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Tabu Search based Brain MR Image Segmentation Using Hidden Markov Random Field Model

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Abstract

In this paper, we propose a hybrid Tabu(HT) Algorithm for segmentation of Brain Magnetic Resonance (MR) images in supervised framewrok. Gaussian Hidden Markov Random Field (GHMRF) is used to model the available degraded image. The *apriori* image MRF model parameters as well as the GHMRF model parameters are assumed to be known. The class labels are estimated using the Maximum *a Posteriori* (MAP) estimation criterion.

1 Introduction

Image segmentation is one of the early vision problem and has a wide application domain. The problem becomes more compounded while segmenting noisy scenes. For the last two decades, stochastic models, specifically, Markov Random Field (MRF) models have been used for image segmentation [1, 2]. MRF provides an unifying tool to provide a link between local and global characteristics of the image with the notion of neighbor structure. Geman and Geman[1] have proposed line fields with MRF models with a view to preserve the edges in noisy images. Often, the segmentation problem is formulated as pixel labelling problem and the pixel labels are estimated using the principle of Maximum *a Posteriori* Estimation (MAP) [1, 2, 3, 4]. Since computation of the MAP estimate is also a difficult problem, by and large, Simulated Annealing (SA) algorithm has been used to obtain the MAP estimates [3, 4]. The model based approaches for image segmentation can be viewed as either *supervised* or *unsupervised*. In the *supervised* framework, the associated model parameters are assumed to be known *a priori* [1, 2, 3, 4]. The problem is usually formulated as a pixel labelling problem using Bayesian approach. The MAP estimates of the pixel labels are obtained using SA algorithm [3]. In [3], Nanda et.al. have proposed a Homotopy Continuation method for MRF model parameter estimation while the image labels are obtained using SA algorithm. In [2], even though ICM converges with in a few iterations, it has the inherent local convergence property. Simulated Annealing was found to be a potential alternative [3, 4] for quite sometime. The notion of Tabu Search can provide an alternate tool for such problems.

Tabu Search proposed by Glover [5, 6, 7] can be viewed as a strategy to solve combinatorial optimization problem and is an adaptive procedure which overcomes the limitations of local optimality. The notions of Tabu Search could be used to devise algorithms for image restoration [4] and segmentation [8]. In [4] a parallel Tabu Search algorithm is proposed. Segmentation of Brain Magnetic Resonance Images (MRI) has attracted attention of many researchers for the last decade [9]. Hidden Markov Model(HMM) have been extensively used for speech recognition[10]. Zhang et.al. [9] has proposed Hidden Markov Random Field (HMRF) model to model the observed degraded image in the context of segmentation.

In this article, we have addressed the problem of image segmentation in *supervised* framework. The *a priori* class is modeled as a MRF model and the observed degraded image is modeled as HMRF model. Specifically, the observed image is modeled as Gaussian Hidden Markov Random Field (GHMRF). In supervised framework, the associated model parameters like mean (μ) and standard deviation (σ) of the Gaussian model are assumed to be known. The estimation of class labels are obtained by the Bayesian approach and the principle of Maximum a Posteriori(MAP) estimation.

A Hybrid Tabu Search based algorithm is proposed to obtained the MAP estimates of the labels. Synthetic images of different classes are generated using Gibb's sampler[1]. The proposed algorithm could be successfully tested for real Brain Magnetic Resonance Images (MRI).

2 Hidden Markov Random Field Model (HMRF)

Hidden Markov Models (HMM) have been applied to the problem of speech recognition [10]. HMMs are defined as stochastic processes generated by a Markov chain whose state sequence can not be observed directly, but only through a sequence of observations. Each observation is assumed to be a stochastic function of sate sequence. Here the underlying stochastic process is a MRF instead of a Markov chain and therefore, not restricted to one dimension. This special case is referred to as Hidden Markov Random Field (HMRF) model [9].

Let the images are assumed to be defined on a discrete rectangular lattice S=N×N. Let X denote the random field associated with the labels of the original image. The label process X is assumed to be MRF with respect to a neighborhood system η and is described by its local characteristics.

$$\begin{aligned} P(X_{i,j} = x_{i,j} \mid X_{k,l} = x_{k,l}, k,l \in S, (k,l) \neq (i.j)) \\ = P(X_{i,j} = x_{i,j} \mid X_{k,l} = x_{k,l}, (k,l) \in \eta) \end{aligned}$$

Since X is a MRF, or equivalently Gibbs distributed, the joint distribution can be expressed as $P(X = x \mid \phi) = \frac{1}{Z} e^{-U(x,\phi)}$, where $Z = \sum_x e^{-U(x,\phi)}$ is the partition function, ϕ denote the clique parameter vector, *U(x,ϕ)* is the energy function and is of the form $U(X,\phi) = \sum_{c(i,j)\in c} V_c(x,\phi)$ is the clique potential. Y is the observed random field. For any potential realization x, the random variables Y_i are conditional independent

$$P(Y \mid X) = \prod_{i \in S} P(y_i \mid x_i) \tag{1}$$

The joint probability of (X,Y) can be expressed as

$$P(Y,X) = P(Y \mid X)P(X) = P(X) \prod_{i \in S} P(y_i \mid x_i)$$

According to the local characteristics of MRF, the joint probability distribution of pair (X_i, Y_i) given the neighborhood configuration of X_{η_i} is

$$P(y_i, x_i \mid x_{\eta_i}) = P(y_i \mid x_i)P(x_i \mid x_{\eta_i}) \tag{2}$$

Thus, the marginal probability distribution of Y_i dependent on θ and X_{η_i}

$$\begin{aligned} P(y_i \mid x_{\eta_i}, \theta) &= \sum_{l \in L} P(y_i, l \mid x_{\eta_i}, \theta) \\ &= \sum_{l \in L} P(y_i, l, \theta) P(l \mid x_{\eta_i}) \end{aligned} \tag{3}$$

where $\theta = \{\theta_l, l \in L\}$. (3) is the Hidden Markov Random Field model, L denotes the number of labels.
With Gaussian distribution, (3) can be expressed as

$$p(y_i \mid x_{\eta_i}, \theta) = \sum_{l \in l} g(y_i, \theta_l) P(l \mid x_{\eta_i}) \tag{4}$$

(4) is refereed to as the Gaussian Hidden Markov Random Field (GHMRF) model, where $g(y_i, \theta_l)$ is the Gaussian probability density function.

3 Image Segmentation

We have addressed the problem of Brain MR image segmentation in both supervised and unsupervised framework. Since in supervised framework the model parameters are assumed to be known, it is required to estimate the pixel labels using the associated model parameters. Let X be the random field associated with the noise free class label and x be the realization of the same. X is modeled as a MRF. Let Y denote the observed image random field and y be the realization of it. Y is modeled as Gauss Hidden Markov Random Field (GHMRF). Let θ be the associated model parameters. In the pixel labeling problem, let x^* denote the true but unknown labeling configuration and $\hat{x}$ denote the estimate for x^*. x^* and $\hat{x}$ are realization of random field X, which is modeled as MRF. The observed image y is a realization of GHMRF. The problem is to recover x^* from the observed image y. The following optimality criterion is considered.

$$\hat{x} = ^{arg\ max}{x} \ P(X \mid Y, \theta) \tag{5}$$

where the model parameters for each class $\theta_l = [\mu_l, \sigma_l]$ are selected on an ad hoc basis. Since X is unknown, the posteriori probability distribution

$P(X \mid Y, \theta)$ can not be evaluated. Hence, using Baye's rule, (5) can be expressed as

$$\hat{x} = ^{arg\ max}_{x} \frac{P(Y \mid X, \theta) P(X)}{P(Y)} \quad (6)$$

Since Y is known. The denominator of (6) is a constant. Thus, (6) can be written as

$$\hat{x} = ^{arg\ max}_{x} P(Y \mid X, \theta) P(X) \quad (7)$$

Since, X is a MRF, the prior probability distribution in (7) is given as $P(X) = \frac{1}{Z'} e^{-U(X)}$. It is also assumed that the pixel intensity y_i follows a Gaussian distribution with parameters $\theta_i = \{\mu_l, \theta_l\}$. Given the class label $x_i = l$,

$$P(y_i \mid x_i) = \frac{1}{\sqrt{2\pi\sigma_l^2}} exp\left(-\frac{(y_i - \mu_l)^2}{2\sigma_l^2}\right) \quad (8)$$

Using the assumption of conditional independence

$$P(Y \mid X) = \prod_{i \in S} P(y_i \mid x_i) \quad (9)$$

$$= \prod_{i \in S} \left[\frac{1}{\sqrt{2\pi}} \left(-\frac{(y_i - \mu_{x_i})^2}{2\sigma_{x_i}^2} - log(\sigma_{x_i})\right)\right]$$

(10) can be expressed as

$$P(Y \mid X) = \frac{1}{Z'} exp(-U(Y \mid X))) \quad (10)$$

$$U(Y \mid X) = \sum_{i \in S} U(y_i \mid x_i) \quad (11)$$

$$= \sum_{i \in S} \left[\frac{(y_i - \mu_{x_i})^2}{2\sigma_{x_i}^2} + log(\sigma_{x_i})\right]$$

and $Z' = (2\pi)^{N/2}$. Using the above, (7) can be expressed as

$$\hat{x} = ^{arg\ max}_{x} \left[\frac{1}{Z'} exp(-U(X)) \frac{1}{Z'} exp(-U(Y \mid X))\right]$$

$$= ^{arg\ max}_{x} exp\left[-\{U(Y \mid X) + U(X)\}\right] \quad (12)$$

(12) is equivalent to minimizing the following

$$\hat{x} = ^{arg\ min}_{x} [U(Y \mid X) + U(X)] \quad (13)$$

The MAP estimate in (13) is obtained by employing the proposed Hybrid Tabu search algorithm.

3.1 Tabu Search

Tabu Search (TS) is a general heuristic search procedure devised for finding a global minimum of a function which may be linear or nonlinear. It was developed by Glover [6] in 1986. This procedure has a flexible memory to keep the information about the past steps of the search and uses it to create and exploit the new solutions in the search space. Initially the Tabu Search algorithm starts from a random point or move X_{ini} and the next point obtained from the set of the feasible solutions by applying simple modification to X_{ini}. This modification is called a 'move' and the next point x_1 is the 'new point'. In order to avoid the algorithm's move to a new point in the search space which has been visited earlier, a Tabu list is introduced, where every successful move is stored. The new move obtained will now be compared with all the earlier moves stored in the Tabu list and if the new move matches with any move in the Tabu list, it is discarded. The use of Tabu list decreases the possibility of cycling,i.e. to a solution that has been visited recently. The Tabu list is updated with the new set of solutions. The best valued solution is selected as the next solution X_{next}. This process helps in overcoming the local minima problem.

3.2 Tabu Search Based Hybrid Algorithm

We have proposed the Tabu Search algorithm which mostly makes the Tabu array with recent moves of minimum energy but also moves with higher energy has been accepted with a probability. This strategy is our aspiration condition. The basic steps of the algorithm to obtain the MAP estimate is as follows.

3.2.1 Hybrid Tabu Algorithm

1. Initialize the initial temperature T_{in}.
2. The Initial Image of the algorithm is the degraded image.
3. A Tabu list, i.e. Tabu image set is created to store the recent moves, i.e. the image estimates of the algorithm. The set is of fixed length.
4. From the current move of the image, the next Tabu image is generated.

i) Perturb $x_{ij}(t)$ with a zero mean Gaussian distribution with a suitable variance.
ii) Evaluate the energy $U_p(x_{ij}(t+1))$ and $U_P(x_{ij}(t))$. If $\Delta f = (U_P(x_{ij}(t+1)) - U_P(x_{ij}(t))) < 0$, assign the modified value as the new value. If $\Delta f > 0$, accept the $x_{ij}(t+1)$ with a probability (if $exp(-\Delta f/T(x) > random(0,1))$.
iii) Repeat step (ii) for all the pixels of the image.

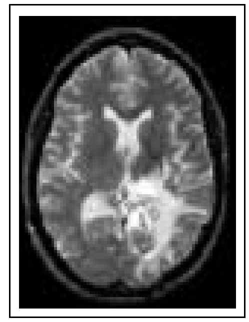
5. Compute the power of the updated image $x(t+1)$ as $Px(t+1)$ and compare it with the powers of the Tabu list named as Tabu energy, if $Px(t+1) > P_{Tabu}$, accept $x(t+1)$ as Tabu image.

6. Aspiration condition: If $Px(t+1) > P_{Tabu}$, accept $x(t+1)$ as Tabu image with some probability.

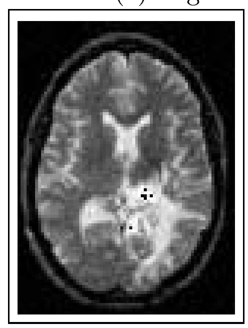
7. Update the Tabu list.

8. Decrease the Temperature according to the logarithmic cooling schedule.

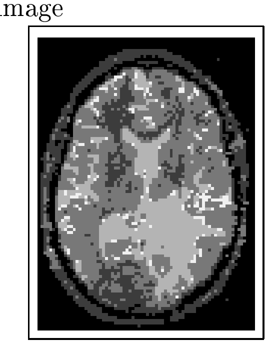
9. Repeat step 4-8 till the stopping criterion is met. In our simulation the stopping criterion is: the parameters do not change for three consecutive combined iterations.



(a)Original Image



(b)Noisy Image



(c)Segmented Image

Fig.1:

Table 1: Parameters for Fig.1

Class	1	2	3	4	5	6
μ	0.052	2.6248	1.5974	3.265	2.0	1.0
σ	0.079	4.86	0.562	0.408	0.257	0.2
δ	0.28					

4 Results and Discussions

We have validated the proposed algorithm with brain MR images. Two brain MR images are shown in Fig.1 and Fig.2 respectively. The degraded images are obtained by adding white Gaussian noise to the original images. The observed degraded image is modeled as GHMRF while the class label is modeled as MRF. In supervised mode, The GHMRF parameters such as μ and σ for different classes are selected on an adhoc basis. The clique potential function for *apriori* MRF class model is given by

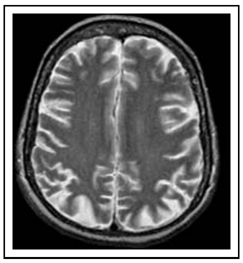
$$\begin{aligned} V_c(x) &= -\delta \quad if \quad |x_i - x_j| \quad = 0 \\ &= \quad \delta \quad if \quad |x_i - x_j| \quad \neq 0 \end{aligned}$$

Where x_i and x_j are the pixel values of the i^{th} and j^{th} pixel respectively.

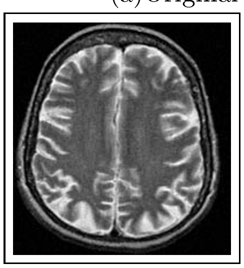
Fig.1(a) shows the real brain MR image of size (74×100) and the noisy image of SNR 25dB is shown in Fig.1(b). The corresponding segmented image is shown in Fig.1(c). The parameters used for this image are: Initial temperature $T_{in} = 0.5$ and Tabu length $K = 10$. Because of noisy case , parameters for six classes as given in Table 1. are used to segment the noisy image. As observed from Fig.1(c) there are four broad classes in the segmented image and thus noisy image could be segmented. The 2nd real image considered is shown in Fig.2(a) and the noisy and segmented images are shown in Fig.2(b) and 2(c) respectively. The parameters used in this case are: $T_{in} = 0.1$ and Tabu length K=5. Because of noisy case, five initial classes are assumed and hence the parameters used for these five classes are shown in Table 2. However, as observed from Fig.2(c), the algorithm could segment into three broad classes and thus proper segmentation could be achieved.

Table 2: Parameters for Fig.2

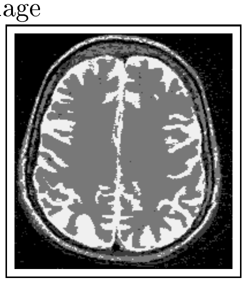
Class	1	2	3	4	5
μ	0.1	0.5	1.5	1.5	2.0
σ	0.5	0.5	0.5	1.5	1.5
δ	0.1				



(a)Original Image



(b)Noisy Image



(c)Segmented Image

Fig.2

5 Conclusion

In this paper we have proposed supervised image segmentation scheme for brain MR images. The problem is formulated as a pixel labelling problem. The class labels are modeled as MRF and the observed image as GHMRF model. The model parameters were selected on an ad hoc basis in supervised framework. The MAP estimates are obtained using the proposed hybrid Tabu algorithm. The scheme could yeild successful results for real brain MR images. In this scheme, the *a priori* model parameter δ is selected on an ad hoc basis. Hence, the proposed algorithm which has a globally convergent attribute may be preferred to the algorithm that requires a proper choice of initial parameters. Currently attempts are made to estimate the model parameter δ together with other parameters.

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