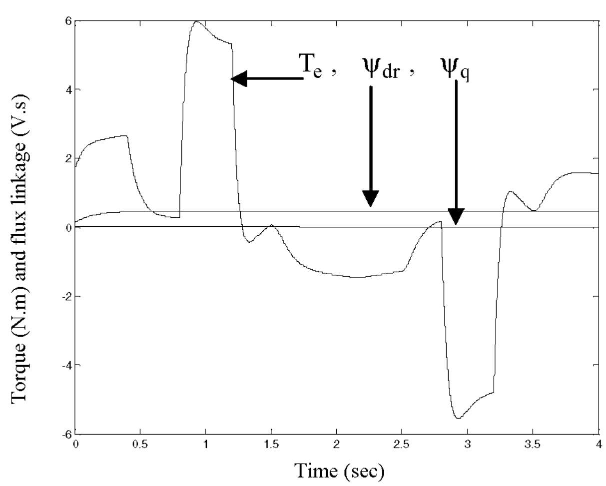

Fig. 4 (b)

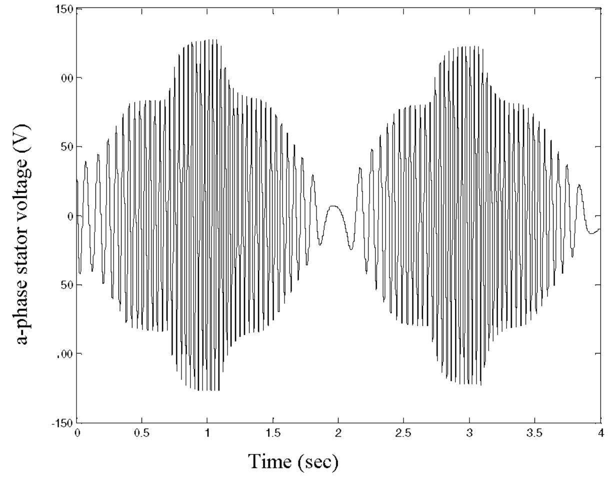

Fig. 4 (c)

Fig. 4. Benchmark test results (a) Speed response, (b) Torque and flux linkage, (c) a- phase stator voltage.

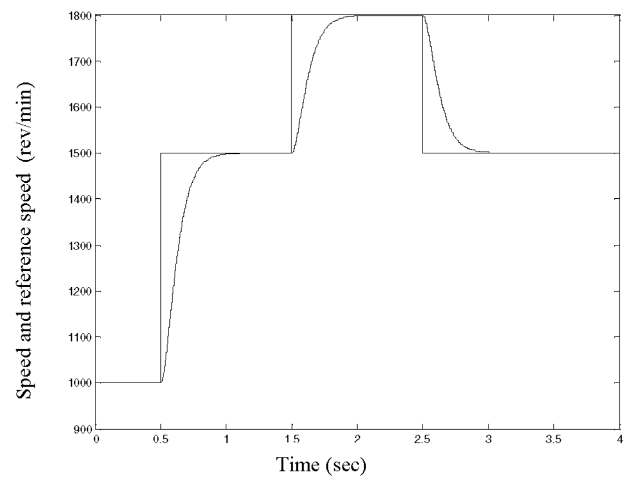

Fig. 5 (a)

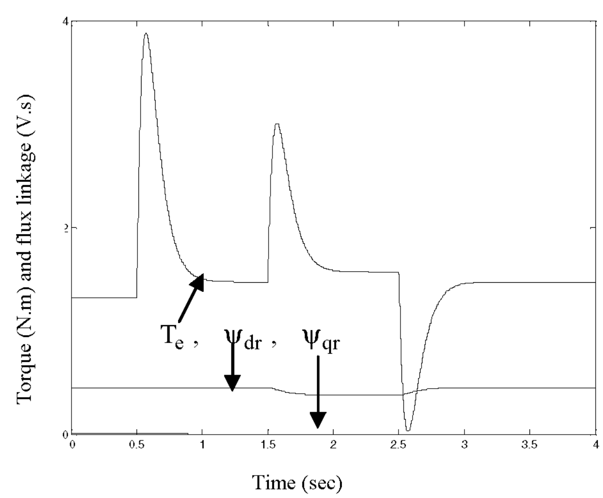

Fig. 5 (b)

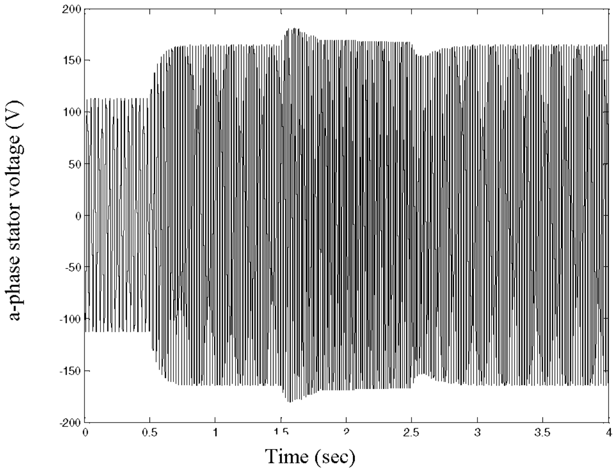

Fig. 5 (c)

Fig. 5. Step change in speed command with flux weakening (a) Speed response, (b) Torque and flux linkage, (c) a- phase stator voltage