

# Sliding Mode Observer Based Tip-Tracking Control of 2-DOF Serial Flexible Link Manipulator

Umesh Kumar Sahu

Department of ETC Engineering

G. H. Raisoni College of Engineering, Nagpur

Email: umesh.sahu@raisoni.net

Dipti Patra

Department of Electrical Engineering

NIT Rourkela, 769008, India

Email: dpatra@nitrkl.ac.in

Bidyadhar Subudhi

Department of Electrical Engineering

NIT Rourkela, 769008, India

Email: bidyadhar@nitrkl.ac.in

**Abstract**—Nowadays, lightweight flexible robot is used in many space robots, because these are more transportable and more maneuverable, unlike the rigid-link robot manipulator. However, due to non-minimum phase behavior and distributed link flexure tip-tracking control of FLM is challenging. Also, inherent model uncertainties and environmental disturbance lead to an error in tip-positioning and link vibration. To deal with these challenges, the composite controller is designed. In which, Sliding Mode Observer (SMO) based new estimation law is proposed, to reconstruct uncertain parameter for accurate tip-tracking control of 2-DOF Serial Flexible Link Manipulator (2DSFLM). The Lyapunov theory is used to investigate both convergence and stability of the proposed control system. The performance and robustness of the developed SMO for 2DSFLM is investigated by pursuing simulation studies. It is observed from the simulation studies that the developed composite control scheme effectively estimates the deviation in tip-tracking performance of 2DSFLM with an improved transient response.

**Keywords:** 2-DOF Serial Flexible Link Manipulator, Assumed Mode Method, Composite Control, Sliding Mode Observer, Tip-tracking Control.

## I. INTRODUCTION

Unlike the industrial robotic manipulators, flexible link manipulators (FLMs) are manufactured with materials of light weights and thin structure for employing the later in specialized applications such as in space robots to reduce the cost of transport. Although owing to thin and light structure, FLM extends many benefits such as lower power consumption, an increase in payload holding capability and high-speed operation, over rigid-link manipulator. However, structural flexible nature of their arms and joints leads to inaccuracy in tip-tracking performance compared with the rigid-link manipulator [1].

As flexible manipulator exhibits rigid and flexible types of motion, the dynamics equation become highly complex. So, accurate dynamic modeling of 2DSFLM is needed for analyzing the dynamics of FLM and controller design. Different models have been applied, to examine the dynamics of 2DSFLM. The simplest and fundamental spring-mass discrete model [2], Euler-Bernoulli Beam Model [3]. PDE model [4] describes the dynamics of FLM into two degrees of freedoms. In FLM, the equation of motion is derived by Lagrangian, Newton-Euler, and Kanes methods. The Newton-Euler algorithm is widely utilized for dynamic modeling of

the rigid-link manipulator, whereas generalized Newton-Euler algorithm [5] is applied for understanding the dynamics of flexible manipulators. Lagrangian is a most common modeling method for flexible manipulator [6], associated with one decomposition method. Finite element method (FEM) [7], assumed mode method (AMM) [8], [9] or lumped parameter method (LPM) [10] are popular decomposition methods that are used to simplify the process of controller design, by reducing the infinite dimensionality of the system to some finite dimensional models. AMM or FEM method uses either the Lagrangian formulation or the Newton-Euler methods for FLM modeling. The AMM is more versatile and less computational complex among three, so AMM has been used with a Lagrangian formulation for dynamic modeling of FLM.

Due to non-minimum phase characteristics of the system, the designing of controller for tip-tracking of 2DSFLM is difficult. Un-modeled dynamics, nonlinearities, unmeasurable noise, etc., are most often considered as major uncertainties in 2DSFLM. Due to these uncertainties, partial state vector or a non-uniform signal received by the controller [11]. In view of estimating uncertain parameters, researches have proposed many observers based estimation methods for FLMs. In [12], proposed an extended disturbance observer based on delta-operator for serial flexible-joint manipulator. In [13], an Extended State Observer (ESO) based on feedback linearization is presented for tip-tracking control of flexible-joint manipulator. In [14], [15], the continuous-time ESO has been developed for estimation of un-modeled dynamics of FLM. But, when the speed of FLMs increases, the methods mentioned above, can not estimate of system states and become more complex that gives an erroneous response. Instead of ESO, SMO can be a better observer because it is appropriate for fast, simple and accurate uncertain parameter estimation with finite time convergence. Also, SMO has various advantages such as robustness to parameter uncertainty, can handle non-linear system and has excellent dynamic properties [16]. In [17], SMO has been used in electric and hybrid vehicles for slip ration control. In two-link robot manipulator high order SMO is utilized for output feedback tracking in [18]. A composite control scheme based on SMO for altitude control system in flexible aircraft in [19]. Recently, an SMO based approach is proposed for adaptive control of the uncertain non-linear

system with actuator faults in [20]. Motivated by the above discussion and based on hypothesis presented in [20], here new state estimation control law is developed based on SMO, to estimate uncertain parameter for tip-tracking control of 2DSFLM. The following are the contributions of this paper:

- 1) Dynamic Modeling of 2DSFLM based on AMM and Lagrangian approach is developed [8], for controller design of 2DSFLM.
- 2) New estimation law is developed based on [20] for simple, fast and accurate estimation of uncertainties present in 2DSFLM.
- 3) SMO and state feedback controller based composite control scheme is designed, in which SMO is adopted for uncertain parameter estimation and state-feedback controller (SFC) was utilized to stabilize the system.
- 4) Simulation is conducted to examine the performance and robustness of the developed control scheme.

The paper is organized as follows: Dynamic modeling of 2DSFLM is presented in In Section-II. Section-III presents the proposed structure of control scheme and problem formulation. In Section-IV, design of sliding mode observer is presented. In Section-V, composite controller is designed. Simulation study are given in the Section-VI, followed by Section-VII that presents the conclusions of this work.

## II. MODELING OF THE 2DSFLM

In FLM, the tip positioning and tracking are very difficult due to distributed link flexure. In this work, it is assumed that the links have constant material properties, motion of the 2DSFLM in the horizontal plane and have uniform cross-sectional area [15]. Fig. 1 shows the schematic diagram of a 2DSFLM, where  $\tau_i$ ,  $\theta_i$ ,  $\delta_i$  and  $y_i(l_i, t)$  are the actual applied torque, joint angle, tip deflection and the tip position of the  $i^{th}$  link.

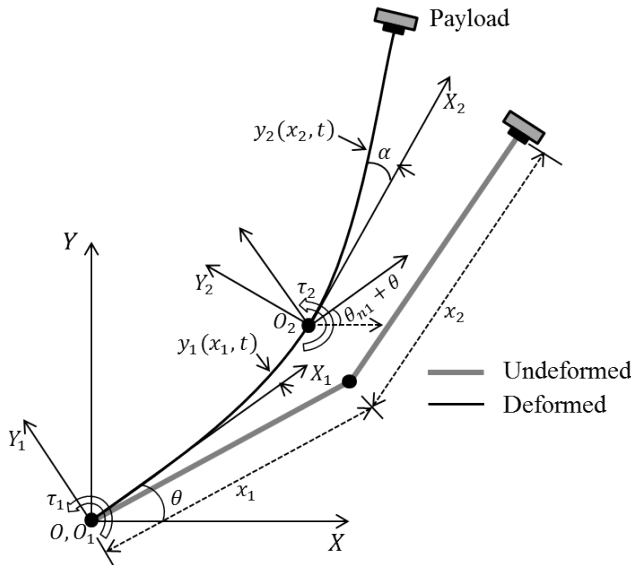


Fig. 1: Schematic diagram of a 2DSFLM

The actual output vector  $y_{pi}$  is output for the  $i^{th}$  link. Hence, the elastic displacement can be written as

$$y_{pi} = \theta_i + \left( \frac{y_i(l_i, t)}{l_i} \right) \quad (1)$$

where,  $l_i$  is the  $i^{th}$  link length.

The 2DSFLM dynamics can be derived by using the energy principle and the Lagrangian formulation technique along with AMM [8]. The 2DSFLM dynamics can be written as

$$M(\theta_i, \delta_i) \begin{bmatrix} \ddot{\theta}_i \\ \ddot{\delta}_i \end{bmatrix} + \begin{bmatrix} c_1(\theta_i, \delta_i, \dot{\theta}_i, \dot{\delta}_i) \\ c_2(\theta_i, \delta_i, \dot{\theta}_i, \dot{\delta}_i) \end{bmatrix} + k \begin{bmatrix} 0 \\ \delta_i \end{bmatrix} + D \begin{bmatrix} 0 \\ \dot{\delta}_i \end{bmatrix} = \begin{bmatrix} \tau_i \\ 0 \end{bmatrix} \quad (2)$$

In (2),  $M$ ,  $c_1$ ,  $c_2$ ,  $k$  and  $D$  are the positive definite symmetric inertia matrix, Coriolis force vector, Centrifugal force vector, stiffness matrix and the Damping Matrix. The state-space form of 2DSFLM dynamics (2) can be rewritten as

$$\begin{aligned} \dot{x}(t) &= Ax_i(t) + Bu_i(t) \\ y(t) &= Cx_i(t) \end{aligned} \quad (3)$$

$$\begin{aligned} A &= \begin{bmatrix} 0_m & I_m \\ -M^{-1}k & -M^{-1}D \end{bmatrix}, B = \begin{bmatrix} 0_{m \times 1} \\ M^{-1}e_1 \end{bmatrix}, \\ c &= [0_m \quad I_m], \\ u_i &= [\tau_i \quad 0]^T, \\ x_i &= [\theta_i \quad \dot{\theta}_i \quad \delta_i \quad \dot{\delta}_i]^T. \end{aligned}$$

where,  $x_i(t) \in \mathbb{R}^{2n}$ ,  $y(t) \in \mathbb{R}^m$  and  $u_i(t) \in \mathbb{R}^n$  are the 2DSFLM state, tip position and input respectively.  $A$ ,  $B$  and  $C$  are matrices with suitable dimensions.

Eq. (2) can be rewritten as

$$M_{11}\ddot{\theta} + M_{12}\ddot{\delta} + D_{11}\dot{\theta} + D_{12}\dot{\delta} + G_1 = \tau \quad (4)$$

$$M_{21}\ddot{\theta} + M_{22}\ddot{\delta} + D_{21}\dot{\theta} + D_{22}\dot{\delta} + G_2 = 0 \quad (5)$$

From (5), one obtains

$$\ddot{\delta} = -M_{22}^{-1}[M_{21}\ddot{\theta} + D_{21}\dot{\theta} + D_{22}\dot{\delta} + G_2] \quad (6)$$

Substituting (6) into (4), gives

$$\begin{aligned} (M_{11} - M_{12}M_{22}^{-1}M_{21})\ddot{\theta} + (D_{11} - M_{12}M_{22}^{-1}D_{21})\dot{\theta} \\ + (D_{12}\dot{\delta} - M_{12}M_{22}^{-1}G_2 + G_1) = \tau \end{aligned} \quad (7)$$

Eq. (7) can be written as

$$P\ddot{\theta} + Q\dot{\theta} + R = \tau \quad (8)$$

The dynamic equation is rewritten by defining two new variables as  $x_1 = \theta$  and  $x_2 = \dot{\theta}$  as

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = P^{-1}(\tau - (Qx_1 + R)) \end{cases} \quad (9)$$

In (9), considering  $-P^{-1}(Qx_1 + R)$  is the un-modeled dynamics and disturbance, which is assumed as new state  $x_3(k)$  for convenience of SMO design. The 2DSFLM dynamics (9) is utilized in design of controller.

### III. PROBLEM FORMULATION

The structure of SMO based tip-tracking control of 2DSFLM is shown in Fig. 2. Given a nonlinear system with disturbance and uncertainties, so the objective is to determine a control input  $u(t)$  such that, the output trajectory  $y(t)$  will track the reference output trajectory with minimum tracking error.

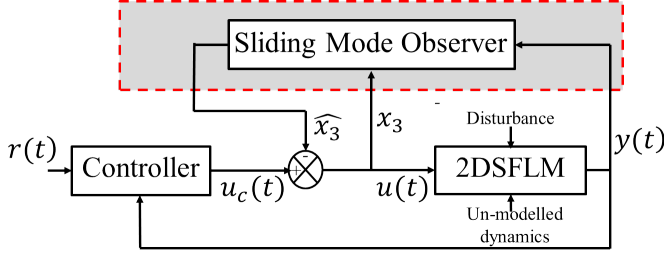


Fig. 2: The structure of proposed control scheme

### IV. SLIDING MODE OBSERVER

The design of new estimation law is presented in this section, by utilizing sliding mode observer technique for estimating the uncertain term  $x_3$ . Initially, upper bound of unknown dynamics  $x_3$  need to be designed, so that proper value of gains can be selected in the following estimation method.

In 2DSFLM, the torque and velocity are finite. More specifically that  $|\dot{\theta}_i| \leq \dot{\theta}_{i\_max}$  and  $|\dot{\tau}_i| \leq \dot{\tau}_{i\_max}$  for the  $i_{th}$  link  $i \in \{1, 2\}$ , while the two positive and known constant and  $\dot{\tau}_{i\_max}$  and  $\dot{\theta}_{i\_max}$  are the maximum rotation velocity and the maximum torque of  $i_{th}$  link respectively.

$$\|\tau\| \leq \sqrt{\sum_{i=1}^n \tau_{i\_max}^2} \quad (10)$$

In (10), it must be clear that faulty applied torque of both the hub should be smaller than the maximum applied torque. Although  $x_3$  is uncertain, it should always bounded by positive and known scalar  $c \in \mathfrak{R}$  respectively.

**Lemma 1.** The unknown dynamics  $x_3$  should remain bounded by known scalar  $c \in \mathfrak{R}$ . That is  $\|x_3\| \leq c$ .

*Proof.* on the basis of (9),

$$\|x_3\| \leq \|P^{-1}(Qx_1 + R)\| \quad (11)$$

It yields substituting the inequality (10) into (11) leads to

$$\|x_3\| \leq \sqrt{\sum_{i=1}^n \tau_{i\_max}^2} = c \quad (12)$$

where,  $\theta_{i\_max}$ ,  $\tau_{i\_max}$  and  $c$  are known.  $\square$

Then, to design a new estimation law, a sliding mode hypersurface or a manifold, i.e., sliding surface is chosen in such a way that trajectory of the system shows required behaviour when restrained to sliding surface. Therefore, sliding surface is introduced as

$$S = x_2 - \xi \quad (13)$$

$$\dot{\xi} = P^{-1}(KS + \alpha_1 \text{sgn}(S) + \alpha_2 S^{\delta_1/\delta_2} + \tau - Qx_1 + R\xi) \quad (14)$$

where,  $\alpha_i$  is the sliding mode gains,  $\delta_i$  and  $K$  are constant satisfying  $K > 0, \alpha_i > 0, \delta_i > 0$  also  $\delta_1 < \delta_2$ .

**Theorem 1.** A nonlinear estimation law can be designed, considering the system (9) as

$$f_r = -KS - \alpha_1 \text{sgn}(S) - \alpha_2 S^{\delta_1/\delta_2} - R\xi \quad (15)$$

To estimate  $x_3$  exactly by  $f_r$ , the gain value  $\alpha_1$  is chosen such that the condition  $\alpha_1 > R$  is satisfied. furthermore, after certain time, the related estimation error  $f_e = f_r - f$  will converge to zero.

*Proof.* for (9), Lyapunov candidate function is chosen as

$$V_1 = 0.5S^T P S \quad (16)$$

Then

$$\begin{aligned} \dot{V}_1 &= S^T P(\dot{x}_2 - \dot{\xi}_2) + 0.5S^T \dot{P} S \\ &= 0.5S^T \dot{P} S + S^T(\tau - Qx_1 + R\dot{\xi}) \\ &= S^T(-KS - \alpha_1 \text{sgn}(S) - \alpha_2 S^{\delta_1/\delta_2} - x_3) \end{aligned} \quad (17)$$

With the gain  $\alpha_1$  chosen such that  $\alpha_1 > R$ , by using the inequality  $\lambda_1 \|x\|^2 \leq x^T P x \leq \lambda_2 \|x\|^2$  and lemma 1 gives.

$$\begin{aligned} V_1 &\leq -K \|S\|^2 - \alpha_1 \|S\| - \alpha_2 S^{\delta_1/\delta_2} - R \|S\| \\ &\leq -K \|S\|^2 - \alpha_2 S^{\delta_1/\delta_2} \\ &\leq \frac{-2K}{\lambda_2} - \alpha_2 \left(\frac{2}{\lambda_2}\right)^{\frac{\delta_1+\delta_2}{2\delta_2}} V_1^{\frac{\delta_1+\delta_2}{2\delta_2}} \end{aligned} \quad (18)$$

Solving inequality (18) can get  $V_1(t) \equiv 0$  for all  $t \geq t_f$

$$t_f = \frac{\delta_2 \lambda_2}{K(\delta_1 + 2\delta_2)} \ln \left( \frac{\frac{2K}{\lambda_2} V^{\frac{\delta_2 - \delta_1}{2\delta_2}}(0) + \alpha_2 \left(\frac{2}{\lambda_2}\right)^{\frac{\delta_1 + \delta_2}{2\delta_2}}}{\alpha_2 \left(\frac{2}{\lambda_2}\right)^{\frac{\delta_1 + \delta_2}{2\delta_2}}} \right) \quad (19)$$

and

$$\dot{S}(t) \equiv 0, t > t_f \quad (20)$$

Uncertainty estimation error  $f_e$  can be rewritten as

$$\begin{aligned} f_e &= f_r - x_3 \\ &= -KS - \alpha_1 \text{sgn}(S) - \alpha_2 S^{\delta_1/\delta_2} - \tau + Qx_1 + RS \\ &= P\dot{S} \end{aligned} \quad (21)$$

Then, it can be determined from (20) and (21) that  $f_e \equiv 0$  for  $t > t_f$ . It implies that  $f_r$  can estimate the un-modeled dynamics and disturbance ( $x_3$ ) with the estimation error converging to zero after a certain time  $t_f$ .  $\square$

## V. COMPOSITE CONTROLLER DESIGN

In Section-IV, new estimation law is designed using SMO to estimate uncertainties present in 2DSFLM. To stabilize the tip-tracking performance of 2DSFLM, following composite control scheme is designed:

$$u(t) = u_c(t) - \hat{x}_3 \quad (22)$$

where,  $\hat{x}_3$  is estimation of  $x_3$  by SMO,  $u_c(t)$  is SFC which is formulated as

$$u_c(t) = Gx(t) \quad (23)$$

where,  $G$  is conventional state-feedback controller gain.

## VI. SIMULATION RESULTS AND DISCUSSIONS

Here, initially 2DSFLM dynamic model is validated which is presented in Section-II then performance of developed composite controller presented in Section-IV and Section-V is analyzed. Table II lists the controller parameters required in simulation study. Table II shows the parameter values for both the links. The quantitative measure Root Mean Square of Error (RMSE) is used for comparing the performance of the developed control scheme, which is given as:

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^N (e(t))^2} \quad (24)$$

TABLE I: Controller Parameter for NC-TLFM

Controller	Control law	Control parameter	Control parameter values
SFC	Eq. (23)	$G_1, G_2$	$G_1 = [14.21; -9.18; 0.68; -0.05];$ $G_2 = [12.15; -8.67; 0.36; -0.11]$
SMO	Eq. (22)	$\alpha_1, \alpha_2$	$\alpha_1 = \text{diag}[0.72, 0.24];$ $\alpha_2 = \text{diag}[0.8, 0.42];$ $\beta_1 = \text{diag}[5, 120];$
ESO	Eq. (7) [14]	$\beta_1, \beta_2, \beta_3$	$\beta_2 = \text{diag}[1000, 1000];$ $\beta_3 = \text{diag}[10, 10]$

In this simulation, sinusoidal signal with  $20^\circ$  amplitude and 0.1 Hz frequency is considered as torque input for both the link of 2DSFLM. Fig. 3 show hub angle response of link-2 obtained through both simulation and experiments. It can be seen from Fig. 3 that the derived 2DSFLM dynamic model resembles to the actual behaviour of physical 2DSFLM.

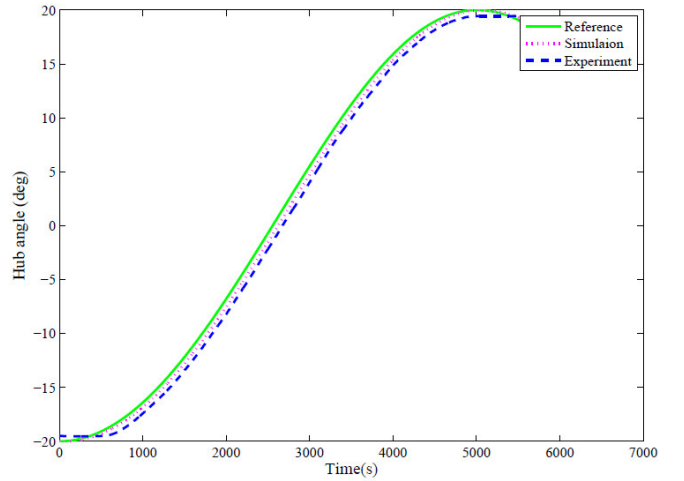


Fig. 3: Link-2 hub angle response

TABLE III: Root mean square error (Simulation)

Response	Link	SFC	SFC with ESO	SFC with SMO
Hub Angular Response	Link-1	3.1132	2.6471	1.0218
	Link-2	3.2648	2.7158	1.1251
Tip Acceleration Response	Link-1	1.3672	0.1117	0.0654
	Link-2	0.1097	0.0999	0.0287

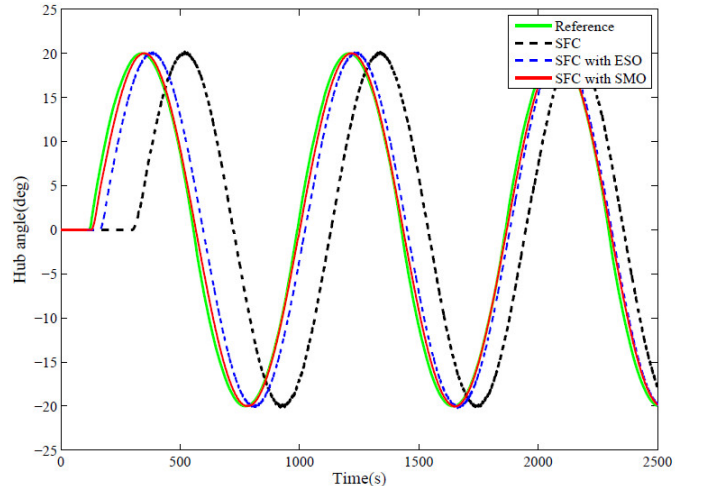


Fig. 4: Tip-tracking and estimation of link-2 (hub angle response)

The tip-tracking and estimation, i.e., hub angle response and tip acceleration response are shown in Fig. 4 and Fig. 5 respectively. Quantitative parameter (RMSE) is calculated from response curves obtained from simulation studies of SFC, SFC with ESO and SFC with SMO, and same is given in Table III. It can be observed from Table III, the RMSE for the SFC with SMO is lower than that obtained for the SFC with ESO.

TABLE II: Parameter values of link-1 and 2

Parameter	Link-1	Link-2
Mass of Link	$m_1 = 0.15268kg$	$m_2 = 0.0535kg$
Link Length	$L_1 = 0.201m$	$L_2 = 0.2m$
Elasticity	$E_1 = 2.0684 \times 10^{11} N/m^2$	$E_2 = 2.0684 \times 10^{11} N/m^2$
Rotor Moment of Inertia	$Jm_1 = 6.28 \times 10^{-6} kg.m^2$	$Jm_2 = 1.03 \times 10^{-6} kg.m^2$
Drive Moment of Inertia	$J_1 = 7.361 \times 10^{-4} kg.m^2$	$J_2 = 44.55 \times 10^{-6} kg.m^2$
Link Moment of Inertia	$I_1 = 0.17043kg.m^2$	$I_2 = 0.0064387kg.m^2$

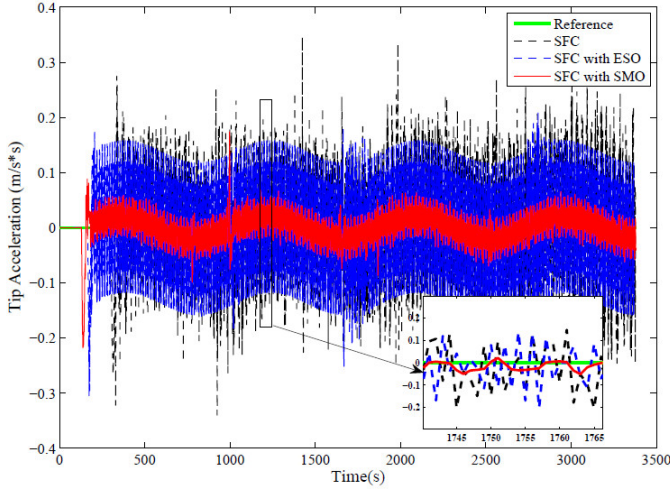


Fig. 5: Tip-tracking and estimation of link-2 (tip acceleration response)

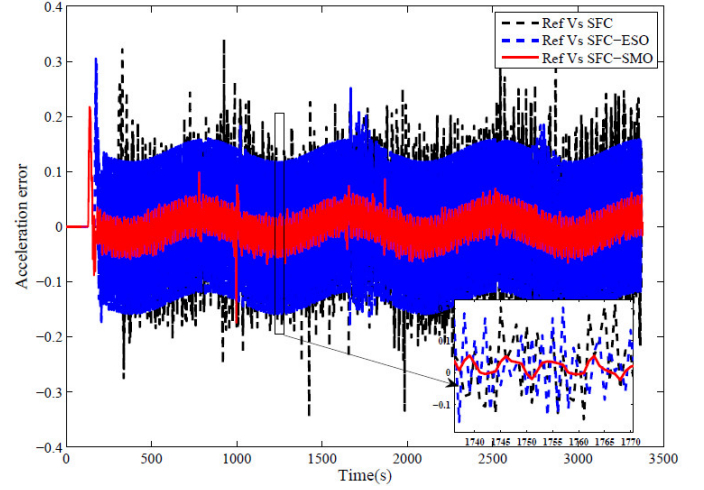


Fig. 7: Link-2 acceleration error

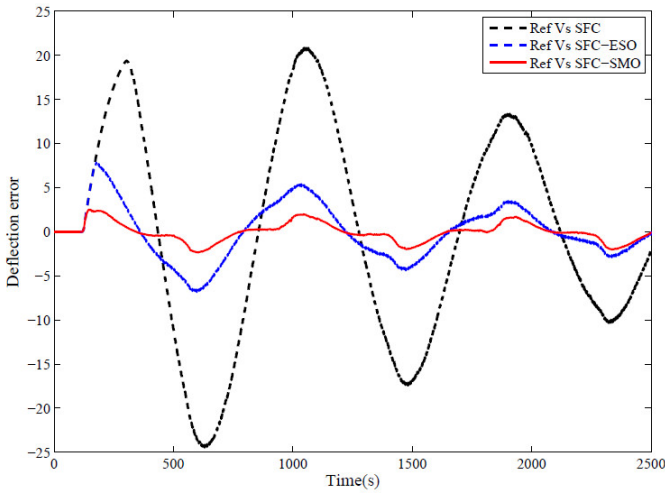


Fig. 6: Link-2 deflection error

Fig. 6 shows the deflection error profile of link-2, from which it is observed that there is a maximum tip deflection error amplitude of 20 mm in case of SFC, 7.1 mm in case of SFC with ESO, but SFC with SMO yields the minimum tip deflection error, i.e., 4 mm. Fig. 7 shows the acceleration error profile for link-2, from which it is observed that tip

acceleration error for SFC is 0.3, 0.13 in case of SFC with ESO, and yields the minimum tip acceleration error of 0.03 for SFC with SMO. It is observed from Fig. 3 - Fig. 7 that, SFC with SMO tracks the desired trajectories with minimum tracking error and gives better tip-tracking performance.

## VII. CONCLUSION

In this work, new composite control scheme based on SMO and state feedback controller (SFC) for tip-tracking control of 2DSFLM have been proposed. In which, new estimation law is designed based on SMO for simple, fast and accurate estimation of un-modeled dynamics and uncertainties in 2DSFLM. From simulation studies, it is observed that the proposed composite controller can provide perfect tip-tracking performance, despite uncertainties and environmental disturbance in the 2DSFLM dynamics. The performance of the developed composite controller is analyzed by simulation studies in MATLAB and SIMULINK environment. In the future, experimental validation of the proposed tip-tracking control scheme needs to be carried out for analyzing performance in a real-time environment. Also, the more theoretical hypothesis needs to be carried out to improve the robustness and stability in flexible manipulator performance.

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