Image segmentation Using Parallel Tabu Search Algorithm and MRF Model

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Abstract

In the paper, we propose Tabu Search (TS) based schemes for image segmentation using Markov Random Field (MRF) model. The segmentation problem is formulated as pixel labeling problem and the MAP estimates of the labels were obtained by the two proposed TS algorithms. The TS algorithm was parallelized to improve the overall performance of the scheme. The performance of the algorithms was compared with Simulated Annealing (SA) algorithm and the algorithms outperformed the SA algorithm. The algorithms were tested for synthetic as well as real images.

1. Introduction

The image segmentation problem is an early computational vision problem which has been addressed for more than three decades [1,2,3]. Different strategies evolved with time to tackle various issues in segmentation. The early work on segmentation is broadly based on thresholding and region growing [1]. Broadly, the approach adopted can be viewed as model based approach and over the last two decades stochastic modeling has been extensively used for image segmentation[2,3,4,5,6]. In many of the recent works the problem of image segmentation is cast as a pixel labeling problem. The label estimates are obtained by adhering to the Maximum a Posterior (MAP) estimation principle.

Specifically, MRF model and its variants are often used to model the images [2, 3, 4, 6]. In [5], a tree MRF model based segmentation scheme is proposed. The authors in [5] employed the MAP estimation principles to obtain the label estimates. Spatio-Temporal MRF model has been used for tracking traffic scenes [4]. The MRF model parameters are either estimated *a priori* thus leading to supervised segmentation scheme, or estimated together with the labels leading to unsupervised schemes. The unsupervised schemes using the notion of evolutionary computation has been proposed to segment textured images [7]. Supervised image segmentation problem using evolutionary computation has been addressed in [8]. Image segmentation based on homotopy continuation method and MRF model has been achieved in [9].

In this paper, we propose two Tabu algorithms for image segmentation in a stochastic frame work. We have formulated the problem as a pixel labeling problem and the label estimates are obtained using the MAP estimation criterion. We have modeled the label process as the MRF model. The MRF model parameters are assumed to be known a priori i.e. they are selected on an ad hoc basis. The MAP estimates of the labels are obtained by our proposed TS algorithm. Furthermore, we have parallelized the algorithm to improve the performance. Performance of the proposed algorithms is compared with the SA algorithm for synthetic as well as real images. In our experiments, it is observed that, the proposed algorithms outperformed the SA algorithm.

2. Image Model

The images are assumed to be defined on a discrete rectangular lattice M=(NxN). Let X denotes the random field associated to the noise free image and Z denotes the corresponding label process. Let z be a realization of Z and the label process Z is modeled as MRF. The observed image y is assumed to be a realization of the random field Y. The label process Z is assumed to be a MRF with respect to a neighborhood system η and is described by its local characteristics.

$$P(Z_{ij} = z_{ij} \mid Z_{kl} = z_{kl}, kl \in (N \times N), (k,l) \neq (i,j))$$
$$= P(Z_{ij} = z_{ij} \mid Z_{kl} = z_{kl}, k,l \in \eta)$$

Since, Z is MRF, or equivalently distributed, the joint distribution can expressed as

$$P(Z = z \mid \phi) = \frac{1}{Z'} e^{-U(z,\phi)} \text{ , where}$$

$Z' = \sum_{z} e^{-U(x,\phi)}$ is the partition function, ϕ denote the clique parameter vector, $U(x,\phi)$ is the energy function and is of the form $U(z,\phi) = \sum_{c:(i,j) \in c} V_c(z,\phi)$, $V_c(z,\phi)$ is the clique potential. We consider the following simple image model.

$$Y_{ij} = Z_{ij} + W_{ij}, \forall (i,j) \in (N \times N) \qquad (1)$$

With a lexicographical ordering (1) will be
$$Y = Z + N$$

we assume the following : (a) W_{ij} is a white Gaussian sequence with zero mean and variance σ^2, (b) W_{ij} is statistically independent of $Z_{kl}, \forall (i,j) \, and \, (k,l)$ belonging to (NxN), (c) Z_{ij} takes any value from the label set M=(1,……,M$_m$) (typically M$_m$ =256). In general, the parameter vector $\theta = [q^T, \sigma^2]^T$.

3. Image Segmentation

The image segmentation problem is formulated as a pixel labeling problem. The label process Z, of the image is modeled as MRF. We consider the degradation model given by (1). The number of regions M and the model parameter vector $\theta = [q^T, \sigma^2]^T$ are assumed to be known. The objective is to obtain the optimal estimate of the realization of the scene labels z* and hence segmentation. This is formulated based on the MAP criterion. We consider the following optimality criterion.

$$\hat{z} = \arg \max_z P(Z = z \mid Y = y, \theta) \qquad (2)$$

Where θ denote the parameter vector, which is assumed to be known a priori, $\hat{z}$ is the MAP estimate of the labels. Since z is unknown in (2), the posterior probability of (2) is unknown. Using this Bayesian approach it can be shown that

$$P(Z = z \mid Y = y, \theta)$$
$$= \frac{P(Y = y \mid Z = z, \theta) P(Z = z \mid \theta)}{P(Y = y \mid \theta)} \qquad (3)$$

Since y is known the denominator is a constant. Using the second assumption i.e. the noise is independent of z and the degradation model, $P(Y = y \mid Z = z, \theta)$ can be expressed as

$$P(Y = y \mid Z = z, \theta) = \frac{1}{(2\pi\sigma^2)^{\frac{N^2}{2}}} e^{\frac{-\|y-z\|^2}{2\sigma^2}}. \text{ Since, } Z$$

is MRF, $P(Z = z \mid \theta) = \frac{1}{z'} e^{-U(z,\phi)}$. With the known parameter vector θ, the problem reduces to

$$\hat{z} = \arg \max_z e^{-\left[\frac{\|y-z\|^2}{2\sigma^2} + U(z,\phi)\right]} \qquad (4)$$

It can be easily seen from (4) that the problem reduces to the following minimization problem.

$$\hat{z} = \min_z \left[\frac{\|y-z\|^2}{2\sigma^2} + U(z,\phi) \right] \qquad (5)$$

This minimization is achieved by the proposed Tabu algorithms and SA.

4. Tabu Search

The conventional optimization algorithm suffers from the problem of local minima trapping. This problem is circumvented by the stochastic optimization algorithms such as Simulated Annealing (SA) and Genetic Algorithm (GA). The stochastic optimization algorithms are computationally intensive. One of the reasons could be attributed to the revisiting of the candidate solutions already visited in the search space. Tabu Search (TS) is an adaptive procedure which incorporates the notion of guided search and avoids revisiting the points in the search space to reach the global optimum [10,11,12]. The application domain encompasses scheduling flow sequencing, traveling sales man problem and integrated

circuit design [10,11,12]. Recently, evolutionary TS has been proposed for cell image segmentation [13]. The proposed algorithm in [13] has been shown to outperform GA. The notion of TS has also been applied to determine the optimal coefficients of the digital filter [14].

In our TS algorithm, the Tabu array consists of recent moves. These moves not only consists of moves that minimizes the energy but also consists of moves with higher energy. Thus moves corresponding to the higher energy has been accepted with probability. This represents our aspiration condition. The basic steps of the algorithm is as follows.

4.1 Tabu Search Algorithm

1. Initialize the initial temperature T_{in}.
2. The initial image for the algorithm is the degraded image.
3. A Tabu list, i.e. Tabu image set is created to store the recent moves, i.e. the image estimates of the algorithm. The set is of fixed length.
4. From the current move or image the next Tabu image is generated.
    i) Perturb $z_{ij}(t)$ with a zero mean Gaussian Distribution with a suitable variance.
    ii) Evaluate the energy $Up(z_{ij}(t+1))$ & $Up(z_{ij}(t))$. If $\Delta f = (Up(z_{ij}(t+1)) - Up(z_{ij}(t))) < 0$, assign the modified value as the new value. If $\Delta f > 0$, accept the $z_{ij}(t+1)$ with a probability.
    iii) Repeat step (i) and (ii) for all the pixels of the image.
5. Compute the power of the updated image z(t+1) as $P_z(t+1)$ and compare it with the powers of the tabu list named as Tabu energy, if $P_z(t+1) < P_{Tabu}$ then z(t+1) is a Tabu image.
6. Aspiration condition: If $P_z(t+1) > P_{Tabu}$, accept z(t+1) as Tabu image with probability
7. Update the Tabu list.
8. Decrease the Temperature according to the logarithmic cooling schedule.
9. Repeat step $4 - 8$ for a fixed number of iterations.

5. Parallel Tabu Search

In the proposed TS algorithm, the next move is decided based on the energy of all possible Tabu moves. This increases a substantial amount of computational burden. The computational burden is reduced by parallelizing the Tabu algorithm. The image is partitioned into a set of sub images, say for a example of size (16x16) and (32x32). The energy can be computed over each sub image simultaneously. The energy for the whole image is the sum of the energies of individual sub-images. The computation of energy of each sub image can be achieved by submitting each job to individual processor. The total energy can be computed in parallel machines. Thus, all possible Tabu moves are determined. If the energy of the next image i.e. the next move is lower than that of all the moves of the Tabu array then the next image is accepted as the next move. If the energy of the next image i.e. next move is higher as compared to all the moves of the Tabu array then the next move is accepted with probability.

This is our aspiration condition which helps the algorithm to overcome the problem of local minima trapping. The detailed steps of the algorithm are as follows.

5.1 Parallel Tabu Algorithm

1. Initialize the initial temperature T_{in}.
2. The initial image for the algorithm is the degraded image.
3. A Tabu list, i.e. Tabu image set is created to store the recent moves, i.e. the image estimate of the algorithm. The set is of fixed length.
4. The image is partitioned into Subimages of size (16x16) and (32 x32).
5. For each sub image
   i) Perturb $z_{ij}(t)$ with a zero mean Gaussian Distribution with a suitable variane.
   ii) Evaluate the energy $U_p(z_{ij}(t+1))$ & $U_p(z_{ij}(t))$. If $\Delta f = (U_p(z_{ij}(t+1)) - U_p(z_{ij}(t))) < 0$,assign the modified value as the new value. If $\Delta f > 0$, accept the $z_{ij}(t+1)$ with a probability.
6. Repeat step 5 for all the subimages.
7. Compute the total energy of the image by adding the energies of the subimages.
8. Compute the power of the updated image $z(t+1)$ as $P_{z(t+1)}$ and compare it with the powers of the tabu list named as Tabu energy if $P_{z(t+1)} < P_{Tabu}$ then $z(t+1)$ is a Tabu image.
9. Aspiration condition : If $P_{z(t+1)} > P_{Tabu}$, accept $z(t+1)$ as Tabu image with probability
10. Update the Tabu list.
11. Decrease the Temperature according to the logarithmic cooling schedule.
12. Repeat step 4 – 9 for certain number of iterations.

6. Results and Discussion

We have considered the following first order MRF model with line field as the *a priori* model of the label process.

$$U(z,\phi,h,v)$$
$$= \sum_{ij} \alpha \left\{ \left(z_{ij} - z_{i,j-1}\right)^2 \left(1 - h_{ij}\right) + \left(z_{ij} - z_{i-1,j}\right)^2 \left(1 - v_{ij}\right) \right\}$$
$$+ \beta \left(h_{ij} + v_{ij}\right) \qquad (6)$$

Where z denotes the realization of the label corresponding to the noise free image $h_{ij}\ and\ v_{ij}$ are the horizontal and vertical line fields. h_{ij} =1 when $|z_{ij} - z_{i,j-1}|$ > threshold or h_{ij} =0. Similarly $v_{ij} = 1$ when $|z_{ij} - z_{i-1,j}|$ > threshold or $v_{ij} = 0$. These line fields help in preserving the edges of the images $\alpha\ and\ \beta$ are the image model parameters. The corresponding a *posteriori* model is

$$U_p\left(x,\phi,h,v\right) = \frac{1}{2\sigma^2} \| y - z \|^2 + U\left(x,\phi,h,v\right) \quad (7)$$

We have considered synthetic as well as real images. Synthetic images having three or four classes have been generated using Gibb's sampler. These are shown in Fig.1(a) and Fig.3(a) respectively. The noisy image of SNR 15dB corresponding to Fig.1(a) is shown in Fig.1(b).

The parameters choosen for Fig.1(b) are $\alpha = 0.02, \beta = 5\ and\ \sigma = 4$. The parameters of the Tabu algorithm are: Initial temperature $T_{in} = 0.1$, the length of the Tabu image array L=10, number of iteration is 300. Fig.1(c) shows the segmented image obtained using the Tabu algorithm. It is observed that the algorithm could segment the image satisfactorily except loosing the sharpness of few edges. Segmented images obtained from SA and parallel Tabu algorithms were very close to that of the Tabu algorithm. For the shake of illustration the results of Fig.2 corresponds to parallel Tabu algorithm. Fig2 shows decay of energy of the algorithm. It is seen from Fig.2 that the energy falls to a value of 55 around 15 iterations for Tabu algorithm while the SA takes approximately 30 iterations. The rate of fall in case of Tabu algorithm is faster than that of the SA. Thus, Tabu algorithm converges faster than that of SA. The decay of energy in case of parallel Tabu is almost identical to that of Tabu algorithm. The reasons could be attributed to the following. (i) Inter processor communication is not taken into account, (ii) The total energy computed by partitioning is very close to the energy computed without partitioning, (iii) The results correspond to serial implementation.

Fig.3(a) shows the original image of three class and the corresponding noisy image is shown in Fig.3(b). All the three algorithms could be successfully tested and hence for the sake of illustration, the result yielded by Parallel Tabu algorithm is reported in Fig.3(c). The model parameters used are $\alpha = 0.02, \beta = 5.5\ and\ \sigma = 8$. The other parameters were same as that of four class image. It is evident from Fig.3(c) that proper segmentation could be achieved. The behavior of the energy in this case is identical to that of the four class image.

We have also validated our algorithms for real images as shown in Fig.4(a) and Fig.6(a). The corresponding noisy images are shown in Fig.4(b) and Fig.6(b). The segmented images are shown in Fig.4(c) and Fig.6(c). It is seen from Fig.4(c) that satisfactory segmentation could be achieved except some distortions in the edges. The model parameters used are $\alpha = 0.019, \beta = 2\ and\ \sigma = 3$. The decay of the energy for three algorithms corresponding to Fig.4 are shown in Fig.5. It is again observed from the Fig.5 that the energy of Tabu algorithm decays at a faster rate than that of SA. Energy curve of Tabu and Parallel Tabu are close to each other. It is further evident that the algorithm converges at around 150 iterations where as SA takes approximately 250 iterations to converge. The rate of fall together with the number of iterations required for convergence, corroborates that the proposed algorithms outperform the SA.

We also tested successfully our algorithms on another real image as shown in Fig.6(a). The corresponding noisy image SNR=30dB is shown in Fig.6(b). The model parameters are $\alpha = 0.02, \beta = 2.5\ and\ \sigma = 5$. It is seen from Fig.6(c) that the algorithm could segment all the parts except a part of the edges is miss classified. The

behavior of the energy exhibited in this case is similar to that of the real object of Fig.4(a). Thus, the two proposed algorithms could be validated for real and synthetic images. In the examples, the performance of Tabu and Parallel Tabu algorithms was superior to that of the SA.

7. Conclusion

We have proposed a Tabu algorithm for image segmentation. The performance of the Tabu algorithm is enhanced by proposing a parallel Tabu algorithm. Both the algorithms could be successfully tested for synthetic as well as real images. In all the cases, the two proposed algorithms outperformed the SA. Since we do not have a parallel machine, the parallel algorithm was implemented in a serial machine. The results reported for parallel Tabu corresponds to the serial implementation. Current work focuses on the parameter estimation of the image exploiting the notions of Tabu search and hence design an unsupervised scheme.

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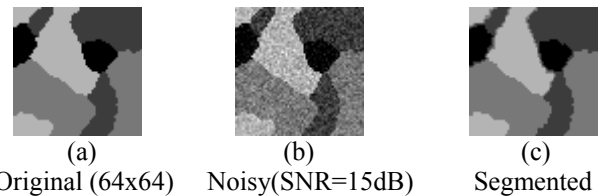
| (a) | (b) | (c) |
| Original (64x64) | Noisy(SNR=15dB) | Segmented |

Fig.1 : Segmentation of synthetic image of four class

Fig.2 : Energy plot for the algorithms corresponding to Fig.1.

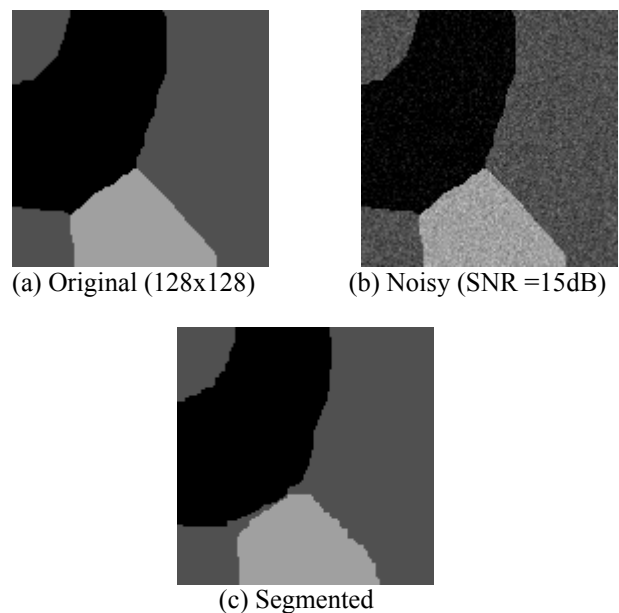| (a) Original (128x128) | (b) Noisy (SNR =15dB) |

(c) Segmented
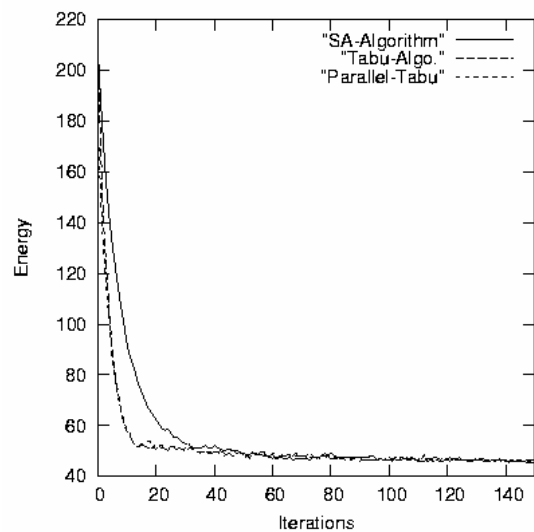
Fig.3 : Segmentation of synthetic image of three class
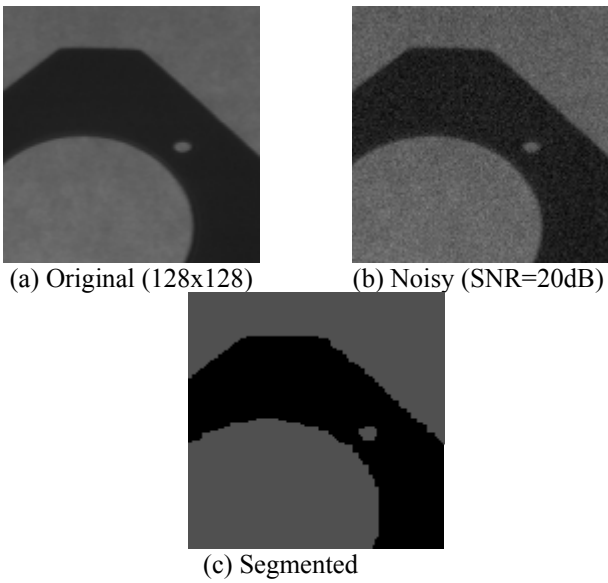
(a) Original (128x128)   (b) Noisy (SNR=20dB)

(c) Segmented

Fig.4 : Segmentation of real image

Fig.5 : Energy plot for the algorithms corresponding to
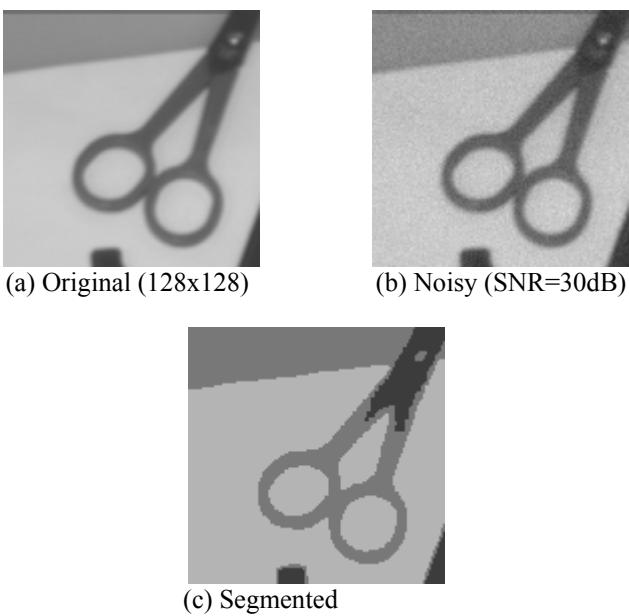   Fig.4

(a) Original (128x128)   (b) Noisy (SNR=30dB)

(c) Segmented

Fig.6 : Segmentation of real image.