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Damping Capacity of Layered and Jointed Copper Structures

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Abstract: This paper considers the mechanism of damping and its theoretical evaluation for layered copper cantilever structures jointed with a number of equispaced connecting bolts under an equal tightening torque. Extensive experiments have been conducted on a number of specimens for comparison with numerical results from the theory. Intensity of interface pressure, its distribution characteristics, dynamic slip ratio and kinematic coefficient of friction at the interfaces, relative spacing of the connecting bolts, and frequency and amplitude of excitation are all found to have an effect on the damping capacity of such structures. It is established that the damping capacity of copper structures jointed with connecting bolts can be improved considerably by increasing the number of layers while maintaining uniform intensity of pressure distribution at the interfaces.

Keywords: Interface pressure, dynamic slip ratio, relative spacing, damping capacity

NOMENCLATURE

a_1	=	Amplitude of vibration of the first cycle
a_{n+1}	=	Amplitude of vibration of the $(n+1)$ th cycle
α'	=	Ratio of static bending stiffness of the layered and jointed cantilever beam to that of a solid one (k/k')
α	=	Dynamic slip ratio
b	=	Width of the specimen
$\mathrm{d}u_r$	=	Incremental relative dynamic slip
D_B	=	Diameter of the connecting bolt
δ_f	=	Logarithmic damping decrement due to interface friction damping
δ_0	=	Logarithmic damping decrement due to material and support damping
δ	=	Logarithmic damping decrement
Δ	=	Deflection due to static load
E_{loss}	=	Energy loss (per cycle) due to interface friction
E_n	=	Energy stored in the system with amplitude of vibration a_1
E_{n+1}	=	Energy stored in the system with amplitude of vibration a_{n+1}
E_f	=	Energy loss arising from interface friction under the joints
E_0	=	Energy loss from material and support damping
E_{net}	=	Energy stored (per cycle) in the system per cycle

E	=	Static bending modulus
F_r	=	Frictional force at the interfaces of the beam in the presence of relative dynamic slip
F_{rM}	=	Maximum frictional force at the interfaces of the beam during vibration
$2h$	=	Thickness of each layer of the cantilever specimen
I	=	Second moment of area
k'	=	Static bending stiffness of the solid cantilever beam
k	=	Static bending stiffness of the layered and jointed beam
l	=	Free length of the layered and jointed beam
m	=	Number of layers
μ	=	Kinematic coefficient of friction
n	=	Number of cycles
N	=	Total normal force
p	=	Interface pressure due to tightening load
P	=	Axial load on the connecting bolt due to tightening
R_B	=	Radius of the connecting bolt
R_M	=	Limiting radius of the influencing zone under each connecting bolt
R	=	Any radius within the influencing zone around the connecting bolt
Ψ	=	Damping ratio
σ_s	=	Surface stress on the jointed structure due to tightening load
T_1	=	Time period
T	=	Tightening torque applied on the connecting bolt
$u(x,t)$	=	Relative dynamic slip between interfaces at a bolted joint in the absence of a friction force
$u_r(x,t)$	=	Relative dynamic slip between interfaces at a bolted joint in the presence of a friction force
u_{rM}	=	Relative dynamic slip between the interfaces at the maximum amplitude of vibration
W	=	Static load
ω_n	=	Natural frequency of vibration
$y(x,t)$	=	Deflection of the beam under vibration
$y(l,0)$	=	Initial free end displacement

1. INTRODUCTION

The study of damping and improvement of damping properties of structural members has become increasingly significant in engineering science for controlling the undesirable effects of vibration while simultaneously enhancing the damping capacity. This study has been taken up primarily in four major areas: Materials science, structural mechanics, vibration control and inspection methods. Damping in vibrating mechanical systems has been subdivided into two classes: Material damping and system damping, depending on the main routes of energy dissipation. Coulomb (1784) postulated that material damping arises due to interfacial friction between the grain boundaries of the material under dynamic condition. Further studies on material damping have been made by Robertson and Yorgiadis (1946), Demer (1956), Lazan (1968) and Birchak (1977). System damping arises from slip and other boundary shear effects at mating surfaces, interfaces or joints between distinguishable parts. Murty (1971) established that the energy dissipated at the support is very small compared to material damping.

As the material damping within the structural members is of low magnitude, various other techniques are used to improve the damping capacity of structures. These are: (i) Use of constrained/unconstrained viscoelastic layers, (ii) fabrication using a multi-layered sandwich construction, (iii) use of stress raisers, (iv) insertion of special high-elasticity inserts in the parent structure, (v) application of spaced damping techniques, (vi) use of a viscous fluid layer, (vii) use of bonded joints and (viii) fabricating layered and jointed structures with welded/riveted/bolted joints.

One of the most important techniques for improving the damping capacity of structures is the use of constrained and unconstrained layers. A variety of work on constrained layer damping has been reported by Grootenhuis (1970), Behera and Nanda (1986), Nakra (1998), Chantalakhana et al. (2000), and Trindade et al. (2002). Appreciable structural damping has been achieved using laminated constructions consisting of alternate layers of an elastic material such as metal and a high damping viscoelastic one such as many thermoplastics. Extensive work has been reported by Plantema (1966), Mead and Markus (1970), Han (1985), Trindade et al. (2001), Yim et al. (2003), and Srikantha et al. (2003) on damping of sandwich beams. It has been established that the damping characteristics of a structural member can be improved considerably by using elastic inserts or pins with good damping properties. Mallik and Ghosh (1973, 1974), and Rahmathullah and Mallik (1979) have shown that the damping capacity can be increased by use of a suitable combination of strip and insert material.

Another technique for the improvement of damping capacity is to move the damping material away from the structure. This technique was first used to damp out noise and vibration in U. S. submarines. The details of studies in this field have been done by Miller and Warnaka (1970). Uno Ingard and Akay (1987) have studied the motion of two plates coupled by a viscous fluid layer and established that the damping produced by the fluid layer increases with layer thickness and also with decreasing frequency. Thornley and Lees (1972) studied the dynamic characteristics of joints with epoxy resin as the interfacial bonding material and established that this bonding increases both the static and dynamic stiffness with a small increase in damping of a joint.

Joints are present in most of the structures and usually over ninety percent of the inherent damping in a fabricated structure originates in the joints. Belgaumkar et al. (1968), Masuko et al. (1973), and Nishiwaki et al. (1978, 1980) have reported extensive work on techniques for improving the damping capacity of welded structures and also established that the damping capacity of a welded machine tool structure is not different from that of a cast one. Anno et al. (1970) have reported that steel plates welded with plug joints show a high damping capacity than those with other forms of welded joints. Pain (1957) has established that riveted joints also improve the damping capacity of structures. Extensive work has been done by Fernlund (1961), Kobayashi et al. (1986), Shin et al. (1991), Masuko et al. (1973), Nishiwaki et al. (1978, 1980), Motosh (1975), Connolly et al. (1965), Mitsunaga (1965) and Law et al. (2004) on the damping of structures with bolted joints. It is generally recognized that the damping capacity of jointed structures may be determined from the frictional loss energy caused by slip at the interfaces between the steel plates. Beards and Williams (1977) have shown that interfacial slip in joints is the major contributor to the inherent damping of most fabricated structures.

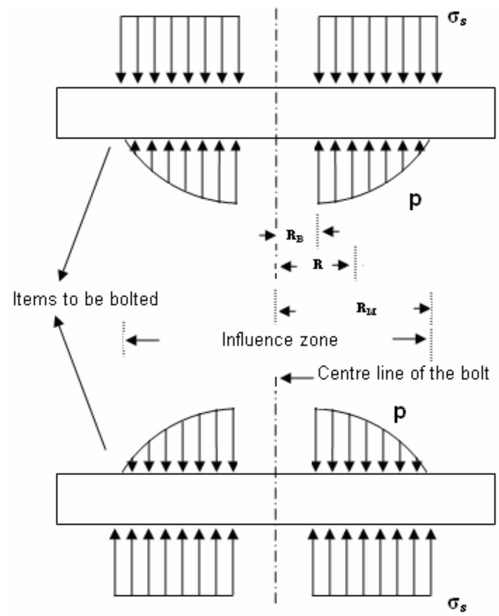

Figure 1. Free-body diagram of a bolted joint showing the influence zone.

Although a considerable amount of work has been reported on the experimental study of damping in welded and riveted structures, no generalized theory has been established. Hence, layered construction jointed with connecting bolts can be used more effectively to achieve a required damping capacity by controlling the influencing parameters. Therefore, attention has to be focused on these influencing parameters in order to maximize the overall damping capacity.

The logarithmic damping decrement, a measure of the damping capacity of layered and jointed structures has been determined using the energy principle, considering the relative dynamic slip and the pressure distribution at the interfaces of the contacting layers. These two vital parameters must be accurately assessed for correct evaluation of the damping capacity of such structures. Previous investigators, including Fernlund (1961), Kobayashi and Matsubayashi (1986), and Shin et al. (1991) have reported on this interface pressure and its distribution characteristics without specifying the spacing of the connecting bolts between the layers. Masuko et al. (1973), Motosh (1975) and Nishiwaki et al. (1978, 1980) have done extensive work assuming uniform intensity of pressure distribution at the interfaces of the layered and jointed structures without considering the actual pattern but instead using Rötschar's pressure cone (1973). Connolly et al. (1965) and Mitsunaga (1965) have reported that the pressure distribution at the jointed interfaces is not uniform but varies almost parabolically, the maximum pressure being at the surface of the bolt hole. Furthermore, Gould and Mikic (1972) and Ziada and Abd (1980) have shown that the pressure distribution at the interfaces of a bolted joint is parabolic in nature and there exists an influence zone in the form of a circle with 3.5 times the diameter of the connecting bolt which is independent of the tightening load applied on it as shown in Figure 1. Nanda (1992) and Nanda and Behera

(1999, 2000) have also done a considerable amount of work on the distribution pattern of the interface pressure and established that it becomes uniform with a separation distance of 2.00211 times the hole diameter between consecutive connecting bolts joining the layered beams. The damping capacity of such structures can be improved substantially by varying the influencing parameters. These include: Intensity and distribution characteristics of the interface pressure, spacing of the connecting bolts, tightening torque applied to them, coefficient of kinetic friction at the interfaces, material used for the structure, dynamic slip ratio and number of layers.

In the present investigation, damping capacity of such layered and jointed structures has been evaluated from analytical expressions developed in the investigation and compared experimentally for copper cantilever beams with two or more layers under different conditions of excitation in order to establish the accuracy of the theory developed.

2. THEORETICAL ANALYSIS

In the case of a layered structure jointed with connecting bolts, the intensity of the interface pressure distribution under each bolt has been assumed to be a non-dimensional polynomial with even powers:

$$p/\sigma_s = A_1 + A_2(R/R_B)^2 + A_3(R/R_B)^4 + A_4(R/R_B)^6 + A_5(R/R_B)^8 + A_6(R/R_B)^{10} \quad (1)$$

Where p, σ_s, R and R_B are the interface pressure, surface stress on the jointed structure due to tightening load, any radius within the influencing zone and the radius of the connecting bolt respectively and A_1, A_2, A_3, A_4, A_5 and A_6 are the constants of the polynomial. These constants were evaluated from the numerical data given by Ziada and Abd (1980) by using Dunn's curve fitting software as: 0.68517E+00, −0.10122E+00, 0.94205E−02, −0.23895E−02, 0.29487E-03 and −0.11262E−04 respectively.

The present work is based on the energy loss due to friction at the interfaces and the strain energy of a cantilever beam, as shown in Figure 2. The energy loss per cycle of vibration (E_f) arising due to friction and relative dynamic slip (u_r) at the interfaces has been calculated using the theory proposed by Nishiwaki et al. (1980):

$$E_f = \oint F_r \mathrm{d}u_r = 2F_{rM}u_{rM} \quad (2)$$

Where F_r, $\mathrm{d}u_r$, F_{rM} and u_{rM} are the frictional force at the interfaces of the beam in presence of relative dynamic slip, incremental relative dynamic slip, maximum frictional force at the interfaces of the beam during vibration and relative dynamic slip between the interfaces at the maximum amplitude of vibration respectively as shown in Figure 3.

The maximum frictional force at the interfaces of the beam under transverse vibration is

$$F_{rM} = \mu N \quad (3)$$

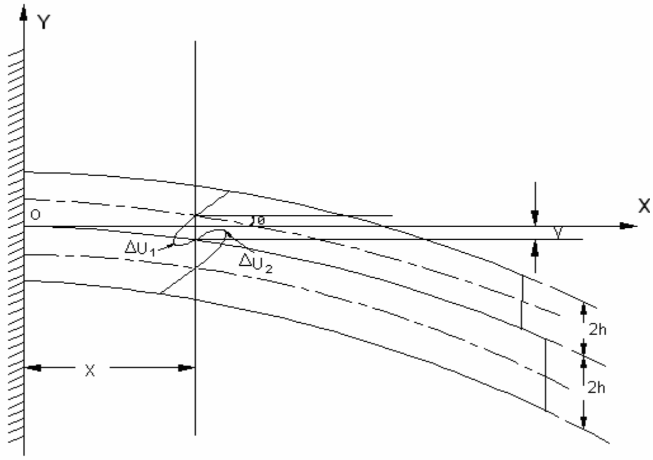

Figure 2. Mechanism of dynamic slip at the interfaces.

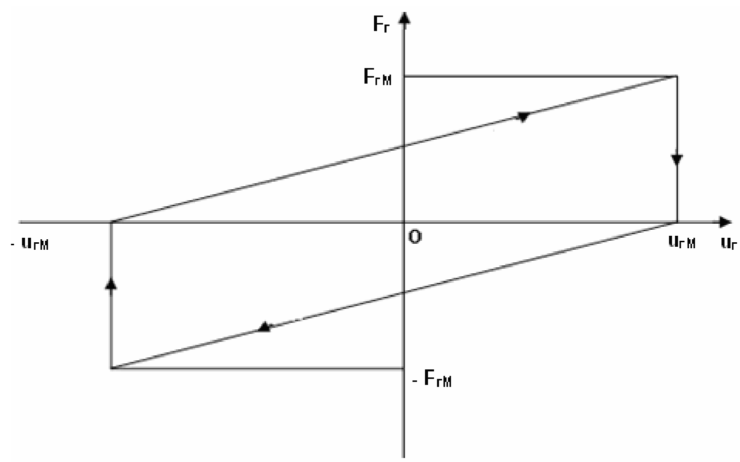

Figure 3. Relationship between the friction force (F_r) and the relative dynamic slip (u_r) during one cycle.

where μ and N are the kinematic coefficient of friction and the total normal force at the interfaces of the layers under each connecting bolt respectively.

This total normal force has been determined by Nanda and Behera (2000) as

$$\begin{aligned} N &= \left[A_1\left\{(R_M/R_B)^2-1\right\}+\{A_2/2\}\left\{(R_M/R_B)^4-1\right\}+\{A_3/3\}\left\{(R_M/R_B)^6-1\right\}\right. \\ &+ \{A_4/4\}\left\{(R_M/R_B)^8-1\right\}+\{A_5/5\}\left\{(R_M/R_B)^{10}-1\right\} \\ &+ \left.\{A_6/6\}\left\{(R_M/R_B)^{12}-1\right\}\right][P/3] \end{aligned} \quad (4)$$

where R_M and P are the limiting radius of the influencing zone under each connecting bolt and the axial load on the connecting bolt due to tightening torque respectively.

The axial load P on the connecting bolt due to tightening torque is given by Shigley (1956) as

$$P = [T/0.2D_B] \tag{5}$$

where T and D_B are the tightening torque and nominal diameter of the connecting bolt respectively.

The vibration of the cantilever beam specimen, as shown in Figure 2, can be expressed as

$$y(x,t) = Y(x)f(t) \tag{6}$$

where the space function, $Y(x) = C_1 \sin \lambda x + C_2 \cos \lambda x + C_3 \sin h\lambda x + C_4 \cos h\lambda x$, and the time function, $f(t) = A \cos \omega_n t + B \sin \omega_n t$, C_1, C_2, C_3, and C_4 are constants to be evaluated from the boundary conditions with the usual notation; $\lambda^4 = \omega_n^2 A\gamma/EIg$, and A and B are constants to be evaluated from the initial conditions.

Using the initial free end displacement, $y(l,0)$, with its boundary conditions for the cantilever beam, the equation for the slope is given by

$$\begin{aligned}[\partial y(x,t)/\partial x] &= -[(\cos \lambda l + \cos h\lambda l)(\cos h\lambda x - \cos \lambda x) \\ &- (\sin \lambda l + \sin h\lambda l)(\sin \lambda x + \sin h\lambda x)] \\ &\times [\lambda y(l,0) \cos \omega_n t][2(\cos \lambda l \sin h\lambda l - \sin \lambda l \cos h\lambda l)]^{-1} \end{aligned} \tag{7}$$

where ω_n is the natural circular frequency of vibration.

The actual relative dynamic slip at the interfaces of a bolted joint at a distance l_i from the fixed end of a layered and jointed cantilever beam is given by

$$u_r(l_i,t) = \alpha u(l_i,t) \tag{8}$$

where α is the dynamic slip ratio (u_r/u_0) and u_0 is the relative dynamic slip between the interfaces in the absence of a friction force at a bolted joint.

If the layered and jointed beam specimen is given an initial displacement at the free end, the relative dynamic slip at the interfaces between the layers, as shown in Figure 2, is given by

$$u_r(l_i,t) = \alpha[\Delta u_1 + \Delta u_2] = 2\alpha h \tan[\partial y(l_i,t)/\partial x] \tag{9}$$

where $2h$ is the thickness of each layer of the cantilever beam.

Modifying equation (7) and combining it with equation (9), the maximum relative dynamic slip under a connecting bolt is found to be

$$
\begin{aligned}
u_{rM} &= [\alpha h][(\cos\lambda l + \cos h\lambda l)(\cos h\lambda l_i - \cos\lambda l_i) \\
&- (\sin\lambda l + \sin h\lambda l)(\sin\lambda l_i + \sin h\lambda l_i)] \\
&\times [\lambda y(l,0)][\sin\lambda l \cos h\lambda l - \cos\lambda l \sin h\lambda l]^{-1}
\end{aligned} \tag{10}
$$

The overall maximum relative dynamic slip for a layered and jointed cantilever beam with q equispaced connecting bolts (having a spacing of 3.5 times their diameter) has been calculated by Nanda (1992) and is given by

$$u_{rM} = \alpha h X_{\text{sum}} \lambda y(l,0) \tag{11}$$

where $X_{\text{sum}} = \left[(\cos\lambda l + \cos h\lambda l)\sum_{i=1}^{q}(\cos h\lambda l_i - \cos\lambda l_i) - (\sin\lambda l + \sin h\lambda l)\sum_{i=1}^{q}(\sin\lambda l_i + \sin h\lambda l_i)\right] \times [\sin\lambda l \cos h\lambda l - \cos\lambda l \sin h\lambda l]^{-1}$

It is assumed that the energy loss of the layered and jointed beam consists of the loss arising from interfacial friction under the joints (E_f) and the loss from material and support damping (E_0). Thus, the logarithmic damping decrement of a layered and jointed beam is expressed as

$$\delta = [(E_f/E_n) + (E_0/E_n)]/2 = \delta_f + \delta_0 \tag{12}$$

where E_n is the energy stored per cycle of vibration due to the initial amplitude of excitation $[y(l,0)]$ and is given by $E_n = [ky^2(l,0)]/2$

The logarithmic damping decrement due to material and support damping (δ_0), being very small compared to the interface friction damping, can be neglected and the equation for the logarithmic damping decrement is thus simplified to

$$\delta \approx \delta_f = E_f/2E_n \tag{13}$$

The energy loss per cycle due to the friction at the interfaces, as given in equation (2), can be modified by combining equations (3) and (11) and the logarithmic damping decrement for such a beam is then found to be

$$\delta = E_f/2E_n = 2\mu N\alpha h X_{\text{sum}}\lambda/ky(l,0) \tag{14}$$

where k is the static bending stiffness of the layered and jointed cantilever beam.

As equation (14) for logarithmic damping decrement is valid for a two-layered jointed cantilever beam, a generalized equation has been developed for a multi-layered and jointed cantilever beam:

$$\delta = 2(m-1)\mu N\alpha h X_{\text{sum}}\lambda/ky(l,0) \tag{15}$$

Where m is the number of layers.

Since direct evaluation of the dynamic slip ratio, α, and kinematic coefficient of friction, μ, were not possible, the product of these two parameters (i.e., $\alpha \times \mu$) has been determined from the experimental results for logarithmic decrement for two-layered copper specimens with 10 mm diameter connecting bolts. For this purpose, equation (14) has been modified to

$$\alpha \times \mu = [ky(l,0)\delta]/[2NhX_{\text{sum}}\lambda] \tag{16}$$

2.1. *Determination of Logarithmic Damping Decrement with Uniform Pressure Distribution at the Interfaces*

As discussed, layered and jointed structures with connecting bolts show parabolic pressure distribution within the influence zone of each bolted joint, resulting in a non-uniform distribution of pressure at the interfaces. In order to obtain a uniform pressure distribution at the interfaces, the consecutive influencing zones must be superimposed by decreasing the spacing of the bolts on the structure. The spacing between bolts required to achieve a uniform pressure distribution at the interfaces has been evaluated with the help of a suitable software package [Nanda and Behera (1999)] and found to be 2.00211 times the diameter of the connecting bolts, the number being independent of the tightening torque on the connecting bolts. The magnitude of the uniform pressure with the above spacing has been determined as shown in Figure 4 and found to be

$$p = 0.671P/3\pi R_B^2 \tag{17}$$

For a layered and jointed beam, the damping ratio, Ψ, is expressed as the ratio between the energy dissipated by the relative dynamic slip between the interfaces and the total energy introduced into the system, and is expressed as

$$\Psi = [E_{\text{loss}}/(E_{\text{loss}} + E_{\text{net}})] \tag{18}$$

where E_{loss} and E_{net} are the energy loss due to interface friction and energy introduced during the unloading process (see Figure 5). For the same cantilever beam with uniform pressure p at the interface, the energy loss due to this frictional force at the interface per half-cycle of vibration may be written as

$$E_{\text{loss}} = \int_0^{\pi/\omega_n} \int_0^{l} \mu pb[\{\partial u_r(x,t)/\partial t\} \mathrm{d}x \mathrm{d}t] \tag{19}$$

However, the energy introduced into the layered and jointed cantilever beam in the form of strain energy per half-cycle of vibration is given by

$$E_{\text{net}} = (3EI/l^3)y^2(l,0) \tag{20}$$

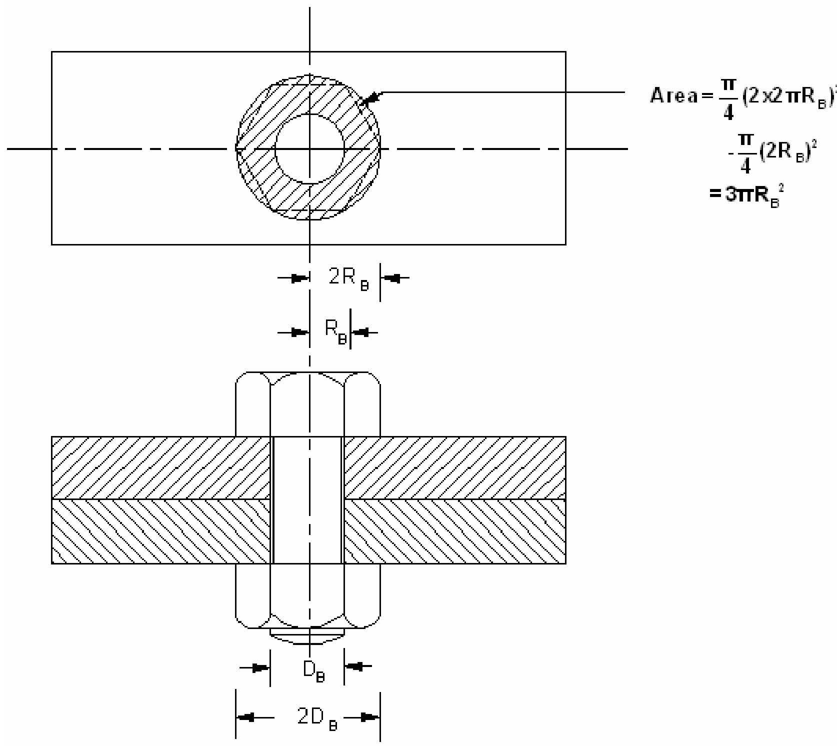

Figure 4. Influence area under a connecting bolt head.

Combining equations (19) and (20), we get

$$E_{\text{loss}}/E_{\text{net}} = \int_0^{\pi/\omega_n}\int_0^{l} \left[\mu pb\{\partial u_r(x,t)/\partial t\}\mathrm{d}x\mathrm{d}t\right] / \left[(3EI/l^3)y^2(l,0)\right] \tag{21}$$

Considering uniform pressure distribution throughout the contact area of the interfaces and assuming the dynamic slip ratio, α, to be independent of both the distance from the fixed end of the cantilever beam and time, equation (21) can be modified to

$$\begin{aligned} E_{\text{loss}}/E_{\text{net}} &= \left[2\mu bhp\alpha/\{(3EI/l^3)y^2(l,0)\}\right] \\ &\quad \times \int_0^{\pi/\omega_n}\int_0^{l}[\partial\{\tan\partial y(x,t)/\partial x\}\mathrm{d}x\mathrm{d}t]/\partial t \end{aligned} \tag{22}$$

Moreover, the slope of the cantilever beams $\partial y(x,t)/\partial x$ being quite small, [$\tan\partial y(x,t)/\partial x$] $\approx \partial y(x,t)/\partial x$, and equation (22) can be further modified to give

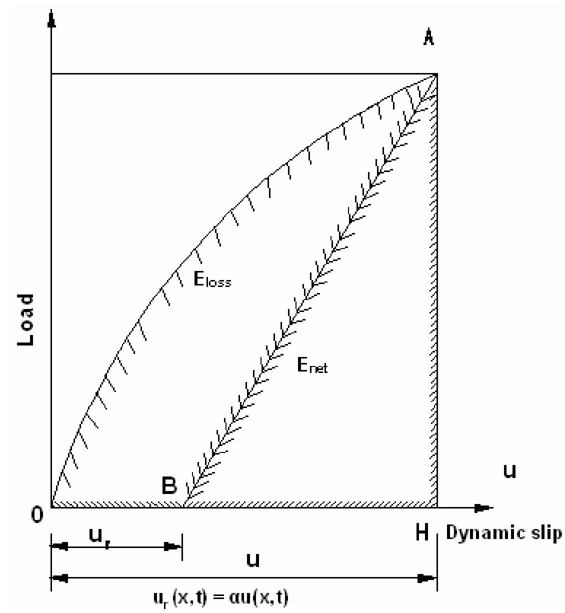

Figure 5. Relationship between u_r and u.

$$E_{\text{loss}}/E_{\text{net}} = \left[2\mu bhp\alpha/\{(3EI/l^3)y^2(l,0)\}\right] \int_0^{\pi/\omega_n} \int_0^{l} \left[\{\partial^2 y(x,t)/\partial x \partial t\} \mathrm{d}x \mathrm{d}t\right] \tag{23}$$

Considering the boundary and the initial conditions of the cantilever beam as $y(l,0) = y_0$ (positive downward deflection) and $\partial y(l,0)/\partial t = 0$ (no initial velocity) respectively, the bending deflection of the beam under vibration can be expressed as

$$y(x,t) = Y(x)\{y_0/Y(l)\}\cos\omega_n t, \tag{24}$$

where $Y(x)$ is the space function and the rest is the time function.

Substituting equation (24) into equation (23), changing the limits of the time interval from 0 and π/ω_n to 0 and $\pi/2\omega_n$, and multiplying the expression by two gives

$$\begin{aligned} E_{\text{loss}}/E_{\text{net}} &= \left[4\mu bhp\alpha/\{(3EI/l^3)y^2(l,0)\}\right] \\ &\times \int_0^{\pi/2\omega_n} \int_0^{l} \partial^2 \left[Y(x)\{y_0/Y(l)\}\cos\omega_n t\right] \mathrm{d}x\mathrm{d}t/[\partial x \partial t] \end{aligned} \tag{25}$$

Using equations (24) and (25) we get

$$E_{\text{loss}}/E_{\text{net}} = \left[4\mu bhp\alpha y(l,0)]/[(3EI/l^3)y^2(l,0)\right] \tag{26}$$

Since $3EI/l^3 = k$, i.e., the static bending stiffness of the layered and jointed beam, equation (26) reduces to

$$E_{\text{loss}}/E_{\text{net}} = [4\mu bhp\alpha]/[ky(l,0)] \tag{27}$$

Equation (18) becomes

$$\Psi = [E_{\text{loss}}/(E_{\text{loss}} + E_{\text{net}})] = 1/[1 + E_{\text{net}}/E_{\text{loss}}] \tag{28}$$

Putting the values of $E_{\text{loss}}/E_{\text{net}}$ from equation (27) into equation (28) we get

$$\Psi = 1/[1 + \{ky(l,0)\}/\{4\mu bhp\alpha\}] \tag{29}$$

The logarithmic damping decrement, δ, is usually expressed as $\delta = \ln(a_n/a_{n+1})$. Assuming that the energy stored in the system is proportional to the square of the corresponding amplitude, the relationship between the logarithmic damping decrement and the damping ratio can be written as

$$\delta = \ln(E_n/E_{n+1})^{1/2} = [\ln\{1/(1-\Psi)\}]/2 \tag{30}$$

where E_n and E_{n+1} are the energy stored in the system with amplitudes of vibration a_1 and a_{n+1} respectively.

In the case that $\Psi \leq 1$, the Maclaurin expansion of equation (30) will yield

$$\delta = \left[\Psi + (\Psi^2/2)\right]/2 \tag{31}$$

Similarly, in order to calculate the logarithmic damping decrement for multi-layered cantilever beams, the number of interfacial layers must be taken into consideration. If m layers are jointed together with connecting bolts to construct a multi-layered cantilever beam so as to have uniform interface pressure, the damping ratio is given by

$$\Psi = 1/[1 + \{ky(l,0)\}/\{4(m-1)\mu bhp\alpha\}] \tag{32}$$

3. EXPERIMENTAL TECHNIQUES AND EXPERIMENTS

In order to compare the numerical results produced by the theory in Section 2 with the actual logarithmic damping decrement of layered and jointed copper beams, a series of experiments were conducted. The experimental set-up with detailed instrumentation is shown in Figure 6. The specimens were prepared from commercial copper strips (detailed in Table 1) by joining two or more layers using equispaced connecting bolts, with the same tightening torque applied to each bolt. The distance between consecutive connecting bolts was either 3.5 and 2.00211 times their diameter depending on whether non-uniform or uniform pressure distri-

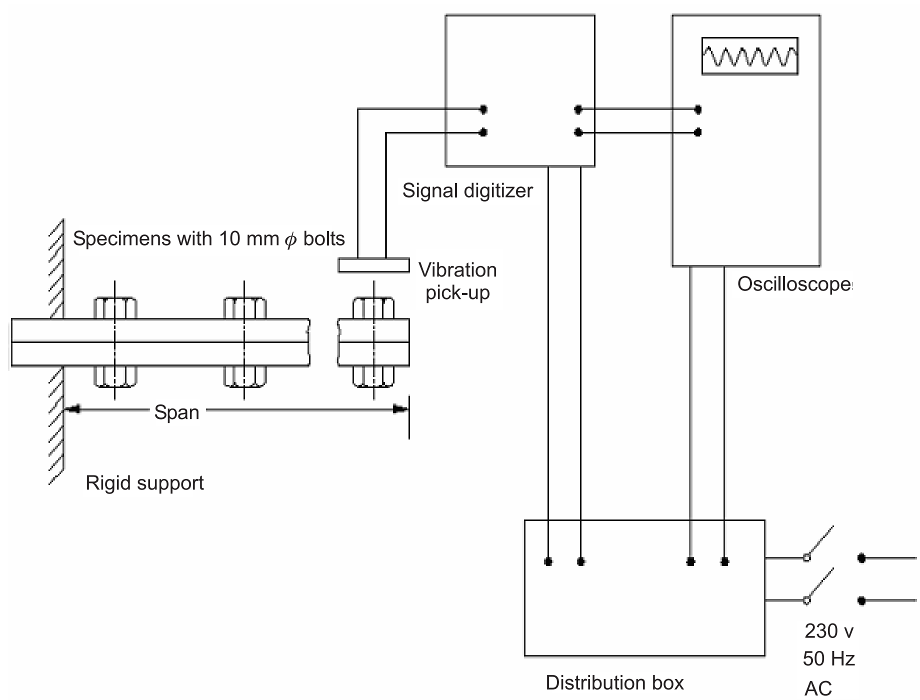

Figure 6. Schematic diagram of experimental set-up with detailed instrumentation.

bution at the interfaces was required. The cantilever lengths of the specimens were varied in order to accommodate the different numbers of connecting bolts given in Table 1.

The specimens are rigidly fixed to the support to obtain perfect cantilever conditions and the first experiments were conducted to determine the bending modulus of elasticity (E) of the specimen materials. Solid cantilever specimens made from the same stock of commercial copper strips were held rigidly at the fixed end and their free end deflection (Δ) measured by applying different static loads (W). From these static loads and the corresponding deflections, the average static bending stiffness (W/Δ) was determined. The bending modulus for the specimen material was then evaluated using the expression $E = [(W/\Delta)(l^3/3I)]$. The average value of E for the copper specimens used in the experiments was found to be 103.5 GN/m^2.

The static bending stiffness (k) of the jointed specimens was determined and found to be always lower than that of an equivalent solid specimen (k'), increasing with increasing tightening torque on the connecting bolts up to a certain limiting value [10.370 N m (7.5 lb ft) for the example shown in Figure 7] beyond which it remains almost constant. The ratio of this bending stiffness (at the limiting tightening torque condition) to the equivalent bending stiffness of a solid cantilever (α') was calculated for all specimens, and the average value of α' for each group of specimens utilized in the numerical analysis.

The logarithmic damping decrement and natural frequency of vibration of all the specimens were determined experimentally at their first mode of free vibration. The tightening torques on all the connecting bolts of the specimens were kept equal for each set of obser-

Table 1. Details of the specimens used in the experiment.

Dimensions of the specimen (thickness × width), (mm × mm)	Diameter of the connecting bolt, (mm)	Number of layers used	Condition of interface pressure	Number of bolts used	Cantilever length, (mm)
3.20 × 35.00				11	385.00
5.60 × 35.00	10	2	non-uniform	10	350.00
12.00 × 37.00				9	315.00
4.80 × 35.00				11	385.00
	10	3	non-uniform	10	350.00
8.40 × 35.00				9	315.00
6.40 × 35.00				11	385.00
	10	4	non-uniform	10	350.00
11.20 × 35.00				9	315.00
3.20 × 40.04				18	360.38
5.60 × 40.04	10	2	uniform	17	340.36
12.00 × 37.00				16	320.34
4.80 × 40.04				18	360.38
	10	3	uniform	17	340.36
8.40 × 40.04				16	320.34
6.40 × 40.04				18	360.38
	10	4	uniform	17	340.36
11.20 × 40.04				16	320.34
6.40 × 40.04	Solid beam			18	360.38

vations and observations made with torque of 3.46, 6.92, 13.84, 20.76, and 27.68 N m (i.e., 2.50, 5.00, 10.00, 15.00 and 20.00 lb ft respectively). The lengths of these specimens were also varied during experimentation. A spring loaded exciter was used to excite the specimens at their free ends. Tests were conducted using various amplitudes of excitation (0.1, 0.2, 0.3, 0.4, and 0.5 mm) for all the specimens tested under the different conditions of the tightening torque on the connecting bolts. The free vibration was sensed with a non-contacting type of vibration pick-up and the corresponding signal was fed to a cathode ray oscilloscope through a digitizer to obtain a steady signal. The logarithmic damping decrement was then evaluated from the measured values of the amplitudes of the first cycle (a_1), last cycle (a_{n+1}) and the number of cycles(n) of the steady signal by using the equation $\delta = \ln(a_1/a_{n+1})/n$. The corresponding natural frequency was determined from the time period (T_1) of the signal by using the relationship $f = 1/T_1$. It was found that the natural frequency of first mode vibration of the layered and jointed beam is always less than that of an equivalent solid one, and increases with an increase in the tightening torque on the connecting bolts up to a limiting value [10.370 N m (7.5 lb ft)] beyond which it remains constant, as shown in Figure 8.

Moreover, in order to study the effect of the spacing between the consecutive connecting bolts on the logarithmic damping decrement, experiments were conducted by varying the distance between them; the results are shown in Figure 9. The distances tried between consecutive connecting bolts were 5.0, 4.0, 3.0, and 2.00211 times the diameter of the connecting bolts.

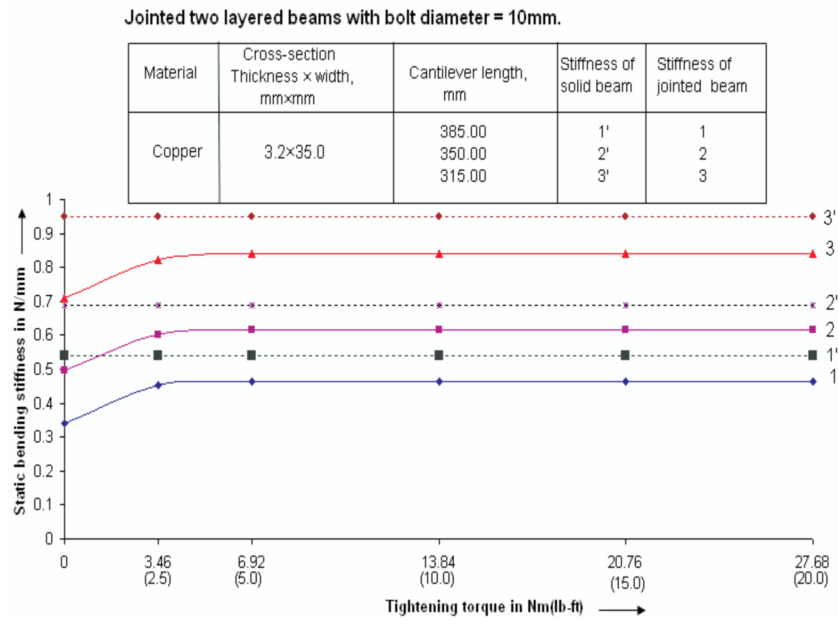

Figure 7. Variation of static bending stiffness with applied tightening torque on the connecting bolts.

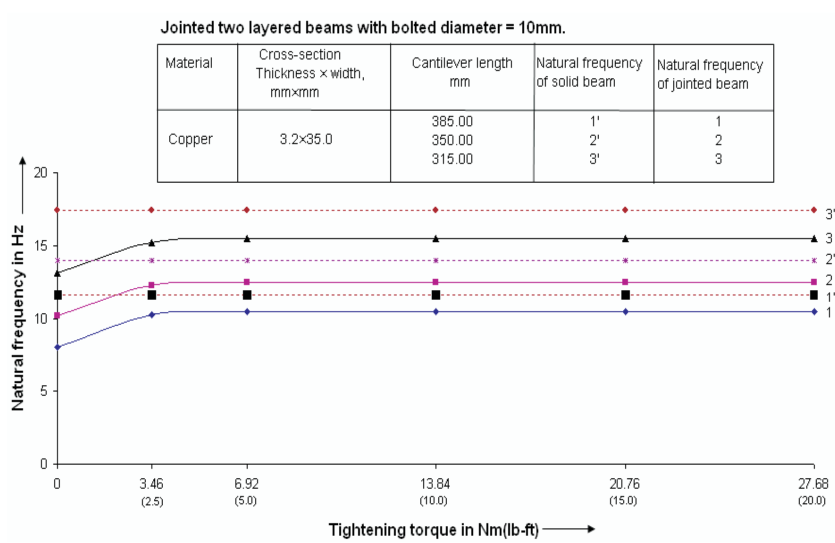

Figure 8. Variation of natural frequency with applied tightening torque on the connecting bolts.

Jointed two layered beams with bolt diameter = 10mm, Amplitude of excitation = 0.1mm.

Material	Cross-section Thickness × width, mm × mm.	Cantilever length, mm	Distance between Consecutive connecting bolts, mm	No.used
Copper	3.20 × 50.00	350.00	50.00	1
	3.20 × 40.00	360.00	40.00	2
	3.20 × 30.00	360.00	30.00	3
	3.20 × 40.04	360.38	20.02	4

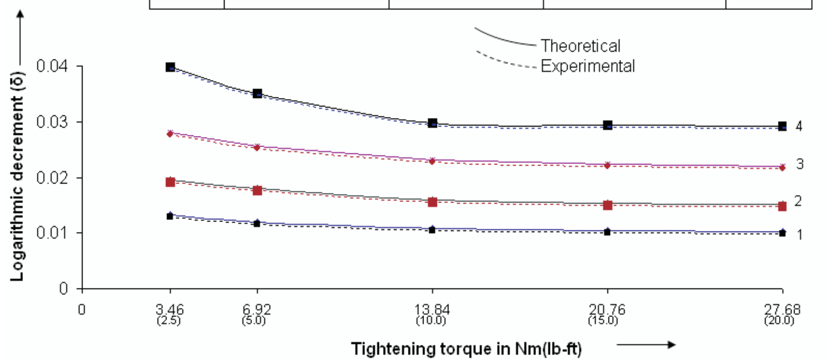

Figure 9. Variation of logarithmic decrement (δ) with applied tightening torque on the connecting bolts.

4. DETERMINATION OF THE PRODUCT OF DYNAMIC SLIP RATIO AND KINEMATIC COEFFICIENT OF FRICTION ($\alpha.\mu$)

The experimental logarithmic damping decrement values for two layered jointed beams with 10 mm diameter connecting bolts under different conditions of excitation have been used to evaluate the corresponding values of the product of dynamic slip ratio and kinematic coefficient of friction using equation (16). The variation in the dynamic slip ratios and natural frequency of the first mode transverse vibration for a particular tightening torque on connecting bolts was determined under different initial amplitudes of excitation. The results have been plotted and one sample is presented in Figure 10. Figure 11 shows a plot of dynamic slip ratios against the applied tightening torque on the connecting bolts for a particular specimen. All these plots have been used in the evaluation of the numerical results for the logarithmic damping decrement of multi-layered jointed beams using equations (15) and (31).

5. COMPARISON OF EXPERIMENTAL AND NUMERICAL RESULTS

The logarithmic damping decrements of three and four layered cantilever specimens with 10 mm diameter connecting bolts have been calculated from equation (15), using the values for the product of dynamic slip ratios and kinematic coefficient of friction from the respective plots discussed in Section 4. Figures 12, 13 and 14 show comparisons of these numerical results with the corresponding experimental ones. It can be seen that the curves are very close to each other, with a maximum variation of 1.53 %, which confirms the accuracy of the values of the product of dynamic slip ratios and kinematic coefficient of friction determined numerically from the experimental results for the logarithmic damping decrement.

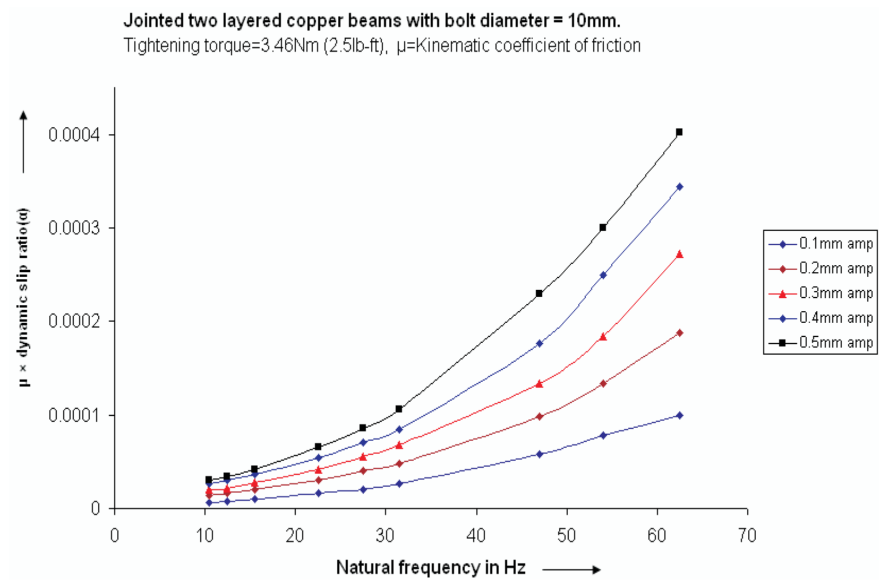

Figure 10. Variation of $\mu \times$ dynamic slip ratio (α) with natural frequency of vibration.

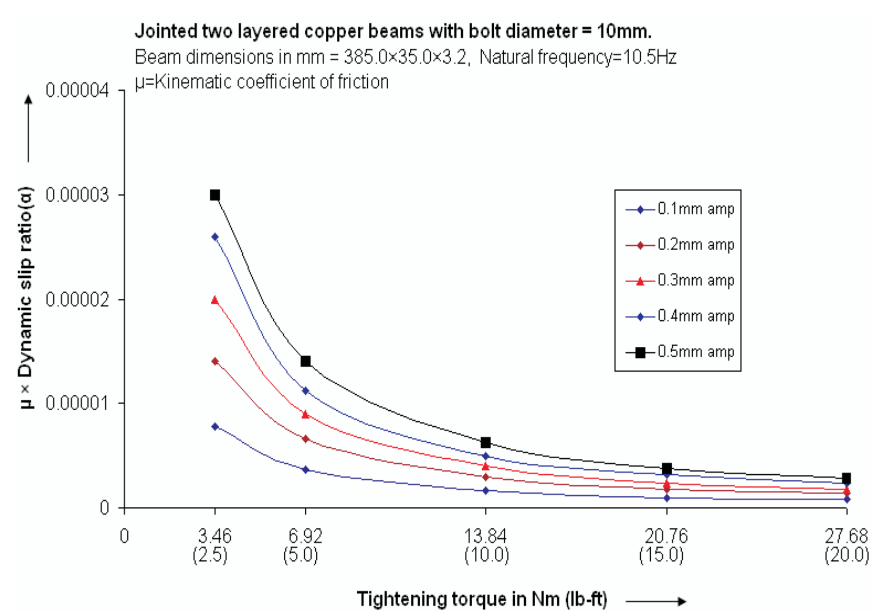

Figure 11. Variation of $\mu \times$ dynamic slip ratio (α) with applied tightening torque on the connecting bolts.

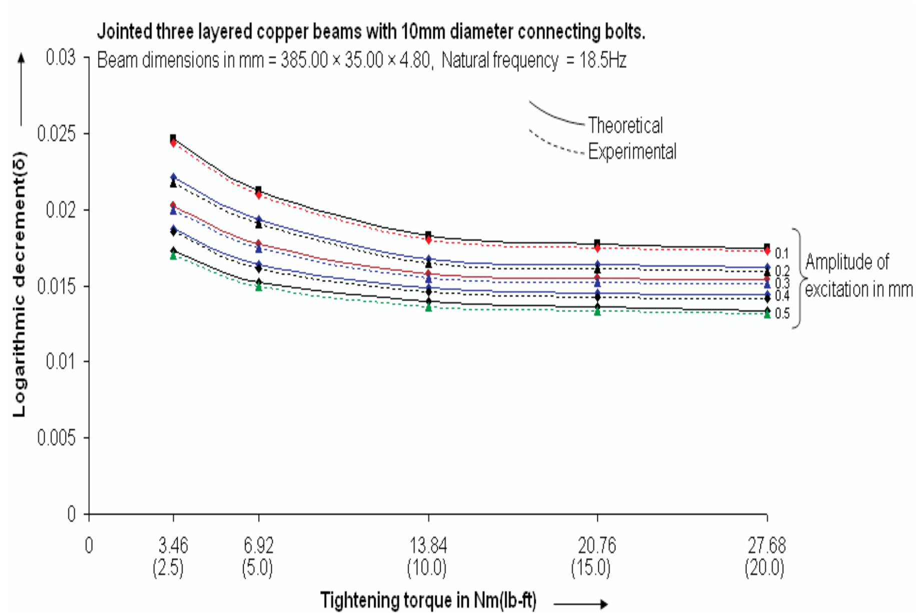

Figure 12. Variation of logarithmic decrement (δ) with applied tightening torque on the connecting bolts.

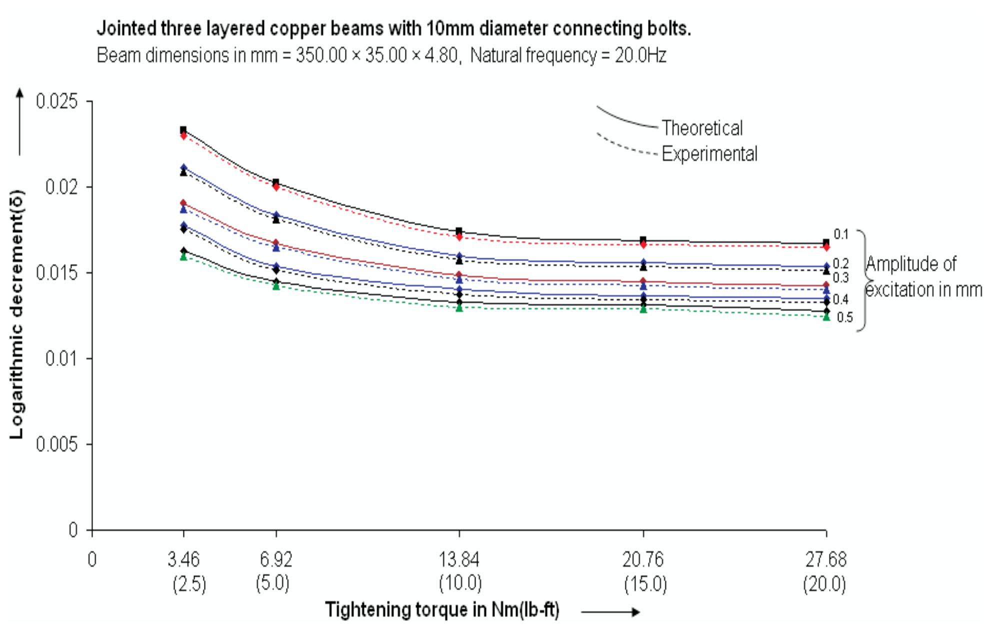

Figure 13. Variation of logarithmic decrement (δ) with applied tightening torque on the connecting bolts.

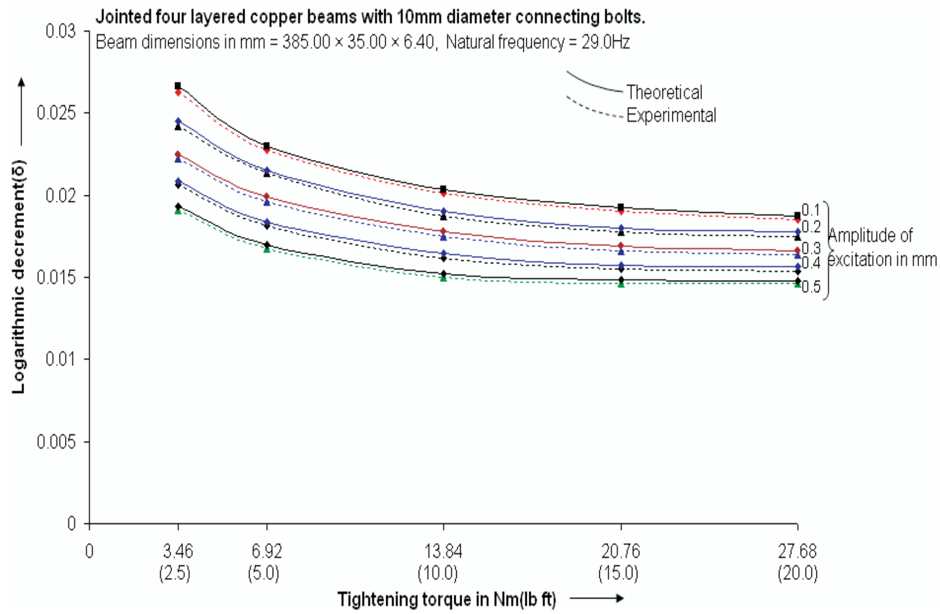

Figure 14. Variation of logarithmic decrement (δ) with applied tightening torque on the connecting bolts.

Numerical results for two, three and four layered jointed cantilever beams with uniform pressure distribution at the interfaces and 10 mm diameter connecting bolts have been determined using equation (31) in order to verify the accuracy of the numerical analysis. These numerical results for logarithmic damping decrement have also been plotted along with the corresponding experimental ones and one such plot for each case has been shown in Figures 15, 16, and 17; again, the plots are very close to each other, with a maximum variation of 1.3 %.

6. CONCLUSIONS

From the theoretical analysis as well as the numerical and experimental results, the following salient points have been observed.

1. The static bending stiffness of a layered and jointed structure is smaller than that of an equivalent solid one and increases with an increase in the tightening torque on the connecting bolts, but remains constant beyond a limiting value of the tightening torque [in this case 10.370 N m (7.50 lb ft)].
2. The natural frequency of the first mode of vibration for a layered and jointed structure is smaller than that of an equivalent solid one and increases with an increase in the tightening torque on the connecting bolts due to the higher static bending stiffness. However, the frequency remains constant beyond a limiting value of the tightening torque [in this case, 10.370 N m (7.5 lb ft)].

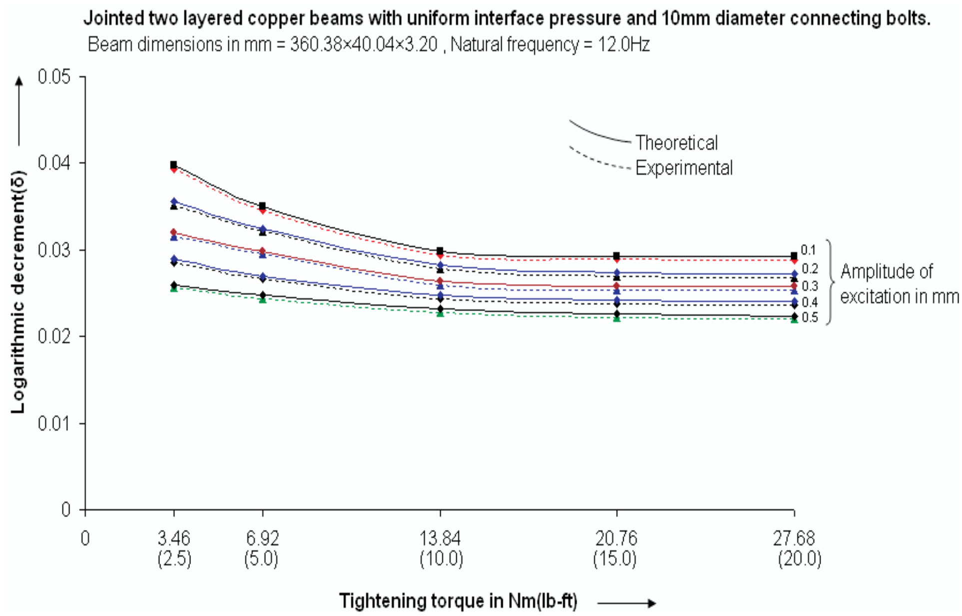

Figure 15. Variation of logarithmic decrement (δ) with applied tightening torque on the connecting bolts.

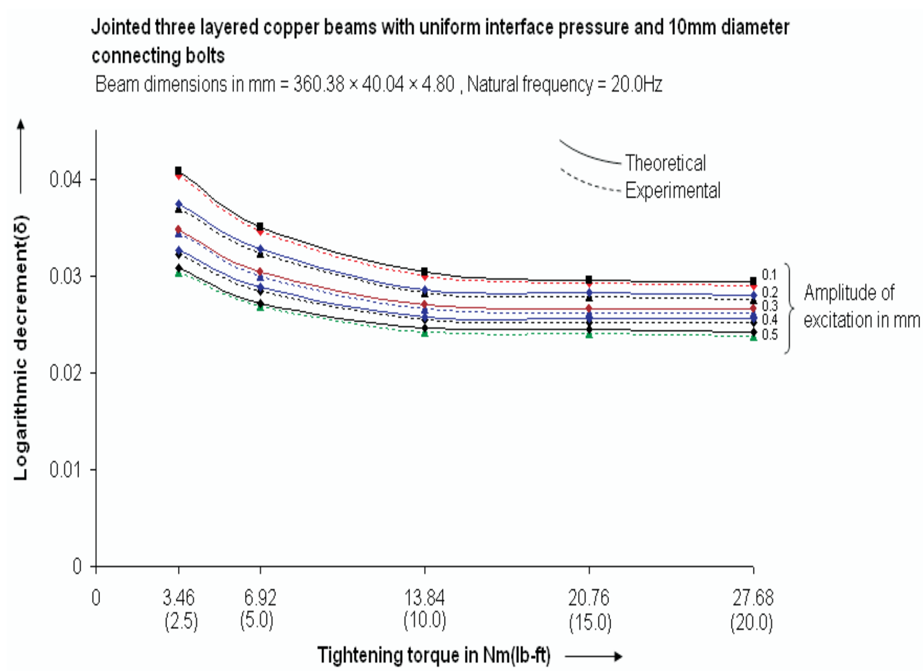

Figure 16. Variation of logarithmic decrement (δ) with applied tightening torque on the connecting bolts.

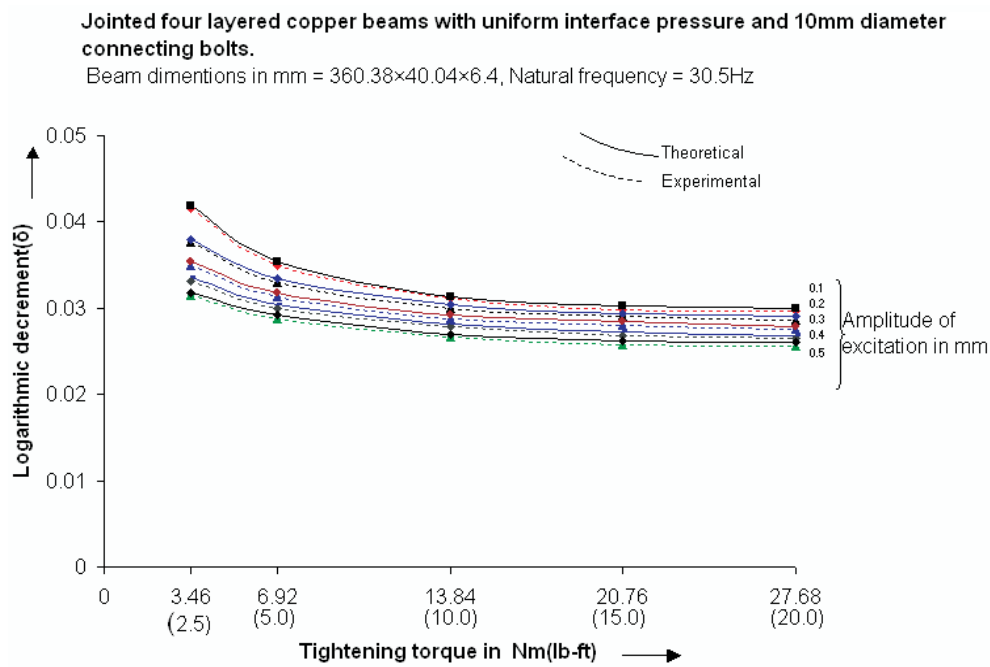

Figure 17. Variation of logarithmic decrement (δ) with applied tightening torque on the connecting bolts.

3. The spread of values for the interface pressure between layers joined by connecting bolts increases with a decrease in the distance between consecutive connecting bolts and attains uniformity throughout the contacting surfaces when the spacing of the connecting bolts is 2.00211 times the diameter of the bolt. The logarithmic damping decrement decreases with any further decrease in the distance between consecutive connecting bolts as the layered and jointed cantilever beam behaves like a solid one.
4. Several parameters play a major role on the damping capacity of layered structures jointed with connecting bolts. They are: (a) Tightening torque applied to the connecting bolts, (b) amplitude of excitation, (c) frequency of excitation, (d) arrangement of connecting bolts, and (e) number of layers.

   (a) Logarithmic damping decrement decreases with an increase in tightening torque on the connecting bolts due to higher interface pressure with lower dynamic slip ratio at the interfaces, which tend to behave increasingly like a solid beam. However, the logarithmic damping decrement increases with an increase in tightening torque in the lower range.
   (b) The logarithmic damping decrement of a layered and jointed structure decreases with an increase in amplitude of excitation as a result of the introduction of higher energy into the system, reducing the proportion of that energy dissipated by interface friction.

(c) The logarithmic damping decrement of a layered and jointed structure decreases with an increase in the natural frequency of vibration due to the increase in static bending stiffness. The dynamic slip ratio increases, but the increase in static bending stiffness of the layered and jointed structure is greater compared to the energy loss through friction at the interfaces, resulting in a decrease of the logarithmic damping decrement. The static bending stiffness increases due to the increase in the cross-section and decrease in the cantilever length of the specimens.

(d) The arrangement of the connecting bolts has an influence on the logarithmic damping decrement of the layered and jointed structure. The logarithmic damping decrement increases with a decrease in distance between consecutive bolts due to an increase in average interface pressure and attains its maximum under the condition of uniform pressure distribution at the interfaces.

(e) The logarithmic damping decrement increases with an increase in the number of layers in a layered and jointed structure because of the increase in the number of interface friction layers, which causes an increase in the energy loss due to interface friction.

Finally, it was established that the damping capacity of the layered and jointed structures can be improved considerably (with a uniform pressure distribution at the interfaces) by using connecting bolts with the minimum possible tightening torque on them, and a large number of layers. This increase in logarithmic damping decrement may be 589.8 % higher than that of an equivalent solid beam. Layered and jointed copper structures, having a higher damping capacity, can be effectively used as beds for machine tools. This will also further enhance the capability of machine tool beds to minimize the stick-slip motion.

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