Comparative Study on Error in MIMO Radar DOA Estimation

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***Abstract*—Hardware implementation for faster real-time signal processing in multiple-input multiple-output (MIMO) radar beamforming by estimating direction of arrival (DOA) using multiple signal classification (MUSIC) is attractive. The above subspace-based method works with a basic principle of eigenvalue decomposition and is found suitable for application-specific integrated circuit (ASIC), field-programmable gate array (FPGA). In that regard, the cyclic Jacobi method is used for computing eigenvalues and eigenvectors, by orthonormal plane rotations in accomplishing eigenvalue decomposition. However, the fasterness and foremost, the accuracy of the DOA estimator realized in such above hardware depends on the involved number of antenna elements in an array, the number of snapshots, and the signal-to-noise ratio. Therefore, the comparative study on DOA estimation error on the above mentioned parameters in MUSIC and QR-based algorithms is presented in this paper.**

***Index Terms*—MIMO, DOA, MUSIC**

I. Introduction

Multiple input multiple output (MIMO) radar [1], in which antenna elements emit multiple independent waveforms at the same time and multiple antennas receive the echo, has drawn attention of researchers in recent years. MIMO radar beamforming involves direction of arrival (DOA) estimation and that can be accomplished using the real-time array signal processing techniques available for applying to radar, sonar, acoustics, and communication systems [2]. However, the estimation of DOA precisely for obtaining accurate beamformation in an antenna array prefers to use high resolution techniques that are realizable in hardwares, field-programmable gate array (FPGA) and application specific integrated circuit (ASIC) for faster applications of MIMO radar.

The methods of estimating DOA of signals using an antenna array can be categorized as conventional and subspace based methods. Conventional methods such as Bartlett and Capon [3] rely on the physical size of the array aperture, resulting in low resolution and precision. Subspace approaches such as multiple signal classification (MUSIC) [4], and estimation of signal parameters via rotational invariance technique (ESPRIT) [5], provide accurate signal direction and are not limited by the physical size of the array aperture, resulting in high resolution.

Attractively, the MUSIC is found to be promising in hardware realization as reported in [6], [7] for various real-time applications. Jacobi algorithm [8], is one of the methods for eigenvalue decomposition (EVD) is used in the above mentioned hardware.

This Jacobi approach employs the Coordinate Rotation Digital Computer (CORDIC) algorithm [9] in hardware implementation for planar rotations. In [10], the approximate Jacobi approach is used, which requires one iteration of full CORDIC. In [11], implemention is done for a scaling free micro rotation CORDIC (μ- CORDIC) processor that executes a specified number of CORDIC iterations. Hence, the speed of the EVD processor is governed by the CORDIC structure as is given in [12]. It can further be optimised, so that the EVD processor provides superior performance which can be quantified as estimation accuracy, the use of number of FPGA resources, and compution time.

Interestingly, the QR decompostion based DOA estimation similar to the search method of MUSIC is reported in [13] which is attarctive in real-time FPGA implementation. Without compromising with estimation accuracy, a required number of antenna arrays, snapshots and sufficient amount of signal-to-noise (SNR) ratio are often required in effective operation of a real-time system after the respective hardware implementation. The efficiency of these approaches is also dependent on the array geometry and signal reception environments [14].

In this paper, a uniform linear array (ULA) is considered with an assumption that received signals are uncorrelated. The performances of MUSIC with cyclic Jacobi approach for EVD and QR decomposition based approach which are suitable for hardware implementation and hence, are examined. The comparative study on average error in DOA estimation for varying number of antenna elements, snapshots and various SNR values is done and the corresponding results are presented.

The remainder of the paper is arranged as follows. Section II introduces signal model for the ULA, Section III describes the DOA estimation algorithms briefly. Simulation results are discussed in Section IV and finally, in Section V conclusion is drawn.

II. Signal Model

MIMO radar having M narrow band receive antennas which form a ULA as is shown in Fig.1. Spacing between elements is d which equals half of the wavelength (λ/2) of the transmitted

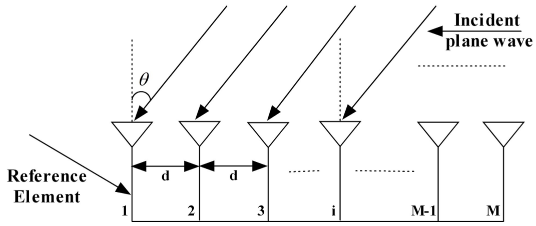

Fig. 1. Uniform linear array

signal. If $x'_1(t)$ is the signal received by the 1^{st} element of the array, the i^{th} array element receives the signal $x'_i(t)$ with a phase delay of

$$\Delta_i = (i-1)\varphi_k(\theta), \text{with } \varphi_k(\theta) = \frac{2\pi d sin(\theta_k)}{\lambda}, \quad \forall i = 1, 2, 3........M \tag{1}$$

Here $\varphi_k(\theta)$ is the spatial frequency of the incidence signal with an arrival angle θ of the k^{th} signal source and the t^{th} snapshot as received signal at i^{th} array element is

$$x'_i(t) = e^{j\Delta_i} s_k(t) \tag{2}$$

$$s_k(t) = e^{-j\omega_k t}, \ \forall\ k = 1, 2, \cdots, L \text{ and } t = 1, 2, \cdots, N \tag{3}$$

where ω_k is the angular frequency of the source signal $s_k(t)$. The signal which comprises L number of frequency components as given in (3) is received at the i^{th} array element and is given by

$$x_i(t) = \sum_{k=1}^{L} s_k(t) e^{j(i-1)\varphi_k(\theta)} + w_i(t), \tag{4}$$

where $w_i(t)$ is the white Gaussian noise with zero mean and σ^2 variance. Defining Steering vector matrix as A with steering vectors $a(\theta_k)$ as its columns can be given as

$$A = [a(\theta_1), a(\theta_2), a(\theta_3), ...a(\theta_L)] \tag{5}$$

$$a(\theta_k) = [1, e^{j\varphi_k(\theta)}, e^{2j\varphi_k(\theta)}...............e^{(M-1)j\varphi_k(\theta)}]^T \tag{6}$$

where T represents transpose operation. Therefore, the received signal can be represented for t^{th} snapshot as

$$X(t) = AS(t) + W(t) \tag{7}$$

where

$$X(t) = [x_1(t), x_2(t), x_3(t)....................x_M(t)]^T \tag{8}$$

$$S(t) = [s_1(t), s_2(t), s_3(t).......................s_L(t)]^T \tag{9}$$

$$W(t) = [w_1(t), w_2(t), w_3(t)....................w_M(t)]^T \tag{10}$$

The signal covariance matrix of $X(t)$ can be given by the expression

$$\begin{aligned} R_x &= E[X(t)X^H(t)] \\ &= \sum_{k=1}^{L} E[|s_k(t)|^2] a(\theta_k) a^H(\theta_k) + \sigma^2 I \end{aligned} \tag{11}$$

where H, $E[.]$ represent Hermitian and expectation operations respectively. I is an identity matrix of size $M \times M$ and $|.|$ represents absolute value of signal.

III. Description of Algorithms

In this section, we briefly describe the DOA estimation methods, QR decompositon, the subspace-based method MUSIC, and also MUSIC with cyclic Jacobi method which are suitable for DOA estimator realization in FPGA. All these methods use signal covariance matrix given in section II.

A. DOA estimation by QR decompositon

The QR decomposition [15], [16], decomposes a matrix into orthogonal matrix Q and upper triangular matrix R. With the matrix Q and searching technique [17], the QR decomposition of R_x may be represented as

$$R_x = QR = \begin{bmatrix} Q_s & Q_n \end{bmatrix} \begin{bmatrix} R_L \\ O \end{bmatrix} \tag{12}$$

The matrices Q, and R can be used to extract information about incident source DOAs. The column vectors of signal and noise space are given by $Q_s = [q(1), q(2), q(3).....q(L)]$ and $Q_n = [q(L+1), q(L+2), q(L+3), ...q(M)]$ respectively. R_L and O represent signal space matrix of size $(L \times M)$ and null matrix of size $(M-L) \times M$. Multiplying (12) with Q^H gives

$$\begin{bmatrix} Q_s^H R_x \\ Q_n^H R_x \end{bmatrix} = \begin{bmatrix} R_L \\ O \end{bmatrix} \tag{13}$$

From (13) we can write

$$Q_n^H A S A^H = O \tag{14}$$

which indicates that $M - L$ column vectors of Q_n are perpendicular to column vectors of A and span the same space, i.e, the noise space is orthogonal to the steering vectors corresponding to the DOAs. The DOAs can be detected by looking for the lowest peaks of $||Q_n^H A(\theta)||$ or by locating the peaks of spatial spectrum given by

$$P(\theta) = 1/A^H(\theta) Q_n^H Q_n A(\theta) \tag{15}$$

B. DOA estimation using MUSIC Algorithm

The MUSIC algorithm provides a spectrum for a spatial angle predicted on the premise that steering vector of the received signal is orthogonal to the noise subspace. The signal's DOA is then determined by locating the peak value in the estimated spectrum. The EVD of covariance matrix, R_x given in (11) can be given as

$$R_x = J_s \wedge_s {J_s}^H + \sigma^2 J_n {J_n}^H \tag{16}$$

where J_s and J_n are signal subspace and noise subspace eigenvectors and $\wedge_s$ is the diagonal matrix of eigenvalues $(\mu_1 \geq \mu_2 ...... \geq \mu_L)$. From [4], the spatial spectrum search function is given by

$$P(\theta) = \frac{1}{\sum_{i=L+1}^{M} |J_i^H a(\theta)|^2} \tag{17}$$

C. MUSIC with cyclic Jacobi method

The cyclic Jacobi method computes eigenvalues and eigenvectors by multiplying both sides of R_x of $M \times M$ dimension with the orthonormal matrices and applying a sequence of orthonormal rotations. The orthogonal matrix $E(p, q, \theta)$ can be given by

$$\begin{array}{cc} \begin{bmatrix} 1 & \dots & 0 & \dots & 0 & \dots & 0 \\ \vdots & & \vdots & & \vdots & & \vdots \\ 0 & \dots & \cos(\theta) & \dots & \sin(\theta) & \dots & 0 \\ \vdots & & \vdots & & \vdots & & \vdots \\ 0 & \dots & -\sin(\theta) & \dots & \cos(\theta) & \dots & 0 \\ \vdots & & \vdots & & \vdots & & \vdots \\ 0 & \dots & 0 & \dots & 0 & \dots & 1 \end{bmatrix} & \begin{array}{c} \\ \\ p \\ \\ q \\ \\ \\ \end{array} \\ \begin{array}{ccccccc} \quad & \quad & p & \qquad\qquad & q & \quad & \quad \end{array} & \end{array} \tag{18}$$

The planar rotation iterative procedure reduces the covariance matrix's (p,q) and (q,p) entries to zero. The off diagonal element of the covariance matrix, r_{pq} is selected by the cyclic Jacobi method sequentially in either row-by-row or column-by-column fashions. For example, if $M = 4$, cycling for cyclic-by-row as follows

$$(p, q) = (1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4)..... \tag{19}$$

From [18], we can obtain the optimal angle for orthogonal rotations as

$$\theta_{opt} = \frac{1}{2} \tan^{-1}\left(\frac{2r_{pq}}{r_{pp} - r_{qq}}\right) \tag{20}$$

With the above optimum angle, the sequence of planar rotations can be given by

$$R_x^1 = E_1^T R_x E_1 \tag{21}$$

$$\vdots$$

$$R_x^z = {E'}^T R_x^{z-1} E' \tag{22}$$

where ${E'}^T = {E_1}^T {E_2}^T \cdots {E_z}^T$ and $E' = E_1 E_2 \cdots {E_z}^T$

Here z denotes the number of iterations, and after z iterations, R_x^z becomes almost diagonal, and these diagonal values are the approximations to the eigenvalues, and corresponding eigenvectors are the columns of E'. The complex covariance matrix and complex search vector are transformed respectively into a real matrix and real search vector using the unitary transformation technique [19]. The unitary matrix is

$$U = \frac{1}{\sqrt{2}} \begin{bmatrix} I & J \\ jII & -jII \end{bmatrix} \cdots \text{when } M \text{ is even} \tag{23}$$

where II is the matrix with all unities on anti-diagonal elements and zeros elsewhere, and both matrices are of size $M/2 \times M/2$.

$$U = \frac{1}{\sqrt{2}} \begin{bmatrix} I & 0 & II \\ 0^T & \sqrt{2} & 0^T \\ jII & 0 & -jI \end{bmatrix} \cdots \text{when } M \text{ is odd} \tag{24}$$

Here I, II are of size $(M-1)/2 \times (M-1)/2$ and 0 represents a $M/2 \times 1$ zero vector. With the unitary matrices (23), (24), by performing UR_xU^H, and considering the real part of the above matrix, the eigenvalues and eigenvectors are computed.

Let the computed eigenvalues of the real part of the UR_xU^H matrix are $\mu_1 \geqslant \mu_2 \geqslant \mu_3 \cdots\cdots \geqslant \mu_M$ and the corresponding eigenvectors are $\{e_1, e_2, e_3..........e_M\}$. To estimate θ_k, k =1,2....L, search function is

$$P(\theta) = \frac{1}{\sum_{i=L+1}^{M} \left|e_i^T a'(\theta)\right|^2} \tag{25}$$

where

$$a'(\theta) = U e^{\frac{-j(M-1)\varphi(\theta)}{2}} a(\theta) \tag{26}$$

IV. Results and Discussion

The simulations were done in MATLAB by considering ULA, and three signal sources at $-70°$,$-30°$ and $50°$.

This simulation looks at three possibilities, each with one parameter changing while the others remain constant. The detailed estimates of three angles for various values of M, N and SNR are given in Table I, II and III respectively.

Fig.2 shows that the MUSIC with cyclic Jacobi method narrows the angular spectrum beam width quite effectively. The performance of the above methods is examined in terms of average error of DOA of the signal as given by

$$\text{Average error} = \frac{\mathit{1}}{L} \sum_{i=\mathit{1}}^{L} (\theta_{actual_i} - \theta_{est_i}) \tag{27}$$

where θ_{actual} and θ_{est} are respectively, the true DOA and estimate of that.

The average error expressed in (27) is computed for the results illustrated in above mentioned Tables I, II and III and for summarizing, the figures are given in Fig.4, Fig.6 and Fig.8.

Case 1: Varying number of array elements, M.

In Fig.3., the peaks at lower number of array elements are not quite precise. As the array grows in size, the peaks become more precise and closely correspond to the angles of arrival. As a result, the angular spectral beam width narrows and the

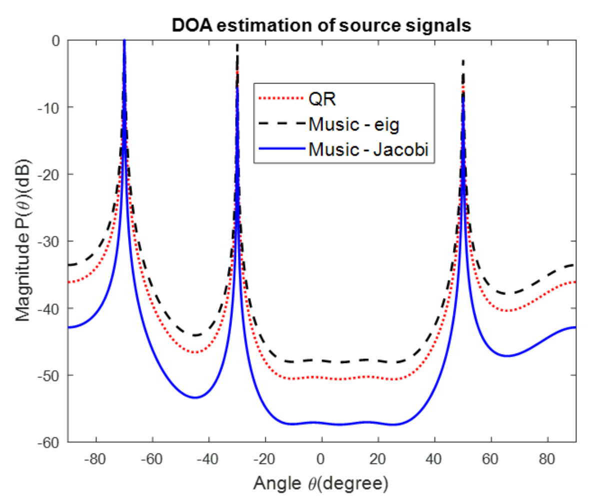

Fig. 2. Angular spectrum

TABLE I
VARIATION WITH NUMBER OF ANTENNA ELEMENTS M

M	DOA (deg) = -70			DOA (deg) = -30			DOA (deg) = 50			Average error (deg)		
	QR	MUSIC-eig	MUSIC-Jacobi	QR	MUSIC-eig	MUSIC-Jacobi	QR	MUSIC-eig	MUSIC-Jacobi	QR	MUSIC-eig	MUSIC-Jacobi
6	-70.0317	-70.0396	-70.0363	-30.0357	-30.0394	-30.0385	49.9628	49.9727	49.9740	0.0349	0.0354	0.0336
10	-69.9787	-69.9818	-69.9811	-29.9962	-29.9978	-29.9977	49.9941	49.9958	49.9954	0.0103	0.0086	0.0082
20	-69.9969	-69.9979	-69.9978	-30.0039	-30.0040	-30.0040	49.9993	49.9994	49.9993	0.0026	0.0023	0.0022
50	-70.0013	-70.0014	-70.0014	-29.9999	-29.9999	-29.9998	49.9995	49.9996	49.9996	0.0006	0.0006	0.0006
70	-69.9994	-69.9996	-69.9996	-29.9996	-29.9996	-29.9996	50.0008	50.0009	50.0008	0.0006	0.0005	0.0005
100	-69.9943	-69.9992	-69.9992	-30.0004	-30.0004	-30.0004	50.0005	50.0005	50.0005	0.0006	0.0005	0.0005

TABLE II
VARIATION WITH NUMBER OF SNAPSHOTS N

N	DOA (deg) = -70			DOA (deg) = -30			DOA (deg) = 50			Average error (deg)		
	QR	MUSIC-eig	MUSIC-Jacobi	QR	MUSIC-eig	MUSIC-Jacobi	QR	MUSIC-eig	MUSIC-Jacobi	QR	MUSIC-eig	MUSIC-Jacobi
50	-70.0317	-70.0396	-70.0363	-30.0357	-30.0394	-30.0385	49.9628	49.9727	49.9740	0.0349	0.0351	0.0336
100	-70.0554	-70.0531	-70.0486	-30.0278	-30.0312	-30.0320	49.9815	49.9881	49.9853	0.0339	0.0323	0.0318
200	-69.9769	-69.9769	-69.9779	-29.9904	-29.9948	-29.9948	49.9598	49.9714	49.9717	0.0243	0.0190	0.0185
500	-70.0145	-70.0160	-70.0159	-29.9936	-29.9969	-29.9969	49.9890	49.9976	49.9829	0.0106	0.0072	0.0072
1000	-70.0226	-69.9992	-69.9992	-29.9914	-29.9947	-29.9947	49.9829	50.0191	50.0190	0.0101	0.0071	0.0070

TABLE III
VARIATION WITH SNR

SNR (dB)	DOA (deg) = -70			DOA (deg) = -30			DOA (deg) = 50			Average error (deg)		
	QR	MUSIC-eig	MUSIC-Jacobi	QR	MUSIC-eig	MUSIC-Jacobi	QR	MUSIC-eig	MUSIC-Jacobi	QR	MUSIC-eig	MUSIC-Jacobi
-10	-72.5018	-69.1500	-69.0995	-29.6912	-31.4331	-31.4331	47.3304	48.2750	48.4511	1.8267	1.3660	1.2942
-5	-70.4795	-70.0041	-70.0520	-29.6919	-30.7144	-30.7144	48.2894	49.2439	49.3291	0.8327	0.4915	0.4791
5	-69.9433	-70.1633	-70.1583	-30.0985	-30.2172	-30.2172	49.5651	49.8225	49.8367	0.1967	0.1860	0.1796
10	-70.0311	-70.1099	-70.1031	-30.0865	-30.1218	-30.1218	49.8154	49.9074	49.9133	0.1007	0.1081	0.1039
20	-70.0317	-70.0396	-70.0363	-30.0357	-30.0385	-30.0385	49.9628	49.9727	49.9740	0.0349	0.0351	0.0336

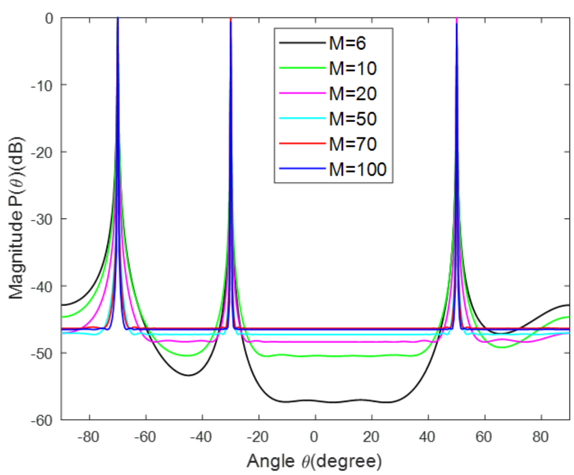

Fig. 3. Variation with number of elements M = 6, 10, 20, 50, 70, 100

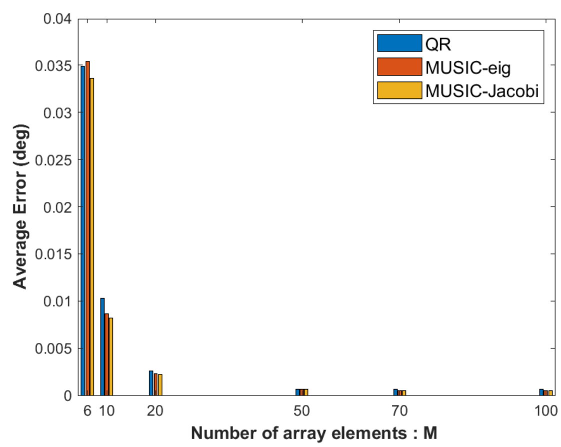

Fig. 4. Average error versus number of array elements M

resolution improves, making DOA extraction in MUSIC easier. Fig.4. shows the variation of average error with the increase of number of array elements. The average error for MUSIC is substantially decreased with the increase of the number of array elements.

Case 2: Varying number of snapshots, N.

When the number of snapshots increased, the resolution improved, resulting in prominent peaks and the array element's direction improved as illustrated in Fig.5. and also the average error decreases with increase in the number of snapshots which can be seen in Fig.6.

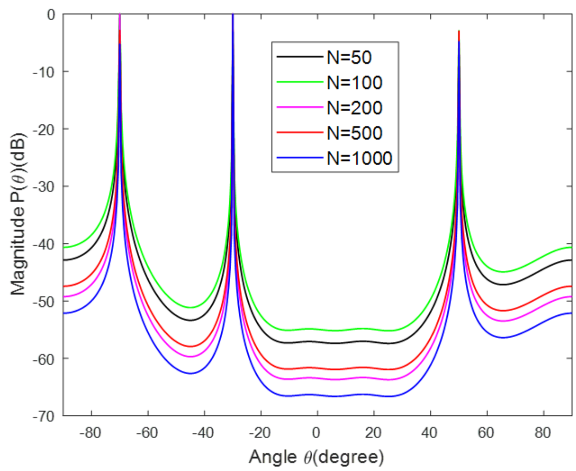

Fig. 5. Variation with snapshots N = 50, 100, 200, 500, 1000

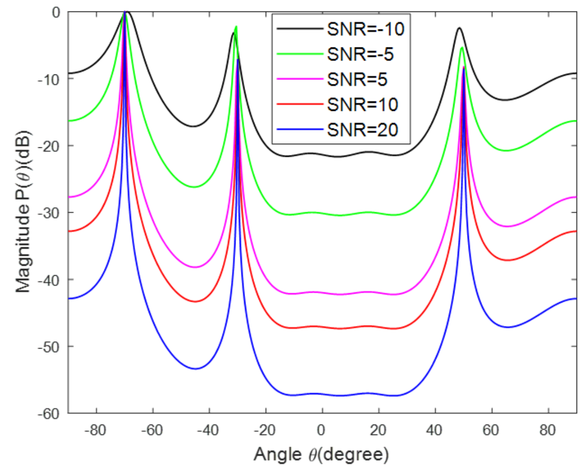

Fig. 7. Variation with SNR (dB) = -10, -5, 5, 10, 20

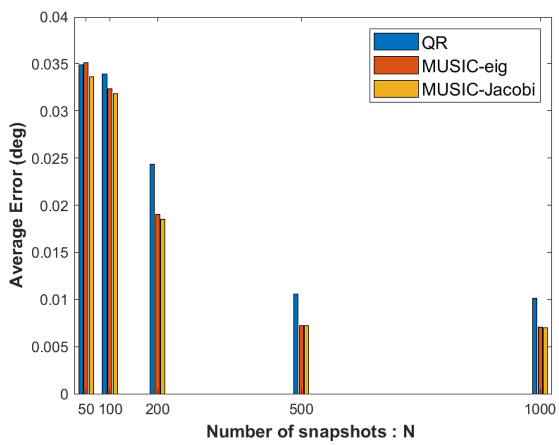

Fig. 6. Average error versus number of snapshots N

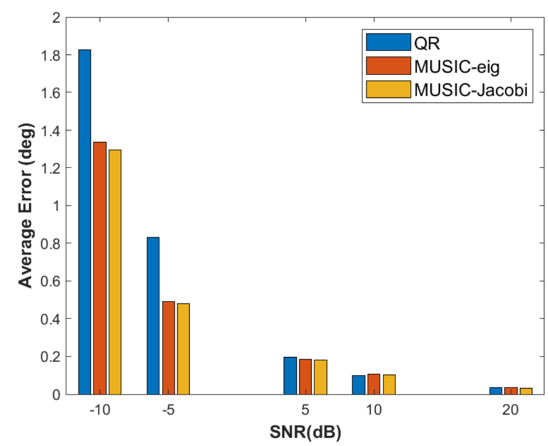

Fig. 8. Average error versus SNR

Case 3: Varying SNR For a low SNR values, the spikes indicating the arrival of a signal from a certain direction are tiny and the response is flat which can be shown in Fig.7. As a result, determining the angles of arrival is challenging. However, when the SNR value is higher, the resolution improves noticeably, and the spikes become more prominent. This is due to the fact that when the SNR decreases, the difference between the eigenvalues associated with the signal and those associated with the noise decreases, causing the peaks to decrease with respect to the noise levels. The difference between the two sets of eigenvalues grows as the SNR increases, and the peaks get higher in comparison to the noise levels and the average error decreases as is depticted in Fig.8.

V. Conclusion

In this work, subspace and QR decomposition based DOA estimation accuracy have been examined. Moreover, MUSIC with cyclic Jacobi method of eigenvalue decomposition is compared with MUSIC which uses in-build eig function of MATLAB. The effectiveness of the DOA estimation algorithms is evaluated by means of computing average error for various values of M, N and SNR. The simulation results indicated that the average error for MUSIC with in-build eig function is better than QR decompositon method, furthermore, the MUSIC algorithm realization with cyclic Jacobi method gives better satisfactory performace than the earlier two and hence, FPGA implemenation of MUSIC is promising in variety of applications, including accoustics, sonar, and communication systems.

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