

## A Fuzzy Sliding Mode Controller for a Field-Oriented Induction Motor Drive

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*This paper presents a robust control technique for a field oriented induction motor drive. Sliding Mode Controller (SMC) and Fuzzy Sliding Mode Controller (FSMC) are designed for the speed loop of the drive. The design steps for both the controllers are laid down clearly. The FSMC uses three-level input membership sets and five-level output membership set of symmetrical triangular shape, nine fuzzy rules, and the Center-of-Gravity defuzzification technique. The performance of the Fuzzy Sliding Mode Controller has been evaluated, through simulation studies, with respect to the conventional sliding mode controller. The chattering free improved performance of the FSMC makes it superior to conventional SMC, and establishes its suitability for the induction motor drive.*

**Keywords:** Field oriented control, Sliding mode controller, Fuzzy sliding mode controller

### NOTATIONS

$v_{ds}$  ( $v_{qs}$ ): the d-axis (q-axis) stator voltage

$i_{ds}$  ( $i_{qs}$ ): the d-axis (q-axis) stator current

$\Psi_{dr}$  ( $\Psi_{qr}$ ): the d-axis (q-axis) rotor flux linkage

$\omega_r$ : mechanical rotor angular velocity,

$\omega_e$ : fundamental supply frequency,

P: number of pole pairs,

$K_T$ : torque constant

$T_e$ : developed torque

$T_L$ : load torque

J: moment of inertia of rotor with load

$\beta$ : viscous friction coefficient (N·m·s/rad)

$\lambda$ : bandwidth of the sliding mode control system

$\eta$ : a positive constant

$\Delta G_{max}$ : maximum error in estimation of G

v: upper bound of command acceleration

$K_{max}$ : gain of the sliding mode controller,

$K_N$  (or  $K_{Fuzz|N}$ ): the fuzzy value of the controller gain

$\bar{K}_{Fuzz|N}$ : defuzzified value of the controller gain

$\mu_{out}$ : degree of membership of output as a function of the fuzzy value of output

\*: denotes command or reference value

### INTRODUCTION

Induction motors fulfill the *de facto* industrial standard, because of their simple and robust structure, higher torque-to-weight ratio, higher reliability and ability to operate in hazardous environment. However, because of the coupling between torque and flux, unlike dc motor, their control is a challenging task. One of the classical methods of induction motor control, by now is the field-oriented control<sup>1</sup>. It leads to decoupling between the flux and torque, thus, resulting in improved dynamic response of torque and speed. But ideal field orientation is obtained if the machine parameters are accurately known under all conditions. If the machine parameters used in the decoupling control scheme can not track their true

values, the efficiency of the motor drive is degraded owing to reduction of torque generating capability and magnetic saturation caused by over excitation. The dynamic control characteristic is also degraded. In addition to this parameter detuning problem, the load torque disturbance and measurement noise also make a robust control technique mandatory, to meet the standards of a high performance drive.

To improve the field oriented control of induction motor under the above mentioned problems and to track complex position and torque trajectories, sliding mode control<sup>2-5</sup> has been proposed. A sliding mode speed controller<sup>2</sup> based on a switching surface is demonstrated. With this switching surface, the stability is guaranteed for the speed control, and insensitivity to uncertainties and disturbances is also obtained. Sliding mode control<sup>3</sup> is applied to position control loop of an indirect vector controlled induction motor drive, without rotor resistance identification scheme. Results are compared with a fixed gain controller. A sliding mode based adaptive input-output linearizing control<sup>4</sup> is presented. The motor flux and speed are separately controlled by sliding mode controllers with variable switching gains. A sliding mode controller with rotor flux estimation<sup>5</sup> is presented. Rotor flux is also estimated using a sliding mode observer. The results are compared with a field oriented controller and an input-output linearizing controller.

Fuzzy logic controller is also used<sup>6</sup> for solving the parameter detuning problem of indirect vector controlled induction motor drive. A fuzzy slip speed estimator<sup>7</sup>, consisting of a fuzzy detuning correction controller and a fuzzy excitation controller, is presented for improving the decoupling characteristics of the drive. An on-line fuzzy tuning technique<sup>8</sup> is proposed for indirect field oriented induction motor drive. It has also been proved<sup>9</sup> that, in principle, certain type of fuzzy logic controller works like a modified sliding mode controller. Fuzzy logic controller and sliding mode controller are combined to formulate the *fuzzy sliding mode controller*<sup>9</sup>, whose application potential is yet to be explored. This *fuzzy sliding mode controller* is expected to be a robust control technique like both sliding mode and fuzzy logic controllers, while being free of the demerit of sliding mode controller, namely

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the chattering of the control input and some of the system states.

This paper investigates the applicability of fuzzy sliding mode controller<sup>9</sup> to a field oriented induction motor drive. Systematic procedure is developed to design sliding mode controller and fuzzy sliding mode controller, and a comparative study is carried out between the two.

## FIELD ORIENTED INDUCTION MOTOR

The dynamic equations of the induction motor in the synchronously rotating d-q reference frame, with stator current and rotor flux components as variables, are considered. The mathematical constraint for field orientated control is:

$$\psi_{qr} = 0 \text{ and } \dot{\psi}_{qr} = 0 \quad (1)$$

Equation (1) is satisfied and field orientation is obtained, when

$$\omega_e = P \omega_r + a_5 i_{qs} / \psi_{dr} \quad (2)$$

When eqn. (2) is satisfied, the dynamic behavior of the induction motor is:

$$\dot{i}_{ds} = -a_1 i_{ds} + a_2 \psi_{dr} + \omega_e i_{qs} + c v_{ds} \quad (3)$$

$$\dot{i}_{qs} = -\omega_e i_{ds} - a_1 i_{qs} - P a_3 \omega_r \psi_{dr} + c v_{qs} \quad (4)$$

$$\dot{\psi}_{dr} = -a_4 \psi_{dr} + a_5 i_{ds} \quad (5)$$

$$T_e = K_T \psi_{dr} i_{qs} \quad (6)$$

where,  $c = L_r / (L_s L_r - L_m^2)$ ,

$$a_1 = c R_s + c R_r L_m^2 / L_r^2,$$

$$a_2 = c R_r L_m / L_r^2, \quad a_3 = c L_m / L_r,$$

$$a_4 = R_r / L_r, \quad a_5 = R_r L_m / L_r$$

Ideally, torque and flux are decoupled under the above condition, resulting in field orientation. However, due to the presence of the motor parameter  $a_5$  in eqn. (2), the indirect field oriented control is highly parameter sensitive. On-line adaptation to achieve ideal field orientation is an important but very difficult issue. Sliding mode control<sup>10</sup> is a good robust control technique against parameter detuning problem. But, it has the demerit, namely chattering of control input and some of the system states. Fuzzy sliding mode control<sup>9</sup> is also a robust control technique like sliding mode control and it does not have the above demerit. The following sections present the design principles of a sliding mode controller (SMC) and a fuzzy sliding mode controller (FSMC) based on the motor eqns. (1) to (6). Their comparative study for the induction motor drive has been carried out.

## DESIGN OF SLIDING MODE CONTROLLER

In sliding mode control, the system is controlled in such a way that the error in the system state (say, speed) always moves towards a sliding surface. The

sliding surface (s) is defined with the tracking error (e) of the state and its rate of change ( $\dot{e}$ ) as variables.

$$s = \dot{e} + \lambda e \quad (7)$$

The distance of the error trajectory from the sliding surface and its rate of convergence are used to decide the control input. The sign of the control input must change at the intersection of tracking error trajectory with the sliding surface. In this way, the error trajectory is forced to move always towards the sliding surface. Once it reaches the sliding surface, the system is constrained to slide along this surface to the equilibrium point. The condition of sliding mode<sup>10</sup> is:

$$\dot{s} \text{sgn}(s) \leq -\eta \quad (8)$$

To design a sliding mode speed controller for the field oriented induction motor drive system, the steps are as follows. The speed dynamic equations are given by:

$$\dot{\omega}_r = g_1 + (T_L / J) \quad (9)$$

$$\text{and, } \dot{\omega}_r = G + u + d \quad (10)$$

where, u is the control input given by:

$$u = K_T \psi_{dr}^* c v_{qs} / J \quad (11)$$

G is a function, which can be estimated from measured values of currents and speed.

$$G = (-\beta g_1 + K_T \psi_{dr}^* g_2) / J \quad (12)$$

$$g_1 = (-\beta \omega_r + K_T \psi_{dr}^* i_{qs}) / J$$

$$g_2 = -(a_1 + a_4) i_{qs} - P \omega_r (1 + a_3 L_m) i_{ds}$$

In eqn. (10), d is the disturbance due to the load torque, and error in estimation of G, which may occur due to measurement inaccuracies.

Substituting (7) and (10) in (8) and simplifying

$$(G + d + \lambda \dot{e} - \dot{\omega}_r^*) \text{sgn}(s) + u \cdot \text{sgn}(s) \leq -\eta \quad (13)$$

To achieve the sliding mode of (8), u is chosen as<sup>10</sup>

$$u = (-\hat{G} - \lambda \dot{e}) - K \cdot \text{sgn}(s) \quad (14)$$

The first term in (14),  $(-\hat{G} - \lambda \dot{e})$  is a compensation term and the second term is the controller. The compensation term is continuous and reflects knowledge of the system dynamics. The controller term is discontinuous and ensures the sliding to occur. From eqns. (13-14), the controller gain, K is derived as<sup>10</sup>

$$K_{\max} \geq (|\Delta G_{\max}| + |d_{\max}| + \eta + v) \quad (15)$$

The controller gain, K is determined using (15) and considering various conditions such as:

- (i) increase in stator and rotor resistance due to temperature rise
- (ii) change in load torque
- (iii) variation in the reference speed

For the induction motor whose rating and parameters are given in Table-1, taking a typical case as (i) 50%

increase in stator and rotor resistance, (ii) change in load torque by 10 N·m in 50 ms (rated torque is 5 N·m), (iii) 50% change in reference (base) speed in 50 ms, the controller gain,  $K_{\max}$  is obtained as  $K_{\max} = 56000 \text{ rad/s}^3$

In a system, where modelling imperfection, parameter variations and amount of noise are more, the value of  $K$  must be large to obtain a satisfactory tracking performance. But larger value of  $K$  leads to more chattering of the control variable and system states. To reduce chattering, a boundary layer of width  $\varphi$  is introduced on both sides of the switching line. Then the control law of (14) is modified as:

$$u = -\hat{G} - \lambda \mathcal{E} - K \cdot \text{sat}(s / \varphi) \quad (16)$$

where,

$$\text{sat}(s / \varphi) = \begin{cases} s / \varphi & \text{if } |s| \leq \varphi \\ \text{sgn}(s) & \text{if } |s| > \varphi \end{cases}$$

This amounts to a reduction of the control gain inside the boundary layer and results in a smooth control signal. The tracking precision is given by:

$$\theta = \varphi / \lambda \quad (17)$$

To have a tracking precision,  $\theta = 1 \text{ rad/s}$ ,

$$\varphi = \theta \lambda = \lambda$$

$$K_{\max} = \varphi \lambda = \lambda^2$$

$$\lambda = \sqrt{K_{\max}} = \sqrt{56.0 \times 10^3} = 236.6 \text{ rad/s} \quad (18)$$

$$\text{and } \varphi = \theta \lambda = 236.6 \text{ rad/s}^2$$

Table – 1 Rating and Parameters of the Induction Motor

Three phase, 50 Hz, 0.75 kW, 220V, 3A, 1440 rpm
Stator and rotor resistances: $R_s = 6.37 \Omega$ , $R_r = 4.3 \Omega$
Stator and rotor self inductances: $L_s = L_r = 0.26 \text{ H}$
Mutual inductance between stator and rotor: $L_m = 0.24 \text{ H}$
Moment of Inertia of motor and load: $J = 0.0088 \text{ Kg} \cdot \text{m}^2$
Viscous friction coefficient: $\beta = 0.003 \text{ N} \cdot \text{m} \cdot \text{s/rad}$

## DESIGN OF FUZZY SLIDING MODE CONTROLLER

The fuzzy sliding mode controller (FSMC) explained here is a modification of the sliding mode controller (eqn. (14)), where the switching controller term,  $-K \cdot \text{sgn}(s)$ , has been replaced by a fuzzy control input as given below.

$$u = (-\hat{G} - \lambda \mathcal{E}) + u_{\text{Fuzz}} \quad (19)$$

$$\text{and } u_{\text{Fuzz}} = -K_{\text{Fuzz}}(e, \mathcal{E}, \lambda) \text{sgn}(s) \quad (20)$$

The gain,  $K_{\text{Fuzz}}$  of the controller is determined from fuzzy rules. The qualitative rules of the fuzzy sliding mode controller are as follows.

- The normalized fuzzy output,  $u_{\text{Fuzz|N}}$  should be negative above the switching line, and positive below it.
- $|u_{\text{Fuzz|N}}|$  should increase as the distance,  $d_1$  between the actual state and the switching line,  $s = 0$ , increases. The distance,  $d_1$  is given by

$$d_1 = \frac{|s|}{\sqrt{1 + \lambda^2}} = \frac{|\lambda e + \mathcal{E}|}{\sqrt{1 + \lambda^2}} \quad (21)$$

- $|u_{\text{Fuzz|N}}|$  should increase as the distance,  $d_2$  between the actual state and the line perpendicular to the switching line increases. The distance,  $d_2$  between the actual state and the line perpendicular to the switching line, is:

$$d_2 = \sqrt{e^2 + \mathcal{E}^2 - d_1^2} \quad (22)$$

The reasons for this rule to be followed are:

- the discontinuities at the boundaries of the phase plane are avoided.
  - the central domain of the phase plane is arrived at very quickly.
- Normalized states,  $e_N$ ,  $\mathcal{E}_N$  that fall out of the phase plane should be covered by the maximum values,  $|u_{\text{Fuzz|N}}|_{\max}$  with the respective sign of  $u_{\text{Fuzz|N}}$ .

The normalized distances,  $d_{1N}$  and  $d_{2N}$  are:

$$d_{1N} = N_1 d_1 \quad \text{and} \quad d_{2N} = N_2 d_2$$

where,  $N_1$  and  $N_2$  are the normalization factors.

These normalized inputs ( $d_{1N}$  and  $d_{2N}$ ) to the fuzzy controller are fuzzified by a three member fuzzy set: { **Z**: Zero, **P**: Positive, **LP**: Large Positive }

The fuzzy set for normalized controller gain (output of the fuzzy controller),  $K_{\text{Fuzz|N}}$  (also denoted as  $K_N$  for brevity) is:

{ **Z**: Zero, **SP**: Small Positive, **MP**: Medium Positive, **VLP**: Very Large Positive }

The membership functions for the normalized inputs are shown in Fig.1(a), and those for the normalized output are shown in Fig.1(b). Linear and symmetrical membership functions are used for ease of realization. Only three-member input sets and five-member output set are chosen, based on engineering experience, so as to have approximately linear transfer characteristics without sacrificing simplicity of the controller. The rule base for the fuzzy controller, consisting of nine rules, is listed in Table-2.

Table – 2 Fuzzy rule base

$d_{1N}$ $d_{2N}$	<b>Z</b>	<b>P</b>	<b>LP</b>
<b>Z</b>	<b>Z</b>	<b>SP</b>	<b>MP</b>
<b>P</b>	<b>SP</b>	<b>MP</b>	<b>LP</b>
<b>LP</b>	<b>MP</b>	<b>LP</b>	<b>VLP</b>

The inference engine performs fuzzy implications, and computes the degree of membership of the output (normalized controller gain) in each fuzzy set using Zadeh AND and OR operations. Then defuzzification is carried out by the Center-of-Gravity method as given in eqn. (23).

$$\bar{K}_{Fuzz|N} = \frac{\int_0^1 \mu_{out} \cdot K_N \cdot dK_N}{\int_0^1 \mu_{out} \cdot dK_N} \quad (23)$$

The defuzzified value,  $\bar{K}_{Fuzz|N}$  is denormalized with respect to the corresponding physical domain,  $K_{Fuzz}$  by the denormalization factor,  $N_u$ .

$$N_u = \frac{K_{Fuzz|max}}{\bar{K}_{Fuzz|N|max}} \quad (24)$$

where,  $\bar{K}_{Fuzz|N|max}$  is the maximum value of defuzzified (but normalized) controller gain, and  $K_{Fuzz|max}$  is the maximum value of the controller gain,  $K_{Fuzz}$ .

Since the sliding mode controller and the fuzzy sliding mode controller, described in this paper, are structurally similar, the maximum gain  $K_{Fuzz|max}$  is taken equal to the gain of the sliding mode controller,  $K_{max}$ , so that comparison of both can be made under similar conditions.

$$K_{Fuzz|max} = 56000 \text{ rad/s}^3$$

For  $N_1 = N_2 = 0.08$  (fixed by engineering judgment and experience), and the above value of  $K_{Fuzz|max}$ , the denormalization factor,  $N_u = 110000$ .

## RESULTS AND DISCUSSIONS

The 3-phase induction motor drive system, whose rating and parameters are given in Table-1, is subjected to various simulation tests with both the above controllers. The simulation study is carried out with a ramp (linear) change in reference speed. The reference speed is linearly increased from 1000 r/min to 1500 r/min in 50 ms, i.e., at a rate 10 (r/min)/ms. The reference d-axis rotor flux linkage is kept at 0.45 V·s, and load torque is kept at zero. The simulation responses of the drive system with sliding mode controller (SMC) are shown in Fig. 2 and those with fuzzy sliding mode controller (FSMC) are shown in Fig. 3. Though the responses with FSMC are generally similar to those with SMC, the speed response has an overshoot of 28 r/min with SMC, but no overshoot is present with FSMC. The q-axis stator voltage increases from initial steady state value of 104 V to final steady state value of 156 V with a peak value of 255 V in SMC and 245 V in FSMC during the transient period. The control input (u) has chattering in SMC, but is free of chattering in FSMC. The q-axis component of stator voltage and current are only affected as they control the torque and hence speed. The field orientation is obvious, as the d-axis stator current and rotor flux remain constant.

To see the chattering-free robust responses of FSMC, the load torque is suddenly increased from 0 to 10 N·m (rated torque is 5 N·m) and then the load is removed after 1 sec. With both SMC (Fig. 4) and FSMC (Fig. 5), there is an instantaneous speed change

of 30 r/min during the change of load. But the drive system recovers to the reference speed of 1000 r/min almost instantaneously. With SMC, the response of current ( $i_{qs}$ ), the q-axis stator input voltage ( $v_{qs}$ ), and the control input (u) have chattering, during the load period. But no such chattering is present in case of FSMC.

## CONCLUSIONS

Sliding mode and fuzzy sliding mode controllers are designed for a field oriented induction motor drive, to have the same maximum controller gain. From the simulation study of both the controllers, it is observed that the control input, the stator input voltage, and some of the states, like speed and stator current, have chattering with sliding mode controller, whereas these are free of chattering with fuzzy sliding mode controller. For the same maximum gain with both the controllers, the speed response is also nearly the same (slightly better in FSMC than SMC), and the stator input voltage is less in case of FSMC compared to SMC. In other words, with fuzzy sliding mode controller, the maximum gain can be increased at the cost of increased stator input voltage, leading to better speed response. So, for chattering-free, robust control of field oriented induction motor drive, fuzzy sliding mode controller is a better choice than sliding mode controller. The number of members in the input and output sets of the fuzzy controller can be increased, so also the number of rules in the fuzzy rule base, so as to closely approximate the linear transfer characteristics within the boundary layer. This would give better performance of the controller at the cost of increased computational time.

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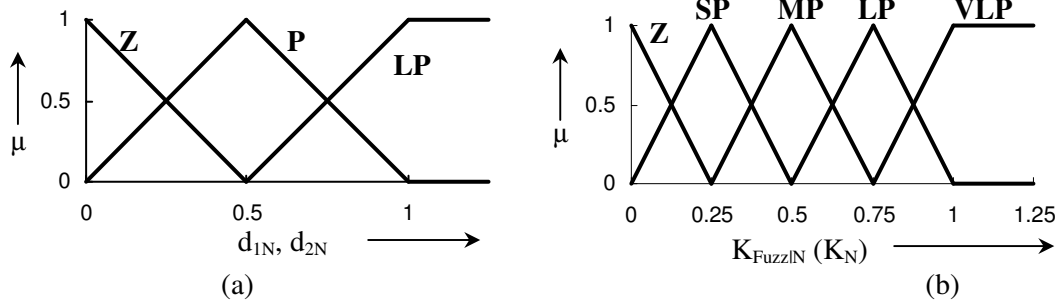


Fig. 1 Membership functions for: (a) normalized inputs, (b) normalized output

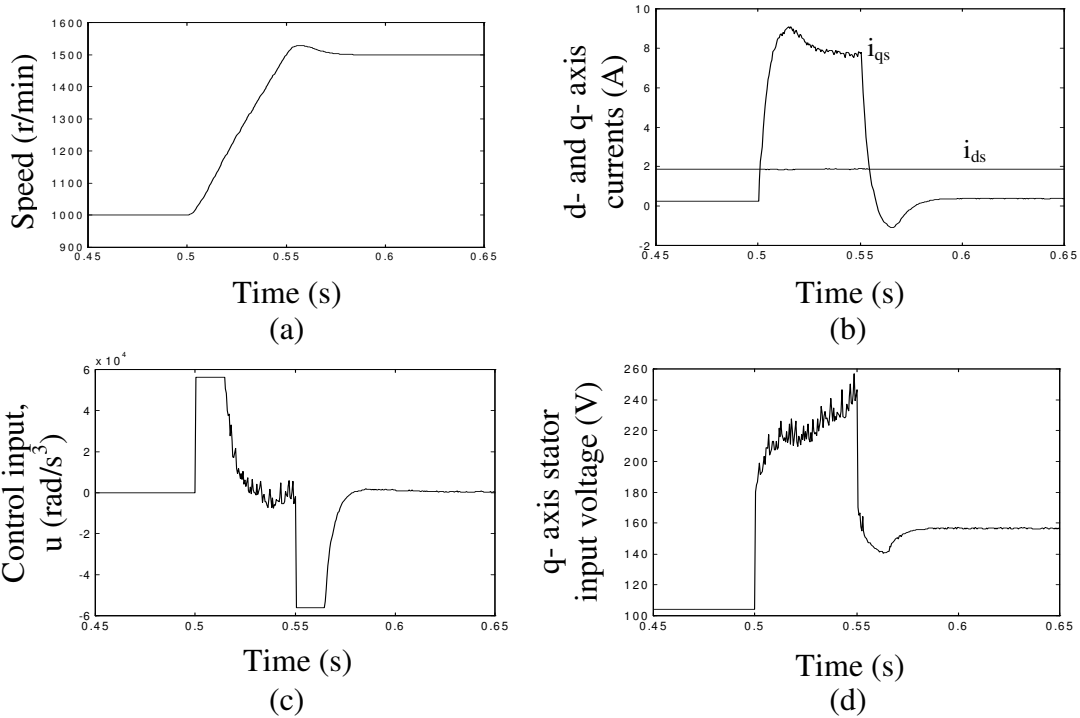
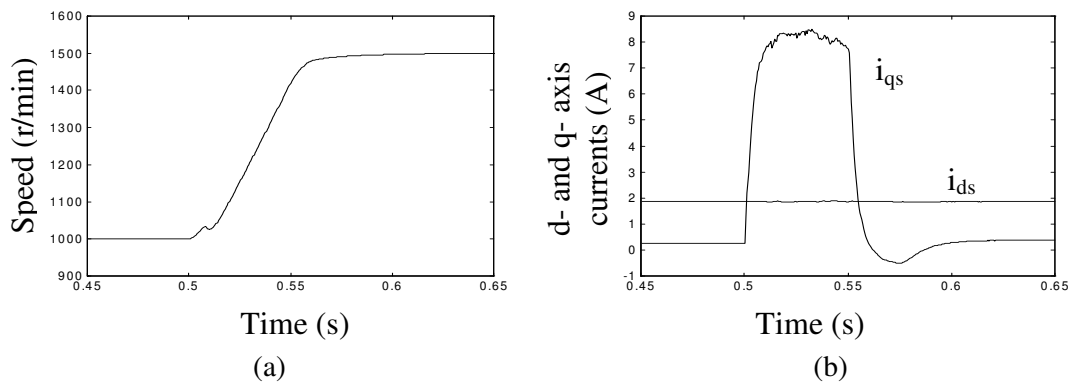


Fig. 2 Simulation responses for ramp (linear) change in reference speed with SMC: (a) Speed, (b) d- and q- axis stator currents, (c) Control input, (d) q- axis stator input voltage



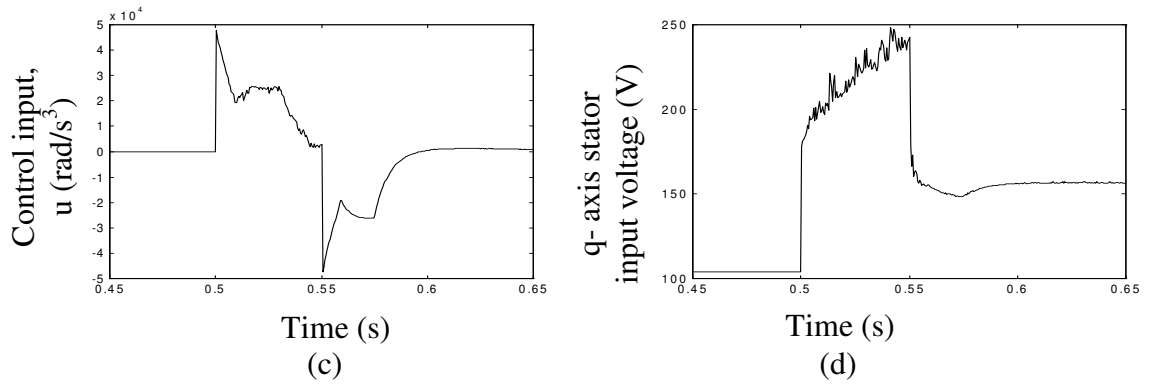


Fig. 3 Simulation responses for ramp (linear) change in reference speed with FSMC: (a) Speed, (b) d- and q- axis stator currents, (c) Control input, (d) q- axis stator input voltage

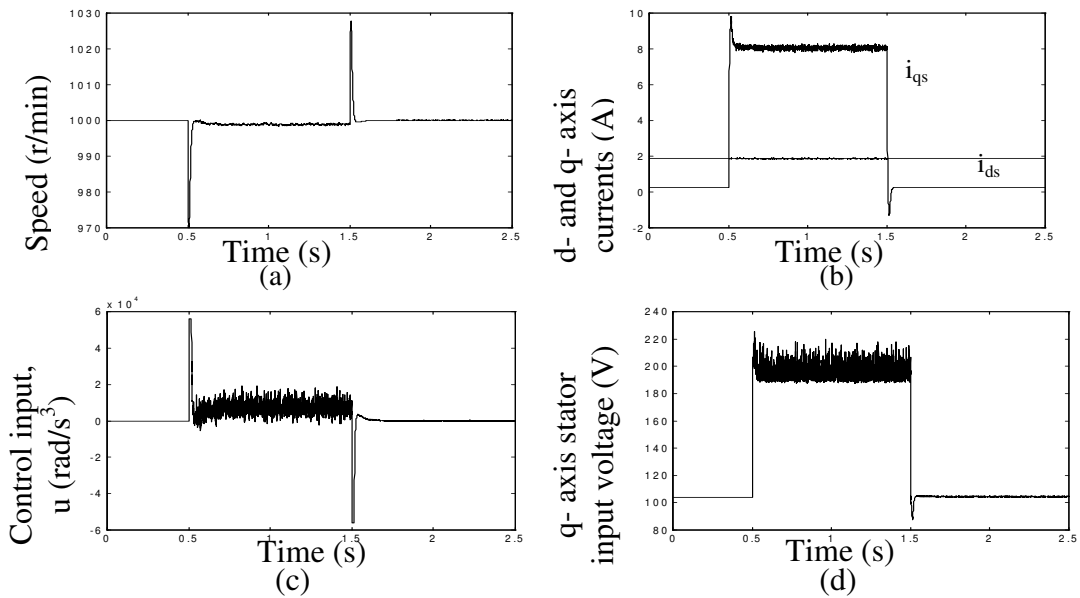


Fig. 4 Simulation responses for step changes in load torque with SMC: (a) Speed, (b) d- and q- axis stator currents, (c) Control input, (d) q- axis stator input voltage

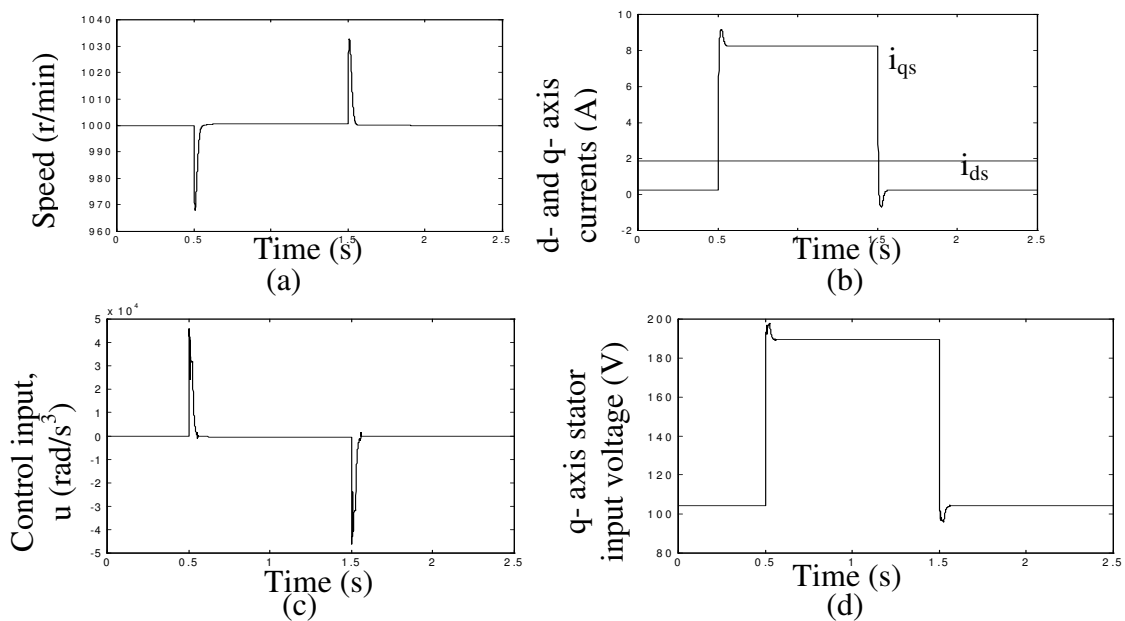


Fig. 5 Simulation responses for step changes in load torque with FSMC: (a) Speed, (b) d- and q- axis stator currents, (c) Control input, (d) q- axis stator input voltage