Aeroelastic instability studies of rotating tapered composite sandwich blades with magnetorheological elastomeric core

Naresh Jonna[1*], and J Srinivas[1]

[1]National Institute of Technology Rourkela, Odisha, India

naresh.jonna11@gmail.com
Srin07@yahoo.co.in

Abstract: The vibration minimization using a certain class of absorption materials is a goal in many recent dynamic systems. Magnetorheological elastomer is one kind of such material, which exhibits variable rheological properties like viscosity due to the magnetic field's influence. Aeroelastic analysis of such rotating structures is crucial in wind turbines and turbomachinery. In this article, aeroelastic stability studies of rotating non-uniform composite sandwich beam embedded with magnetorheological elastomer in supersonic flow are presented. Magnetorheological core layer is sandwiched between the composite face layers in this beam. For composite layers, the problem is formulated using beam theory, the magnetic field-dependent complex modulus is used to determine the magnetorheological core layer's constitutive behavior and aerodynamic pressure is modeled with piston theory. Lagrange's equation is applied to obtain the equations of motion for the sandwich beam and solved by the finite element technique. The published values of the natural frequencies are compared and the influence of rotational speed, aerodynamic pressure, setting angle, intensity of magnetic field, hub radius, taper ratio parameters, and ply orientation on the fundamental frequency are studied. The results reveal that by changing specific composite beam parameters, the flutter boundaries of the rotating beam may be expanded.

Keywords: Aeroelastic· Sandwich composites· Magnetorheological elastomer· Rotating beam·

1 Introduction

Composite structures are replacing traditional structural materials due to its high stiffness, strength-to-weight ratio, durability, and low maintenance cost. They're employed in a wide range of engineering applications, including as aviation, robot arms, spacecraft, and wind turbine blades. When these materials are utilized in spinning applications like robotic arms, helicopter rotor blades, wind turbine blades, satellite antennas, jet engine blades, airplane propellers, and other turbo-machinery applications, may affected by the centrifugal forces which results in vibrations. The most effective technique to control excessive vibration is to incorporate dampening into the structures. Constrained-layer damping (CLD) is a frequently used approach for vibration reduction of thin-walled beam structures. This structural dampening method relies on the addition of additional layers to structure in order to minimize mechanical vibration and noise.

Magnetorheological (MR) smart materials have gotten a lot of interest in recent years because of their unique and superior intelligent features in a wide variety of technical applications. Magnetic fields make these materials exceedingly sensitive. The rheological characteristics of these materials change quickly when they are exposed to an external field. They have reversible qualities, and by withdrawing the external fields, they can revert to their former state. Magnetorheological materials are suitable options for increasing the dynamic stability of structures because of their semi-active control capabilities.

The dynamic characteristics of rotating under supersonic flow have been studied by several researchers. S.A.Fazelzadeh et al. [1] studies vibrational analysis of functionally graded blades using DQM under high temperatures subjected to supersonic flow. Aeroelastic characteristics of functionally graded GPLRPC composites using extended Galerkin method is explored by Bahaadini and Saidi [2] in supersonic flow. S.Farhadi et al. [3] investigates the flutter of a rotating rectangular plate based on Kane dynamic method subjected to supersonic airflow. Majidi-Mozafari et al. [4] investigated the rotating cylindrical shells made of graphene nanoplatelets reinforced functionally graded, under supersonic flow condition for various distribution patterns. Many researchers also studied on the rotating beams [5-8]. A book on finite element analysis of rotating beams is written by Ganguli [9] with examples to understand the basic concepts.

For aerospace applications, weight reduction is the primary criteria without affecting the performance of the components. As a result, composite materials are used in rotating components. Several researchers analysed the rotating composite beams. In this regard, the impact of pre-setting and pre-twist angles on the bending coupled vibrations of rotating thin-walled composite beams was investigated by oh et al. [10]. Jafari-Talookolaei[11] obtained mode shape and natural frequency analytically for rotating composite beam, by taking into account the influence of numerous factors including material anisotropy, delamination, hub radius, fibre angles, and rotation speeds. Multiple beam theories, such as the Reddy theories Euler, and Timoshenko, were used by Aksencer and Aydogdu [12] to explore the rotating composite beam vibration characteristics. Rafiee et al. [13] reviewed the scientific literature on composite beams and blades in depth, which are rotating. The review focused on numerical, analytical, and semi-analytical examinations of the dynamics, vibration, and control of rotating composite beams and blades.

In recent years, many researchers have focused on magnetorheological sandwich structures. The MR sandwich beam’s free and forced vibration characteristics are studied by several authors [14-18] numerical and experimentally. Focusing on the rotating MR sandwich beam. The dynamic study of a rotating uniform magnetorheological sandwich beam exposed to periodic loads axially was examined by Nayak et al. [19]. Navazi et al. [20] explored rotating sandwich beam vibrations. They studied the natural frequency and loss factor in relation to magnetic field, taper ratio, and rotation speed. Bornassi et al. [21, 22] considered rotating MRE sandwich beam by using the assumed mode approach to investigate the torsional vibrations of tapered beam and edgewise free vibration for uniform beam with rectangular cross section. The influence of several parameters on natural frequencies, such as magnetic field, taper ratio, setting angles, hub radius, rotation speed, and core layer thickness, are also investigated in depth.

Aeroelastic characteristics of the magnetorheological based structures is studied by applying aerodynamic pressure on the body, which is done using first order piston theory [23]. The first-order piston theory was applied to analyse the MR sandwich beam in supersonic airflow. Asgari et al. [24, 25] investigated the flutter instabilities of partially and fully treated magnetorheological fluid sandwich beams under supersonic flow. The magnetic field intensity, constraining layer, and core layer thicknesses all have an influence on the beam's critical aerodynamic pressure. Bornassi et al. [26, 27] explored the influence of MR fluid on the aeroelastic stability of turbomachines. As per their findings, using this material as the blade core improves the flutter speed of turbomachines. In another study, the bending-torsion flutter of a blade cascade is examined for MRE sandwich beam by assuming subsonic and incompressible flow based on Whitehead unsteady theory. Eshaghi [28, 29] studies the flutter of magnetorheological sandwich plates that have been fully and partially treated. The optimal positions of MR pockets in magnetorheological sandwich plates are determined in order to maximize the bounds of flutter for the MR sandwich plate.

Although few works on the aeroelastic characteristics of MR sandwich structures have appeared in recent years. In these works, the face layers of the sandwich beam are modeled using isotropic materials. In the present study, composite materials are used as the face layers of the tapered sandwich beam. Dynamic analysis and flutter stability issues of such rotating blade is rarely found. The mathematical modeling is explained in the next section.

2 Mathematical modelling

A rotating magnetorheological core embedded composite sandwich beam with length ‘L’ placed with the hub radius ‘R’ at setting angle θ as shown in figure 1. At the root of the beam, a local Cartesian coordinate system is set, with axes running along the beam's length and breadth axes, respectively. The beam is modelled as three layers sandwich beam with top and bottom layers as composite laminates where middle layer is MR elastomer and the beam is double tapered along width and height. At the root of the beam, the thicknesses of three layers are h_{t0}, h_{c0} and h_{b0}, respectively and width b_0. The subscripts t, c, b, 0 denotes the top, core, bottom and root of the beam respectively.

Certain assumptions are considered for mathematical modelling. Due to the low modulus of elasticity of MR material in contrast to that of the composite layers, the axial stress in the magnetorheological core is negligible. During deformations, there is no delamination or slippage between the three layers due to perfect bonding. Because of the incompressibility of the MR core, the transverse deflections of the three layers are the same. Shear stresses in face layers are regarded insignificant due to their thinness, whereas shear stresses in core layers are evaluated

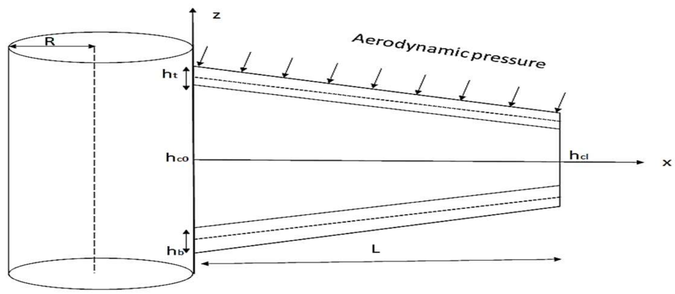

Fig. 1. Rotating MR tapered beam

The thickness of the face sheets of an MR composite sandwich beam remains constant, but only the core is tapered, and the structure's width, i.e. both face layers and core layer, is tapered. The width and thickness variations over the length of the beam may be expressed as follows:

$$b = b_0\left(1 - \beta\frac{x}{L}\right) \; and \; h_t = h_b, h_c = h_{c0}\left(1 - \alpha\frac{x}{L}\right) \quad (1)$$

Where 'β' indicate width ratio of the structure and 'α' is the taper ratio for MR core thickness, respectively.

According to above assumptions and considering the Euler-Bernoulli beam theory, the composite face layers displacement field can be stated as:

$$\mathrm{w}(\mathrm{x,z,t}) = \mathrm{w}_0(\mathrm{x,t}) \quad (2)$$

$$u_{t,b}(\mathrm{x,z,t}) = u_{0,t,b}(\mathrm{x,t}) + \mathrm{z}\frac{\partial w_0}{\partial x}(\mathrm{x,t}) \quad (3)$$

where, w_0 and u_0 are the displacements components of the beam along the z- and x-axis respectively at any point on the mid-plane.

The relationship between the strain–displacement for the MR composite sandwich beam is given by,

$$\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = \frac{u_t - u_b}{h_c} + \frac{D}{h_c}\frac{dw}{dx} \quad (4)$$

Where $d = h_c + 0.5(h_t + h_b)$

The force and moment resultants for the composite laminated beam can be expressed [30] as

$$\begin{bmatrix} N_x \\ M_x \end{bmatrix} = \begin{bmatrix} A_{11} & B_{11} \\ B_{11} & D_{11} \end{bmatrix}\begin{bmatrix} \varepsilon_x \\ \kappa_x \end{bmatrix} \quad (5)$$

where, A_{11}, B_{11}, and D_{11} are the extension, coupling and bending stiffness parameters given as:

$$(A_{11}, B_{11}, D_{11}) = b\sum_{k=1}^{n}\bar{Q}\int_{h_{k-1}}^{h_k}(1, z, z^2)dz \quad (6)$$

Here n is the number of layers. $\bar{Q}$ is the transformed stiffness matrix.

The strain energy for the MR composite sandwich beam can be expressed as:

$$U_s = \frac{1}{2}\int_0^L b\left[(A_{11}^1 + A_{11}^3)\left(\frac{\partial u_0}{\partial x}\right)^2 dx + 2(B_{11}^1 + B_{11}^3)\left(\frac{\partial u_0}{\partial x}\right)\left(\frac{\partial^2 w_0}{\partial x^2}\right) + (D_{11}^1 + D_{11}^3)\left(\frac{\partial^2 w_0}{\partial x^2}\right)^2\right] dx + \frac{1}{2}\int_0^L G^* A_c \gamma_{xz}^2 \quad (7)$$

Where G^* is the complex shear modulus of magnetorheological elastomer core.

The kinetic energy of a composite laminated beam is given by

$$T = \frac{1}{2}\int_0^L b\left[I_0\left(\left(\frac{\partial u_0}{\partial t}\right)^2 + \left(\frac{\partial w_0}{\partial t}\right)^2\right) + I_2\left(\frac{\partial w_0}{\partial t}\right)^2 + \frac{1}{2}\int_0^L I_2\gamma_{xz}^2\right] \quad (8)$$

Where $(I_0, I_2) = \sum_{k=1}^{n}\int_{h_{k-1}}^{h_k} \rho(1, z^2)dz$

The work done by rotating beams owing to centrifugal forces may be stated as

$$W_R = -\frac{1}{2}\int_0^L p(x)w'^2 dx + \frac{1}{2}\int_0^L \rho A\Omega^2 \sin^2(\theta) w^2 dx \quad (9)$$

where θ is the setting angle Ω is the rotational speed, and p(x) is the centrifugal force of the rotating beam which is obtained as

$$p(x) = \Omega^2 \int_x^L \rho A(R + \bar{x})\, d\bar{x} \quad (10)$$

The non-conservative work done due to aerodynamic pressure (ΔP) is given according to linear piston theory as

$$W_A = b\int_0^L \Delta P\, w(x,t)dx \quad (11)$$

$$\Delta P = -\frac{\rho_a V_\infty^2}{\sqrt{M_\infty^2 - 1}}\frac{\partial w}{\partial x} = \frac{\lambda E_1 h^3}{L^3}\frac{\partial w}{\partial x} \quad (12)$$

Here, $\lambda = \frac{\rho_a V_\infty^2}{\sqrt{M_\infty^2 - 1}}\frac{L^3}{E_1 h^3}$ is a constant. Flutter boundary is obtained by using non-dimensional aerodynamic pressure (λ) parameter.

The sum of the work done by centrifugal forces and the work done by aerodynamic pressure is the total work done on the composite beam. The energy equations (7) and (8) are substituted into Lagrange's equation (13) to acquire the composite MR sandwich beam's governing equations of motion

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial U}{\partial q_i} = Q_i \quad (13)$$

The finite element method is used to solve obtained equations of motion with four degrees of freedom beam element. The axial and transverse displacement functions are written in terms of shape functions and displacement vectors as

$$u_{t,b}(x,t) = N_u(x)d(t) \tag{14}$$

$$w(x,t) = N_w(x)d(t) \tag{15}$$

Where N_u (x) and N_w (x) are shape functions and d(t) = [u_{ti} u_{bi}, w_i, θ_i, u_{ti+1}, u_{bi+1}, w_{i+1}, θ_{i+1}] ^{T} is displacement vector.

The formulated governing equations of motion for undamped laminated sandwich beam embedded with magnetorheological fluid is

$$[M]\{\ddot{d}\} + [K]\{d\} = 0 \tag{16}$$

where [M] is the mass matrix and [K] is the stiffness matrix and is the nodal force vector.

3 Results and Discussion

The aeroelastic characteristics of rotating tapered composite sandwich beams embedded with magnetorheological elastomer is studied by varying setting angle, rotation speed, hub radius, and ply orientations. Before studying the effectiveness of the model should be verified. The cantilever sandwich beam with uniform cross section is considered for validation. The correctness of the current modelling method is demonstrated by comparing the first three frequencies obtained by the developed model with those found in the literature [19 20] by varying magnetic field is listed in table1. For further validation of aeroelastic characteristics of the sandwich beam which is compared with the asgari [25] where critical aerodynamic pressure is obtained by varying magnetic fields as shown in fig 2.

Table 1. laminated magnetorheological sandwich beam natural frequencies.

Mode Number	Formulation	Natural frequencies			
	Magnetic field	0	0.2	0.4	0.6
1	Ritz method	14.19	15.52	16.36	16.59
2		51.13	58.4	64.32	66.23
3		104.28	120.59	134.58	139.23
1	Galerkin method	14.43	15.77	16.64	16.87
2		51.84	59.15	65.15	67.11
3		105.05	121.12	135.02	139.73
1	Present	14.24	15.59	16.45	16.67
2		51.39	58.9	65.09	67.1
3		104.36	120.92	135.29	140.12

Table 1 and Figure 2 show that the results of the current formulation are in good accord with those of other authors. After the validation of present method with the previous literature studies. Now considering the material properties of graphite epoxy as listed in the table-2 and the layup configuration [0/90/90/0/MR/0/90/90/0] of a composite laminated MR rotating beam is considered with dimensions of length 400mm, width 30mm and a thickness of 1 mm each for face layers. The core layer thickness is 10mm at the root of the tapered beam.

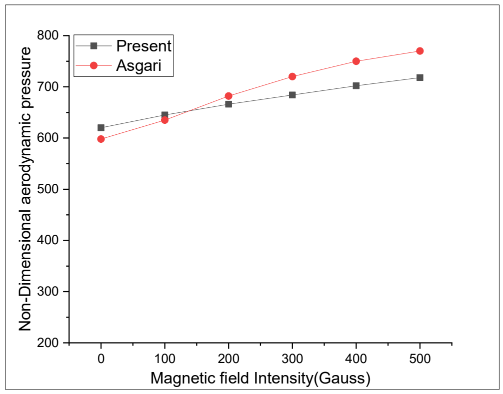

Fig. 2. Comparison of critical aerodynamic pressure with magnetic field for C-C condition

Table 2. Properties of rotating tapered composite beam.

Properties		Value
Longitudinal Youngs Modulus	E_1	131GPa
Transverse Youngs Modulus	E_2=E_3	10.34GPa
Shear Modulus in xy and yz direction	G_{12}=G_{23}	6.895GPa
Shear Modulus in xz direction	G_{13}	6.205Gpa
Poisson's Ratio in xy and yz direction	ν_{12}=ν_{13}	0.22
Poisson's Ratio in xz direction	ν_{23}	0.49
Density of composite layer	ρ_t=ρ_b	1627
Density of MR layer	ρ_c	3500

The influence of aerodynamic pressure on the natural frequencies is studied which is shown in the figure 3 by considering uniform and non-rotating beam with setting angle 90O and radius of the hub is zero. From results it can be observed that first natural frequency is decreasing and reaches zero and that indicates flutter instability boundary has been reached. The divergence type instability occurred for the sandwich beam as explained in the asgari and Kouchakzadeh [25] paper, for clamped-free beams.

As shown in fig.4 rotation of the beam is influencing on the non-dimensional aerodynamic pressure. as the non-dimensional rotational speed is increased then natural frequency increases and loss factor decreases, which indirectly increases the flutter stability region.

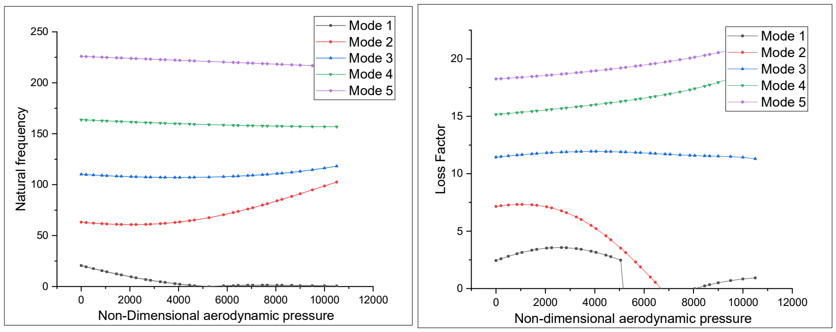

Fig. 3. Comparison of natural frequencies and non-dimensional aerodynamic pressure

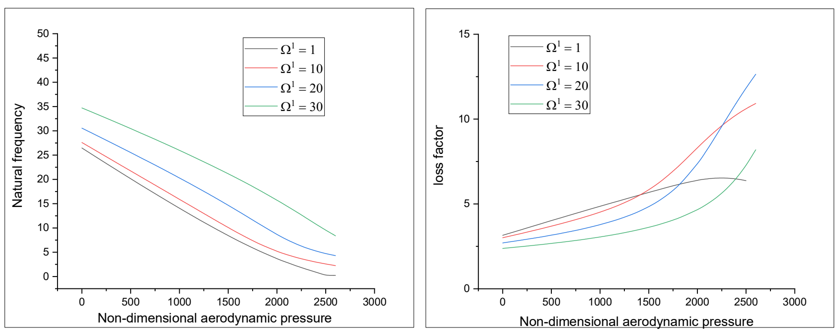

Fig. 4. Comparison of natural frequencies and non-dimensional aerodynamic pressure by varying rotational speed

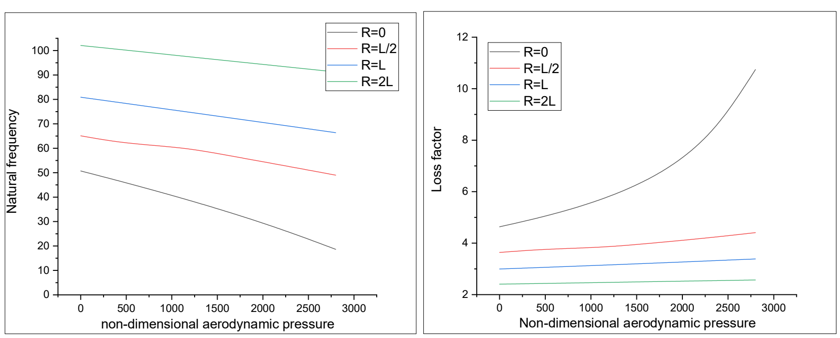

Fig. 5. Comparison of natural frequencies and non-dimensional aerodynamic pressure by varying hub radius

The influence of hub radius and aerodynamic pressure on the natural frequencies and loss factors are shown in the figure 5. Natural frequencies rise as the radius of the hub increases, but the loss factor decreases. Effect of radius of the hub on aerodynamic pressure is moderate. The impact of the taper ratio on natural frequencies and aerodynamic pressure is also explored and shown in figure 6. The natural frequencies and loss factors are increasing when the taper ratio is increased. But natural frequency is decreasing when aerodynamic pressure is increased whereas loss factor is increasing linearly with respect to aerodynamic pressure. here we can choose taper ratio based on the design requirement.

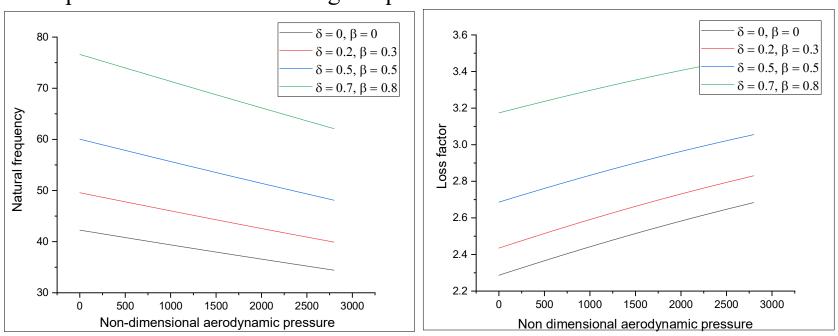

Fig. 6. Comparison of natural frequencies and non-dimensional aerodynamic pressure by varying taper and width ratio.

Figure 7 shows the influence of setting angle on the rotational speed and natural frequency. From the figure it can be observed that as the rotational speed is increased the natural frequencies are increased and loss factor is decreased for all setting angles.

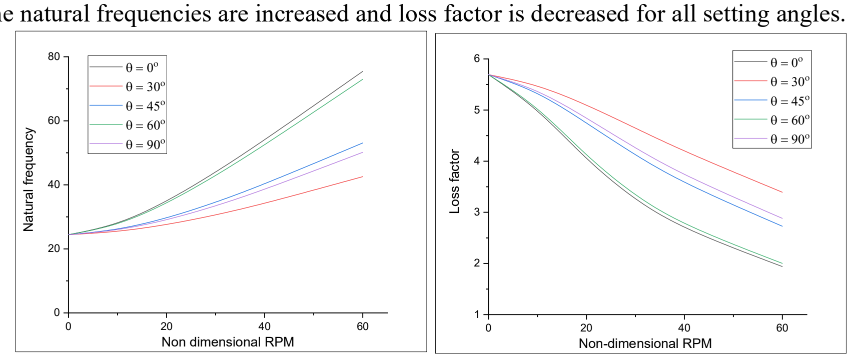

Fig. 7. Comparison of natural frequencies and non-dimensional rotational speed by varying setting angle.

Natural frequencies are changing drastically by changing the setting angle at constant rotational speed. This can be detailly visualize in the figure 8. It is observed from the results that natural frequency decreases from zero setting angle to 30 degree setting angle after that again it increases up to 60 degree setting angle further it decreases. It is following a flow pattern. Same can be seen in the loss factor results with respect setting angle.

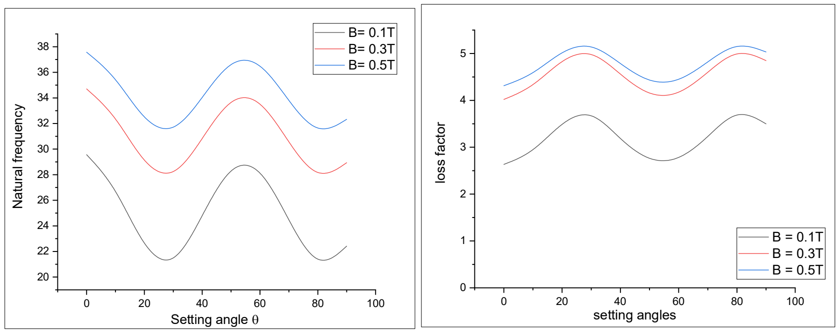

Fig. 8. Comparison of natural frequencies and setting angle by varying magnetic field

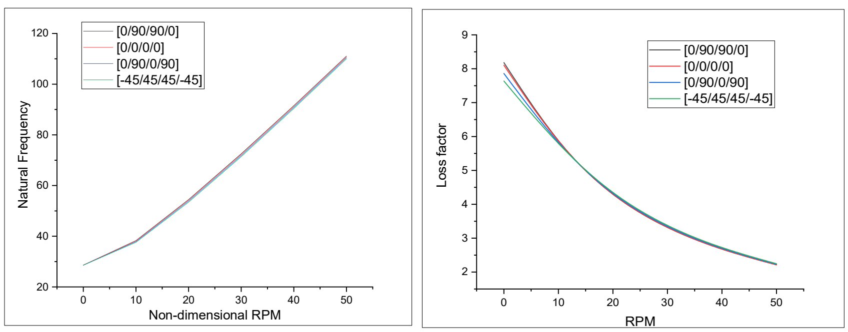

Fig. 8. Comparison of natural frequencies and setting angle by varying magnetic field

Effect of ply orientation on the rotational speed and natural frequency are presented in the figure 9. Here, four types of layups considered they are symmetric cross ply, unidirectional ply, unsymmetric cross ply, angle ply orientations. From results we observe that natural frequency increases while increasing rotational speed but there is not much difference between these four lay-up configurations.

4 Conclusion

A rotating tapered magnetorheological composite sandwich beam's aeroelastic characteristics are investigated. The magnetorheological composite sandwich beam is mathematically modelled. The finite element approach is used to solve the equations of motion. Natural frequencies are compared against the reported in the literature to validate the given formulation by varying magnetic field intensity. Further, it is validated for critical aerodynamic pressure for sandwich beam for clamped-clamped condition by varying magnetic field.

Various parametric studies such as rotational speed, setting angle, taper ratio, magnetic field, ply orientations were performed to study their influence on natural frequency, loss factor and aerodynamic pressure. According to the results obtained, the following main conclusions can be drawn:

- The aerodynamic pressure is increased on cantilever tapered sandwich beam then divergence type of flutter instability is occurred where first mode natural reaches zero axis.
- As the rotational speed is increases, the natural frequency also increases which indicates stiffness of the beam is increased with respect to rotational speed. Further, rotational speed is proportional to flutter instability boundary.
- By rising the hub radius, flutter stability, natural frequencies of tapered magnetorheological composite sandwich beam increases and respective loss factor decreases.
- With constant thickness and variable width composite layers, increasing the breadth of the core layer and taper ratio along the thickness results in an increase in natural frequency and a drop in the corresponding loss factor.
- When the rotational speed is increased, the setting angle causes the loss factor to decrease and the natural frequency to increase. But natural frequency increases and decreases for certain setting angles which follows a periodic motion.
- The change in lay-up orientations of tapered composite MR sandwich beam results in slight change in natural frequency and loss factor.

Therefore, rotating tapered magnetorheological composite sandwich beam characteristics can be altered by applying external magnetic field under aerodynamic pressure. As a future work, optimized size and location will be obtained, pressure is replaced with incompressible and unsteady flow to perform nonlinear problem. Main motive is to control the vibration levels within a frequency band in semi-automatic way.

References

1. Fazelzadeh, S. A., Malekzadeh, P., Zahedinejad, P., & Hosseini, M. (2007). Vibration analysis of functionally graded thin-walled rotating blades under high temperature supersonic flow using the differential quadrature method. Journal of sound and vibration, 306(1-2), 333-348. doi.org/10.1016/j.jsv.2007.05.011
2. Bahaadini, R., & Saidi, A. R. (2018). Aeroelastic analysis of functionally graded rotating blades reinforced with graphene nanoplatelets in supersonic flow. Aerospace Science and Technology, 80, 381-391. doi.org/10.1016/j.ast.2018.06.035
3. Farhadi, S., & Hosseini-Hashemi, S. H. (2011). Aeroelastic behavior of cantilevered rotating rectangular plates. International Journal of Mechanical Sciences, 53(4), 316-328. doi.org/10.1016/j.ijmecsci.2011.01.013
4. Majidi-Mozafari, K., Bahaadini, R., & Saidi, A. R. (2021). Aeroelastic flutter analysis of functionally graded spinning cylindrical shells reinforced with graphene nanoplatelets in supersonic flow. Materials Research Express, 8(11), 115012. doi.org/10.1088/2053-1591/ac2ce4
5. Banerjee, J. R., & Kennedy, D. (2014). Dynamic stiffness method for inplane free vibration of rotating beams including Coriolis effects. Journal of Sound and Vibration, 333(26), 7299-7312. doi.org/10.1016/j.jsv.2014.08.019

6. Kim, H., Yoo, H. H., & Chung, J. (2013). Dynamic model for free vibration and response analysis of rotating beams. Journal of Sound and Vibration, 332(22), 5917-5928. doi.org/10.1016/j.jsv.2013.06.004
7. Sabuncu, M., & Evran, K. (2005). Dynamic stability of a rotating asymmetric cross-section Timoshenko beam subjected to an axial periodic force. Finite elements in analysis and design, 41(11-12), 1011-1026. doi.org/10.1016/j.finel.2004.12.004
8. Sabuncu, M., & Evran, K. (2006). Dynamic stability of a rotating pre-twisted asymmetric cross-section Timoshenko beam subjected to an axial periodic force. International Journal of Mechanical Sciences, 48(6), 579-590. doi.org/10.1016/j.ijmecsci.2006.01.010
9. Ganguli, R. (2017). Finite element analysis of rotating beams. Physics Based Interpolation. Springer, Singapore. doi.org/10.1007/978-981-10-1902-9
10. Oh, S. Y., Song, O., & Librescu, L. (2003). Effects of pretwist and presetting on coupled bending vibrations of rotating thin-walled composite beams. International Journal of Solids and Structures, 40(5), 1203-1224. doi.org/10.1016/S0020-7683(02)00605-4
11. Jafari-Talookolaei, R. A. (2015). Analytical solution for the free vibration characteristics of the rotating composite beams with a delamination. Aerospace Science and Technology, 45, 346-358. doi.org/10.1016/j.ast.2015.06.009
12. Aksencer, T., & Aydogdu, M. (2015). Flapwise vibration of rotating composite beams. Composite Structures, 134, 672-679. doi.org/10.1016/j.compstruct.2015.08.130
13. Rafiee, M., Nitzsche, F., & Labrosse, M. (2017). Dynamics, vibration and control of rotating composite beams and blades: A critical review. Thin-Walled Structures, 119, 795-819. doi.org/10.1016/j.tws.2017.06.018
14. Lara-Prieto, V., Parkin, R., Jackson, M., Silberschmidt, V., & Kęsy, Z. (2009). Vibration characteristics of MR cantilever sandwich beams: experimental study. Smart Materials and structures, 19(1), 015005. doi.org/10.1088/0964-1726/19/1/015005
15. Nayak, B., Dwivedy, S. K., & Murthy, K. S. R. K. (2011). Dynamic analysis of magnetorheological elastomer-based sandwich beam with conductive skins under various boundary conditions. Journal of Sound and Vibration, 330(9), 1837-1859. doi.org/10.1016/j.jsv.2010.10.041
16. Rajamohan, V., Rakheja, S., & Sedaghati, R. (2010). Vibration analysis of a partially treated multi-layer beam with magnetorheological fluid. Journal of Sound and Vibration, 329(17), 3451-3469. doi.org/10.1016/j.jsv.2010.03.010
17. Sun, Q., Zhou, J. X., & Zhang, L. (2003). An adaptive beam model and dynamic characteristics of magnetorheological materials. Journal of Sound and Vibration, 261(3), 465-481. doi.org/10.1016/S0022-460X(02)00985-9
18. Yeh, Z. F., & Shih, Y. S. (2006). Dynamic characteristics and dynamic instability of magnetorheological material-based adaptive beams. Journal of Composite Materials, 40(15), 1333-1359. doi.org/10.1177%2F0021998306059715
19. Nayak, B., Dwivedy, S. K., & Murthy, K. S. R. K. (2014). Dynamic stability of a rotating sandwich beam with magnetorheological elastomer core. European Journal of Mechanics-A/Solids, 47, 143-155. doi.org/10.1016/j.euromechsol.2014.03.004
20. Navazi, H. M., Bornassi, S., & Haddadpour, H. (2017). Vibration analysis of a rotating magnetorheological tapered sandwich beam. International Journal of Mechanical Sciences, 122, 308-317. doi.org/10.1016/j.ijmecsci.2017.01.016
21. Bornassi, S., Navazi, H. M., & Haddadpour, H. (2018). Edgewise bending vibration analysis of a rotating sandwich beam with magnetorheological elastomer core. International Journal of Structural Stability and Dynamics, 18(11), 1850134. doi.org/10.1142/S0219455418501341

22. Bornassi, S., & Navazi, H. M. (2018). Torsional vibration analysis of a rotating tapered sandwich beam with magnetorheological elastomer core. Journal of Intelligent Material Systems and Structures, 29(11), 2406-2423. doi.org/10.1177%2F1045389X18770864
23. Ashley, H., & Zartarian, G. (1956). Piston theory-a new aerodynamic tool for the aeroelastician. Journal of the Aeronautical Sciences, 23(12), 1109-1118. doi.org/10.2514/8.3740
24. Asgari, M., & Kouchakzadeh, M. A. (2016). Aeroelastic characteristics of magneto-rheological fluid sandwich beams in supersonic airflow. Composite Structures, 143, 93-102. doi.org/10.1016/j.compstruct.2016.02.015
25. Asgari, M., Rayyat Rokn-Abadi, M., Yousefi, M., & Haddadpour, H. (2019). Aeroelastic analysis of a sandwich panel with partially treated magneto-rheological fluid core. Journal of Intelligent Material Systems and Structures, 30(1), 140-154. doi.org/10.1177%2F1045389X18803462
26. Bornassi, S., Navazi, H. M., & Haddadpour, H. (2018). Aeroelastic instability analysis of a turbomachinery cascade with magnetorheological elastomer based adaptive blades. Thin-Walled Structures, 130, 71-84 doi.org/10.1016/j.tws.2018.05.010.
27. Bornassi, S., Navazi, H. M., & Haddadpour, H. (2020). Coupled bending-torsion flutter investigation of MRE tapered sandwich blades in a turbomachinery cascade. Thin-Walled Structures, 152, 106765. doi.org/10.1016/j.tws.2020.106765
28. Eshaghi, M. (2020). The effect of magnetorheological fluid and aerodynamic damping on the flutter boundaries of MR fluid sandwich plates in supersonic airflow. European Journal of Mechanics-A/Solids, 82, 103997. doi.org/10.1016/j.euromechsol.2020.103997
29. Eshaghi, M. (2020). Optimal core design of MR based smart panels subject of flutter instability. Mechanics Based Design of Structures and Machines, 1-20. doi.org/10.1080/15397734.2020.1843488
30. Reddy, J. N. (2003). Mechanics of laminated composite plates and shells: theory and analysis. CRC press. doi.org/10.1201/b12409