# Investigating Cylindrical Dielectric Resonator Antenna using Imperfect Wall

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***Abstract-* Theoretical investigation on Cylindrical Dielectric Resonator Antenna (CDRA) using imperfect wall boundary condition is presented here for $TM^z$ modes for the first time, hitherto unreported. Approximate expression for eigenfunctions and eigenvalues are given here for different modes. Simple formula is given to predict the resonant frequency. Numerical and graphical both solutions are presented here. Theoretical results on resonant frequency are compared with measured data and data obtained using 3D EM simulator HFSS to show the accuracy of our theory.**

**Keywords—Dielectric Resonator Antenna (DRA), resonant frequency**

## I. INTRODUCTION

In the modern wireless communication system, antenna plays an important role. Out of several antennas, Dielectric Resonator Antenna (DRA) is a favorite due to its light weight, high radiation efficiency, low surface wave loss, high gain, etc. Extensive theoretical and experimental investigations have been reported on hemispherical DRA [1-2]. Closed form green's function has been obtained using the mode matching technique for hemispherical DRAs. On the other hand, time consuming surface (and/or volume) integral technique has been used to investigate the Cylindrical DRA (CDRA) [3]. Limited analytical solution is available on CDRA.

S. A. Long *et al* have modeled the CDRA as a dielectric cavity, bounded by Perfect Magnetic Conductor (PMC) in 1983 [4]. Closed form approximate analytical solutions for eigenfunction and eigenvalue have been given therein for TE and TM modes. Due to use of PMC as the boundary condition, the analysis of HEM mode was not possible. Some curve fitted empirical formulae to predict the un-normalized wave number of the CDRA for lower order $HEM_{11\delta}, TE_{01\delta}, TM_{01\delta}$ and $TE_{011+\delta}$ modes are reported in [2, 5]. Those empirical formulae are valid for a certain value (or range of) relative permittivity ($\varepsilon_r$) and/or aspect ratio ($a/d$ where $a$ and $d$ are the radius and height of the CDRA respectively). Further, those formulae are mainly valid for $\varepsilon_r \geq 20$. For low value of $\varepsilon_r$, for example, $\varepsilon_r = 10$ which is widely preferred by modern antenna researchers, those empirical formulae give a large error. Therefore, we need a closed form analytical solution to predict the resonant frequency for different modes of the CDRA.

In this work, theoretical investigation on Cylindrical Dielectric Resonator Antenna (CDRA) is presented for the first time, hitherto unreported using imperfect wall. For theoretical investigations, the CDRA is modeled as a dielectric cavity and all surfaces of the CDRA are modeled as Imperfect Magnetic Conductors (IPMC). It is also assumed that the maximum energy is stored in the CDRA and the energy is decaying exponentially outside the CDRA. For theoretical simplification, only $TM^z$ mode is investigated. After matching tangential electric and magnetic fields at the surfaces of the CDRA, two transcendental equations are obtained which are solved simultaneously to predict the resonant frequency for different modes. Theoretical resonant frequency are compared with measured data as found in the literature and data obtained using 3D numerical EM simulator HFSS. It is found that our theoretical data are in excellent agreement with measured and simulated data.

## II. ANTENNA CONFIGURATION

Antenna geometry of the Cylindrical DRA (CDRA) having radius $a$, height $d$ and relative permittivity $\varepsilon_r$ is shown in Fig. 1 and the CDRA is placed on a metallic plate at $z = 0$ surface. It is assumed that the dielectric is isotropic and homogeneous.

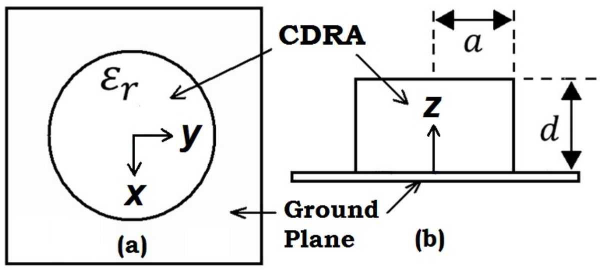


Fig. 1Antenna configuration (a) Top view (b) Side view

## III. THEORY

In this section, theoretical investigation on CDRA is presented systematically. To remove the effect of metallic ground plane, image theory is applied first. This process gives an isolated CDRA having height $h = 2d$ as shown in Fig. 2. For theoretical investigations, it is assumed that the maximum fields are confined inside the CDRA and the fields are decaying exponentially outside the CDRA.

To investigate the CDRA, all regions are divided into four regions as shown in Fig. 3. This approximate theory does not include the corner shaded regions as shown in Fig. 3. Regions II, III and IV have air media whereas the region I has dielectric media. To simplify the analysis, only $TM$ mode is investigated here. Therefore, we can easily assume $m = 0$ case, because $m = 0$ separate the problem by $TE$ and $TM$ modes, otherwise $HEM$ modes will exists [6,7].

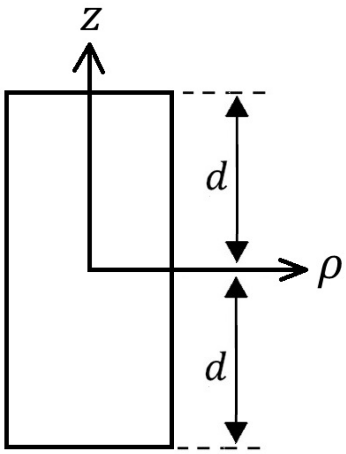


Fig. 2 Side view of the isolated Cylindrical DRA having height $h = 2d$

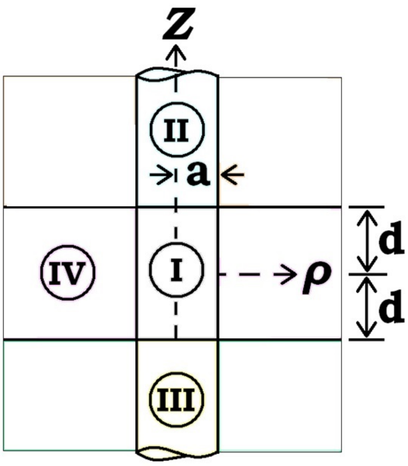


Fig. 3 Equivalent problem, showing different regions

The fields in different regions for $TM^z$ mode can be expressed as:

$$E_\rho = \frac{1}{j\omega\mu\varepsilon}\frac{\partial^2\psi_m}{\partial\rho\partial z} \qquad H_\rho = \frac{1}{\mu\rho}\frac{\partial\psi_m}{\partial\varphi}$$
$$E_\varphi = \frac{1}{j\omega\mu\varepsilon}\frac{1}{\rho}\frac{\partial^2\psi_m}{\partial\varphi\partial z} \qquad H_\varphi = -\frac{1}{\mu}\frac{\partial\psi_m}{\partial\rho} \qquad (1)$$

$$E_z = \frac{1}{j\omega\mu\varepsilon}\left(\frac{\partial^2}{\partial z^2} + K^2\right)\psi_m \qquad H_z = 0$$

**Solutions in Reg. I:**

$$\psi_{1m} = AJ_0(k_{\rho1}\rho)cos(0\varphi) \times [B_o cos(k_z z) + B_1 sin(k_z z)] \qquad (2.a)$$

**Solutions in Reg. II:**

$$\psi_{2m} = J_0(k_{\rho1}\rho)cos(0\varphi) \times B_2 \times e^{-\gamma_z(z-d)} \qquad (2.b)$$

**Solutions in Reg. III:**

$$\psi_{3m} = J_0(k_{\rho1}\rho)cos(0\varphi) \times B_3 \times e^{\gamma_z(z+d)} \qquad (2.c)$$

**Solutions in Reg. IV:**

$$\psi_{4m} = DK_0(k_{\rho4}\rho)cos(0\varphi) \times [B_o cos(k_z z) + B_1 sin(k_z z)] \qquad (2.d)$$

Here, $A, B_0, B_1, B_2, B_3$ and $D$ are amplitude and other notations are carrying their usual meaning.

*A. Matching the fields at $\rho = a$ surface*

Equating $E_{z1} = E_{z4}$ at $\rho = a$ $(where\ |z| \leq d)$, we obtain:

$$\Rightarrow P_1 k_{\rho1}^2 AJ_m(k_{\rho1}a) = P_4(-k_{\rho4}^2)DK_m(k_{\rho4}a) \qquad (3.a)$$

where

$$P_1 = -jQ_1 \quad ; \quad Q_1 = 1/\omega\mu_1\varepsilon_1$$

$$P_4 = -jQ_4 \quad ; \quad Q_4 = 1/\omega\mu_4\varepsilon_4$$

In a similar way, equating $H_{\varphi1} = H_{\varphi4}$ at $\rho = a$, we obtain:

$$\Rightarrow \frac{1}{\mu_1}Ak_{\rho1}J'_m(k_{\rho1}a) = \frac{1}{\mu_4}Dk_{\rho4}K'_m(k_{\rho4}a) \qquad (3.b)$$

Eliminating $A$ and $D$, we get:

$$\left[\frac{1}{\mu_1}\frac{J'_m(x)}{x} + \frac{1}{\mu_4}\frac{P_1}{P_4}\frac{K'_m(y)}{y}\frac{J_m(x)}{K_m(y)}\right] = 0 \qquad (4)$$

where $k_{\rho1}a = x$ and $k_{\rho4}a = y$.

*B. Matching the fields at $z = \pm d$ surface*

To evaluate $B_o, B_1, B_2$ and $B_3$, we have to match the tangential electric and magnetic fields inside and outside of the CDRA. Equating the fields at $z = +d$, we obtain:

$$\Rightarrow B_2 = -R_2 \frac{k_z}{\gamma_z}[-B_o sin(k_z d) + B_1 cos(k_z d)] \qquad (5.a)$$

$$\Rightarrow B_o S_3 \; + \; B_1 S_4 = 0 \qquad (5.b)$$

where

$$R_2 = \frac{P_1}{P_2} \qquad (5.c)$$

$$S_3 = \left[\frac{1}{\mu_1} cos(k_z d) - \frac{1}{\mu_2}\frac{k_z}{\gamma_z} R_2 sin(k_z d)\right] \qquad (5.d)$$

$$S_4 = \left[\frac{1}{\mu_1} sin(k_z d) + \frac{1}{\mu_2}\frac{k_z}{\gamma_z} R_2 cos(k_z d)\right] \qquad (5.e)$$

In a similar way, matching the fields at $z = -d$, we obtain:

$$\Rightarrow B_3 = R_2 \frac{k_z}{\gamma_z}[B_o sin(k_z d) + B_1 cos(k_z d)] \qquad (6.a)$$

$$\Rightarrow B_o S_3 \; - \; B_1 S_4 = 0 \qquad (6.b)$$

To have non-trivial solution of eq. (5.b) and (6.b), det( ) must be zero. Therefore, we can write:

$$\det(\;) = \begin{bmatrix} S_3 & S_4 \\ S_3 & -S_4 \end{bmatrix} \qquad \Rightarrow S_3 S_4 = 0 \qquad (7)$$

### C. Resonant Frequency

The resonant frequency of an isolated source free Cylindrical DRA (CDRA) for arbitrary $TM^z$ mode is evaluated from separation equation as:

$$k_{\rho 1}^2 + k_z^2 = k_1^2 = \varepsilon_{r1} k_o^2 \qquad (8.a)$$
$$-k_{\rho 4}^2 + k_z^2 = k_4^2 = k_o^2 \qquad (8.b)$$
$$k_{\rho 1}^2 - \gamma_z^2 = k_{2,3}^2 = k_o^2 \qquad (8.c)$$

Equations (4), (7) and (8) solved simultaneously to obtain the solution for different $TM_{0np}^z$ modes.

## IV. Results

To evaluate the resonant frequency $(f_r)$ for different $TM_{0np}^z$ modes, we have to compute free-space wave number $k_o$ which is equal to $(2\pi f)/c$ where $c$ is the velocity of light in free-space. Now, eq. (4) is a function $x(= k_{\rho 1} a)$ and $y(= k_{\rho 4} a)$ and they are related with $k_z$ by eq. (7) and (8). These three equations (i. eq. (4), (7) and (8)) are solved simultaneously to find the un-normalized cut-off wavenumber $k_o a\ (= z, \text{say})$. Once $z(= k_o a)$ is obtained, $k_o$ can easily be obtained. Fig. 4 depicts the graphical solution of the cut-off frequency (here, resonant frequency). Theoretical resonant frequency for different $TM_{0np}^z$ modes of the CDRA are compared with data obtained using 3D EM simulator HFSS [8] as shown in Table I.

For experimental validation, we have collected the measured data as found in the open literature. This is shown in Table II. It is found that our theory is in excellent agreement with experimental data and data obtained using 3D EM simulator.

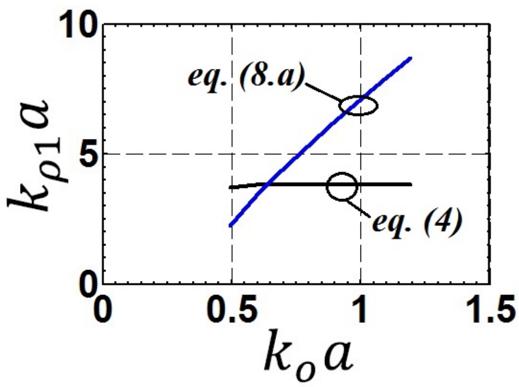


Fig. 4 Graphical solution of cut-off frequency for $TM_{011}^z$ mode of a CDRA ($a = 10mm, d = 5mm, \varepsilon_r = 60$)

TABLE I
COMPARISON OF RESONANT FREQUENCY WITH SIMULATED DATA

| Mode | a (mm) | d (mm) | $\varepsilon_r$ | Reso. Freq. (GHz) | |
|---|---|---|---|---|---|
| | | | | HFSS [8] | Theory |
| $TM_{011}$ | 15 | 10 | 10 | 4.21 | 4.337 |
| $TM_{011}$ | 20 | 15 | 30 | 1.83 | 1.873 |
| $TM_{011}$ | 10 | 5 | 60 | 3.11 | 3.041 |

TABLE II
EXPERIMENTAL VERIFICATION OF RESONANT FREQUENCY

| Mode | Ref. | a (mm) | d (mm) | $\varepsilon_r$ | Reso. Freq. (GHz) | |
|---|---|---|---|---|---|---|
| | | | | | Mea. | Theo. |
| $TM_{011}$ | [1] | 5.145 | 2.255 | 79 | 5.41 | 5.464 |
| $TM_{011}$ | [1] | 6.415 | 2.81 | 38 | 6.13 | 6.305 |
| $TM_{011}$ | [9] | 5.25 | 2.3 | 38 | 7.524 | 7.704 |

## V. Conclusion

In this work, Cylindrica Dielectric Resonator Antenna (CDRA) is investigated by modeling the surfaces of the CDRA as imperfect wall. It is assumed that the maximum field is confined inside the CDRA and the fields are decaying exponentially out-side the CDRA. After that, the tangential electric and magnetic fields are matched at the surfaces of the CDRA to find the eigenfunction. It is found that our theoretical results on resonant frequency for different

$TM^{z}_{0np}$ modes are in close agreement with experimental data and data obtained using 3D EM simulator HFSS. For simplicity, only $TM$ mode is investigated here, whereas the technique is general and can be extended to $TE$ and/or $HEM$ modes also.

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