Fractional reduced differential transform method based solutions of time-fractional seventh-order Sawada–Kotera–Ito equation

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Abstract

Nonlinear fractional differential equations (NLFDEs) are widely used to describe various phenomena in different fields of science and engineering such as physics, chemistry, acoustics, control theory, finance, economics, mechanical, civil, electrical engineering and also in social sciences. Applications of NLFDEs can also be found in turbulence, fluid dynamics, and nonlinear biological systems. NLFDEs are believed to be potent tools to define real-world problems more accurately than the integer-order differential equations. In this investigation, we have applied fractional reduced differential transform method (FRDTM) to obtain the solution of time-fractional seventh-order Sawada–Kotera–Ito (SK-Ito) equation. The novelty of the FRDTM is that it does not require any discretization, transformation, perturbation, or any restrictive conditions. Moreover, this method requires less computation compared to other methods. Computed results are compared with the existing results for the special cases of integer as well as non-integer orders. The present results are in good agreement with the existing solutions. Here, the fractional derivatives are considered in the Caputo sense. Further, convergence analysis of the results with an increasing number of terms of the solution has also been studied.

Keywords: Sawada-Kotera-Ito equation; FRDTM; fractional derivative, semi-analytical methods

MSC (2010) subject classification: 26A33, 74J30, 35A20, 74H10

1. Introduction and motivation

During the last few years, the subject of fractional derivatives and integrals of arbitrary order has been of great interest for many scientists who are working in this field. These new definitions of the fractional-order derivatives and fractional differential equations, especially fractional partial differential equations (FPDF) have gained increasing popularity for possible applications in many areas of science and engineering. For example, signal processing [1], fluid mechanics [2,3], option pricing problems [4,5,6], electrochemistry [7] and structural dynamics [8,9]. Thus, the need for proper numerical and analytic methods that can deal with such models is important.

One of the most important methods that deals with these fractional models are the fractional reduced differential transform method (FRDTM). This method has its inherent advantages and disadvantages. One of the most important advantages of this method is that it does not require any linearization or discretization for the model problem and can reduce the size of the computation. Thus, the use of this method and its modifications for solving differential equations and especially fractional order differential equations due to these advantageous have increased tremendously. Arshad et al. [10] used the FRDTM for solving some FPDE, including the fractional Zakharove-Kuznetsov equation. Also, El-Sayed et al. [11] adapted this method for solving a fractional model of projectile motion in a quadratic resistant medium as an application for the use of fractional order derivative in engineering models. Thabet et al. [12] proposed a conformable fractional differential transform method for solving different conformable fractional PDE achieving good results. A fractional-order integrodifferential equation with nonlocal boundary conditions has been solved in [13] using the FRDTM converting this equation into a system of algebraic equations and solving it to get the unknowns. In addition, many other fractional models have been solved using this method including fractional differential-algebraic equations [14], time-fractional gas dynamics equation [15], fractional Riccati equation [16,17], multiterm time-fractional diffusion equations [18], irrational order fractional equations [19], Bagley–Torvik equation [20], fuzzy fractional dynamical model of marriage [21], system of fractional differential equations [22] and nonlinear fractional Klein-Gordon equation [23].

Due to the importance of PDE in the modeling of different areas of science, there have been many methods that have been proposed for their solution. Among these models with a particular interest in fluid mechanics is the Sawada–Kotera–Ito equation. This model is used to describe the motion of long waves in shallow water under gravity and also in the modeling of nonlinear optics. Many researchers applied efficient and powerful techniques for solving various forms of these equations. For example, Koonprasert et al. [24] applied the Riccati equation mapping method for solving the fifth-order fractional Sawada-Kotera equation. Also, Naher et al. [25] adapted the Exp-function method in order to construct a traveling wave solution for the fifth-order Sawada-Koter equation. For more details about the methods and models of these equations see [26-32] and references therein.

The generalized time-fractional KdV equation of seventh-order is written as follows [33]

$$\frac{\partial^\alpha \psi}{\partial t^\alpha} + m_1\psi^2\psi_x + m_2\psi_x^3 + m_3\psi\psi_x\psi_{xx} + m_4\psi^2\psi_{xxx} + m_5\psi_{xx}\psi_{xxx} + m_6\psi_x\psi_{xxxx} + m_7\psi\psi_{xxxxx} + \psi_{xxxxxxx} = 0 \qquad (1)$$

where $m_i, i = 1(1)7$ are the nonzero parameters. Sawada and Kotera [34], and later, Ito [35] found that for $m_1 = 252, m_2 = 63, m_3 = 378, m_4 = 126, m_5 = 63, m_6 = 42$ and $m_7 = 21$. Now, the time-fractional seventh-order SK–Ito equation is given by [36,37]

$$\frac{\partial^\alpha \psi}{\partial t^\alpha} + 252\psi^2\psi_x + 63\psi_x^3 + 378\psi\psi_x\psi_{xx} + 126\psi^2\psi_{xxx} + 63\psi_{xx}\psi_{xxx} + 42\psi_x\psi_{xxxx} + 21\psi\psi_{xxxxx} + \psi_{xxxxxxx} = 0 \tag{2}$$

Our primary interest in this paper is to investigate the following time-fractional seventh-order SK–Ito equation subject to proper initial conditions (IC) which have been taken into consideration as

$$\begin{cases} \frac{\partial^\alpha \psi}{\partial t^\alpha} + 252\psi^2\psi_x + 63\psi_x^3 + 378\psi\psi_x\psi_{2x} + 126\psi^2\psi_{3x} + 63\psi_{2x}\psi_{3x} + 42\psi_x\psi_{4x} \\ \qquad + 21\psi\psi_{5x} + \psi_{7x} = 0, \\ \psi(x,0) = \frac{4}{3}a^2\left(2 - 3\tanh^2(ax)\right). \end{cases} \tag{3}$$

where a is constant and $\psi_{kx} = \partial^k \psi / \partial x^k$.

To the best of the authors' knowledge, the results that are presented in this work are the first to be introduced for solving time-fractional SK-Ito equation using FRDTM.

The outline of the paper is organized as in section 2; we present the basic concepts of fractional calculus. Section 3 is devoted to present the main steps for applying the FRDTM. In Section 4, the method is demonstrated on the time-fractional SK-Ito equation. Some numerical examples are presented in Section 5. Finally, Section 6 is devoted to the conclusion of this study.

2. Basics on fractional calculus

In this section, we will introduce the basic definitions for the fractional calculus that will be needed [38-40]. These definitions are as follows

Definition 2.1:

The Riemann-Liouville (R-L) fractional differential operator D^α of order α is described as

$$D^\alpha \phi(x) = \begin{cases} \frac{d^m}{dx^m}\phi(x), & \alpha = m \\ \frac{1}{\Gamma(m-\alpha)}\frac{d^m}{dx^m}\int_0^x \frac{\phi(t)}{(x-t)^{\alpha-m+1}}dt, & m-1 < \alpha < m \end{cases} \tag{4}$$

where $m \in Z^+$, $\alpha \in R^+$ and

$$D^{-\alpha}\phi(x) = \frac{1}{\Gamma(\alpha)}\int_0^x (x-t)^{\alpha-1}\phi(t)\,dt, \; 0 < \alpha \le 1. \tag{5}$$

Definition 2.2:

The R-L fractional order integral operator J^{α} is described as

$$J^{\alpha}\phi(x)=\frac{1}{\Gamma(\alpha)}\int_{0}^{x}(x-t)^{\alpha-1}\phi(t)dt, \quad t>0,\ \alpha>0. \tag{6}$$

Following Podlubny [40], we have

$$J^{\alpha}t^{m}=\frac{\Gamma(m+1)}{\Gamma(m+\alpha+1)}t^{m+\alpha}, \tag{7}$$

$$D^{\alpha}t^{m}=\frac{\Gamma(m+1)}{\Gamma(m-\alpha+1)}t^{m-\alpha}. \tag{8}$$

Definition 2.3:

The Caputo fractional differential operator ${}^{C}D^{\alpha}$ of order α is described as

$${}^{C}D^{\alpha}\phi(x)=\begin{cases}\dfrac{1}{\Gamma(m-\alpha)}\displaystyle\int_{0}^{x}\frac{\phi^{m}(t)}{(x-t)^{\alpha-m+1}}dt, & m-1<\alpha<m,\\ \dfrac{d^{m}}{dt^{m}}\phi(t), & \alpha=m.\end{cases} \tag{9}$$

Definition 2.4:

(a) $D_{t}^{\alpha}J_{t}^{\alpha}\phi(t)=\phi(t)$ (10)

(b) $J_{t}^{\alpha}D_{t}^{\alpha}\phi(t)=\phi(t)-\sum_{k=0}^{m}\phi^{(k)}(0^{+})\frac{t^{k}}{k!}$, for $t>0$ and $m-1<\alpha\leq m$.

Next, we will introduce the necessary steps for adapting the fractional reduced differential transform method (FRDTM).

3. Fractional reduced differential transform method (FRDTM)

In this section, we will illustrate the necessary steps for the FRDTM [41-45]. First, let us consider a function of (n+1) variables $\phi(t,x_1,x_2,..,x_n)$ such that

$$\phi(t,x_1,x_2,..,x_n)=\phi_1(x_1)\phi_2(x_2)...\phi_n(x_n)h(t), \tag{11}$$

Hence from the properties of one-dimensional differential transform method [18]

$$\phi(t,x_1,x_2,..,x_n)=\left(\sum_{i_1=0}^{\infty}\phi_1(i_1)x_1^{i_1}\right)\left(\sum_{i_2=0}^{\infty}\phi_2(i_2)x_2^{i_2}\right)\cdots\left(\sum_{i_n=0}^{\infty}\phi_n(i_n)x_n^{i_n}\right)\left(\sum_{j=0}^{\infty}h(j)t^j\right)$$

$$=\sum_{i_1=0}^{\infty}\sum_{i_2=0}^{\infty}\cdots\sum_{i_n=0}^{\infty}\sum_{j=0}^{\infty}\phi(i_1,i_2,...,i_n,j)\,x_1^{i_1}x_2^{i_2}\ldots x_n^{i_n}t^j\ , \tag{12}$$

where $\phi(i_1,i_2,...,i_n,j)=\phi_1(i_1)\phi_2(i_2)\ldots\phi_n(i_n)h(j)$ is denoted as the spectrum of $\phi(t,x_1,x_2,..,x_n)$. Also, $\phi(t,x_1,x_2,..,x_n)$ is called the original function and $\phi_k(x_1,x_2,..,x_n)$ is called the transformed function or T-function. Thus, we need the following lemma.

Lemma 1: Let us consider an analytic and continuously differentiable function $\phi(t,x_1,x_2,..,x_n)$ to $(n+1)$ variables $t,x_1,x_2,..,x_n$ in the domain of our interest, then FRDTM of $\phi(t,x_1,x_2,..,x_n)$ can be written as

$$\phi_k(x_1,x_2,..,x_n)=\frac{1}{\Gamma(1+\alpha k)}\left[D_t^{\alpha k}\phi(t,x_1,x_2,..,x_n)\right]_{t=t_0}\ \text{for}\ k=0,1,2,.... \tag{13}$$

Then, applying the inverse transform of $\phi_k(x_1,x_2,..,x_n)$, we get

$$\phi(t,x_1,x_2,..,x_n)=\sum_{k=0}^{\infty}\phi_k(x_1,x_2,..,x_n)(t-t_0)^{\alpha k}\ . \tag{14}$$

From Eqs. (13) and (14), we obtain

$$\phi(t,x_1,x_2,..,x_n)=\sum_{k=0}^{\infty}\frac{1}{\Gamma(1+\alpha k)}\left[D_t^{\alpha k}\phi(t,x_1,x_2,..,x_n)\right]_{t=t_0}(t-t_0)^{\alpha k}\ . \tag{15}$$

In particular, at $t_0=0$, Eq. (15) reduces to the following equation

$$\phi(t,x_1,x_2,..,x_n)=\sum_{k=0}^{\infty}\frac{1}{\Gamma(1+\alpha k)}\left[D_t^{\alpha k}\phi(t,x_1,x_2,..,x_n)\right]_{t=0}t^{\alpha k}\ . \tag{16}$$

From the above definition, it is observed that the idea of FRDTM is derived from the power series expansion of a function. Then the inverse transformation of the set of values $\{\phi_k(x_1,x_2,..,x_n)\}_{k=0}^{n}$ gives approximate solution as

$$\tilde{\phi}_n(t,x_1,x_2,..,x_n)=\sum_{k=0}^{n}\phi_k(x_1,x_2,..,x_n)t^{\alpha k}\ , \tag{17}$$

where n is the order of the approximation solution and the exact solution may be written in the form

$$\phi(t, x_1, x_2, .., x_n) = \lim_{n\to\infty} \tilde{\phi}_n(t, x_1, x_2, .., x_n), \tag{18}$$

We need the following theorems for better understanding of the proposed method

Theorem 1:

Let $\phi(x,t), \xi(x,t)$ and $\zeta(x,t)$ are three analytical functions such that $\phi(x,t) = R_D^{-1}[\phi_k(x)]$, $\xi(x,t) = R_D^{-1}[\xi_k(x)]$ and $\zeta(x,t) = R_D^{-1}[\zeta_k(x)]$. Hence we reach the following properties

(i) If $\phi(x,t) = c_1\xi(x,t) \pm c_2\zeta(x,t)$, then $\phi_k(x) = c_1\xi_k(x) \pm c_2\zeta_k(x)$, where c_1 and c_2 are constants.

(ii) If $\phi(x,t) = a\,\xi(x,t)$, then $\phi_k(x) = a\,\xi_k(x)$.

(iii) If $\phi(x,t) = x^m t^n$, then $\phi_k(x) = x^m\,\delta(k-n)$ where $\delta(k) = \begin{cases} 1, & k = 0 \\ 0, & k \neq 0 \end{cases}$.

(iv) If $\phi(x,t) = x^m t^n \xi(x,t)$, then $\phi_k(x) = x^m \xi_{k-n}(x)$.

(v) If $\phi(x,t) = \xi(x,t)\zeta(x,t)$, then $\phi_k(x) = \sum_{i=0}^{j} \xi_i(x)\zeta_{j-i}(x) = \sum_{i=0}^{j} \zeta_i(x)\xi_{j-i}(x)$.

(vi) If $\phi(x,t) = \xi(x,t)\zeta(x,t)\varsigma(x,t)$, then $\phi_k(x) = \sum_{j=0}^{k}\sum_{i=0}^{j} \xi_i(x)\zeta_{j-i}(x)\varsigma_{k-j}(x)$.

where R_D^{-1} is the inverse reduced differential transform.

Theorem 2:

(i) If $\phi(x,t) = \frac{\partial^m}{\partial x^m}\xi(x,t)$, then $\phi_k(x) = \frac{\partial^m}{\partial x^m}\xi_k(x)$.

(ii) If $\phi(x,t) = \frac{\partial^{n\alpha}}{\partial t^{n\alpha}}\xi(x,t)$, then $\phi_k(x) = \frac{\Gamma(1+(k+n)\alpha)}{(1+k\alpha)}\xi_{k+n}(x)$.

To explain the basic implementation of FRDTM, we have considered the following equation in the operator form

$$L\phi(x,t) + R\phi(x,t) + N\phi(x,t) = h(x,t), \tag{19}$$

with initial condition

$$\phi(x,0) = g(x), \tag{20}$$

Where $L = \frac{\partial^\alpha}{\partial t^\alpha}$ and R, N are linear, nonlinear differential operators, respectively and $h(x,t)$ is an inhomogeneous source term. Then, by using Theorem 2 property No. (ii) and Eq. (13), Eq. (19) reduces to

$$\frac{\Gamma(1+\alpha k+\alpha)}{\Gamma(1+\alpha k)}\phi_{k+1}(x)=H_k(x)-R\phi_k(x)-N\phi_k(x), \text{ for } k=0,1,2.... \tag{21}$$

where $\phi_k(x)$ and $H_k(x)$ are the differential transformed form of $\phi(x,t)$ and $h(x,t)$, respectively.

Appling FRDTM on the initial conditions, we obtain

$$\phi_0(x)=g(x), \tag{22}$$

Using Eq. (21) and Eq. (22), $\phi_k(x)$ for $k=1,2,3,....$ can be determined.

Then by taking the inverse transformation of $\{\phi_k(x)\}_{k=0}^{n}$ gives n-term approximate solution as

$$\phi_n(x,t)=\sum_{k=0}^{n}\phi_k(x)t^{\alpha k}\ , \tag{23}$$

So, the analytical result of Eq. (23) is written as $\phi(x,t)=\lim_{n\to\infty}\phi_n(x,t)$.

Also, one may see the reference [45] for the convergence analysis of the present method theoretically.

4. Implementation of FRDTM on time-fractional seventh-order SK-Ito Equation

Applying FRDTM on both sides of Eq. (3) and using the appropriate theorem, the following recurrence relations for the problem and its initial condition is obtained

$$\begin{cases} \psi_{k+1}(x)=-\frac{\Gamma(1+\alpha k)}{\Gamma(1+\alpha k+\alpha)}\begin{pmatrix} 252\sum_{j=0}^{k}\sum_{i=0}^{j}\psi_i\psi_{j-i}\frac{\partial}{\partial x}\psi_{k-j}+63\sum_{j=0}^{k}\sum_{i=0}^{j}\frac{\partial}{\partial x}\psi_i\frac{\partial}{\partial x}\psi_{j-i}\frac{\partial}{\partial x}\psi_{k-j}+378\sum_{j=0}^{k}\sum_{i=0}^{j}\phi_i\frac{\partial}{\partial x}\phi_{j-i}\frac{\partial}{\partial x}\phi_{k-j} \\ +126\sum_{j=0}^{k}\sum_{i=0}^{j}\psi_i\psi_{j-i}\frac{\partial^3}{\partial x^3}\psi_{k-j}+63\sum_{i=0}^{k}\frac{\partial^2}{\partial x^2}\psi_i\frac{\partial^3}{\partial x^3}\psi_{k-i}+42\sum_{i=0}^{k}\frac{\partial}{\partial x}\psi_i\frac{\partial^4}{\partial x^4}\psi_{k-i}+ \\ 21\sum_{i=0}^{k}\psi_i\frac{\partial^5}{\partial x^5}\psi_{k-i}+\frac{\partial^7}{\partial x^7}\psi_k \end{pmatrix} \\ \psi_0(x)=\frac{4}{3}a^2\left(2-3\tanh^2(ax)\right) \end{cases} \tag{24}$$

Then, by plugging the transformed initial condition of Eq. (24) into the transformed main equation of Eq. (24) for k=0,1,2,.., we obtain the following values of $\{\psi_k(x)\}_{k=1}^{n}$

$$\psi_1(x)=\frac{512\sinh(ax)a^9}{\Gamma(1+\alpha)\cosh(ax)^9}\left(\cosh(ax)^6-63\cosh(ax)^4+315\cosh(ax)^2-315\right), \tag{25}$$

$$\psi_2(x)=\frac{65536a^{16}}{\Gamma(1+2\alpha)\cosh(ax)^{16}}\begin{pmatrix}\cosh(ax)^{14}-\frac{16383}{2}\cosh(ax)^{12}+888615\cosh(ax)^{10}-\\ \frac{61756695}{4}\cosh(ax)^8+87162075\cosh(ax)^6-\frac{411486075}{2}\\ \cosh(ax)^4+212837625\cosh(ax)^2-\frac{638512875}{8}\end{pmatrix}, \tag{26}$$

$$\psi_3(x)=\frac{8388608\sinh(ax)a^{23}}{\Gamma(1+3\alpha)\cosh(ax)^{23}}\begin{pmatrix}\cosh(ax)^{20}-1048575\cosh(ax)^{18}+\frac{3919486725}{2}\cosh(ax)^{16}\\ -270957109500\cosh(ax)^{14}+8631545214600\cosh(ax)^{12}-\\ \frac{203448548578275}{2}\cosh(ax)^{10}+\frac{2224322216891775}{4}\cosh(ax)^8-\\ 1569896281453500\cosh(ax)^6+\frac{4716494753095125}{2}\cosh(ax)^4-\\ \frac{3573102085678125}{2}\cosh(ax)^2+\frac{2143861251406875}{4}\end{pmatrix} \tag{27}$$

Continuing likewise, all other values of $\{\psi_k(x)\}_{k=4}^{n}$ maybe calculated. So the n^{th} order approximate solution of Eq. (3) maybe written as follows

$$\tilde{\psi}_n(x,t)=\sum_{k=0}^{n}\psi_k(x)t^{\alpha k}\ . \tag{28}$$

5. Numerical results and discussion

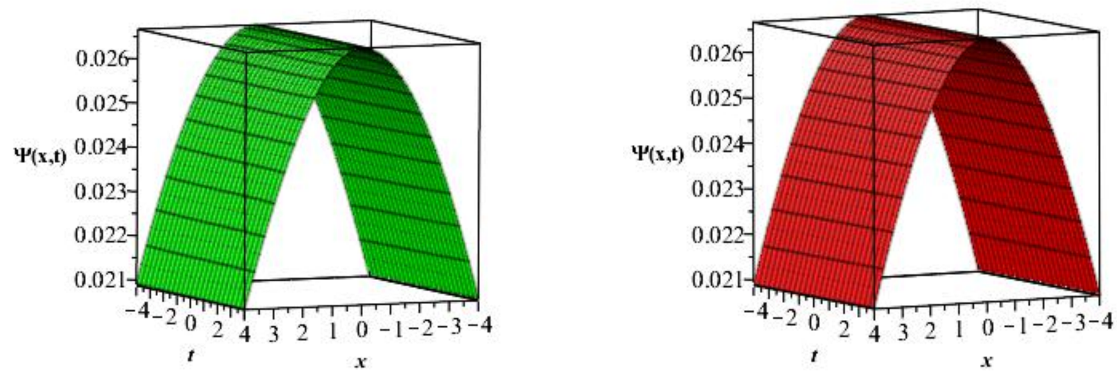
In this section, the approximate solution of time-fractional seventh-order SK-Ito equation using FRDTM has been studied. Here, all the numerical calculations have been computed by truncating the infinite series to a finite number of terms $(n=3)$. In this article, all the figures and tables are included by considering the values of the parameter as $a=0.1$ [36]. The achieved outcomes are compared with the solution of Arora and Sharma [36] and El-Sayed and Kaya [37] for $\alpha=1$, which shows the validation of the present study. Calculated results are displayed in terms of plots. Tables 1 shows the difference between the exact solution and present solution of the absolute errors. Comparison of the present result of Eq. (3) with the existing results at $\alpha = 1$ are demonstrated in Table 2. Solution plot of Eq. (3) has been compared with the exact solution plot, which is depicted in Figure 1. Similarly, Figure 2 gives the plots of Eq. (3) at different values of α (=0.2, 0.5, 0.7 and 0.9). Convergence analysis of the present solution by taking the increasing number of terms have been portrayed in Figure (3) for the Eq. (3) at $\alpha = 1$ and $\alpha = 0.5$. It is also noted that here we have used third-order approximate solution throughout the computations and we have achieved a good approximation with the exact solution of the titled problems. Improved approximation solutions have been obtained if we increase the order of the approximation that is increasing the number of terms of solution.

Table 1. Absolute error of third-order (n=3) approximate solution in comparison with the exact solution [36] $\psi(x,t) = \frac{4}{3}a^2\left(2 - 3\tanh^2\left(ax - \frac{256a^6 t}{3}\right)\right)$ at $a = 0.1, \alpha = 1$ for Eq. (3).

x/t	0.1	0.3	0.5	0.7	1.0
0.1	-4.2262E-08	-1.4787E-07	-2.7889E-07	-4.3209E-07	-6.9640E-07
0.3	-1.1847E-07	-3.7243E-07	-6.4123E-07	-9.1548E-07	-0.1316E-05
0.5	-1.9280E-07	-5.9085E-07	-9.9327E-07	-0.1385E-05	-0.1922E-05
0.7	-2.6416E-07	-8.002E-07	-0.1331E-05	-0.1838E-05	-0.2514E-05
1.0	-3.6344E-07	-0.1091E-05	-0.1803E-05	-0.2477E-05	-0.3374E-05

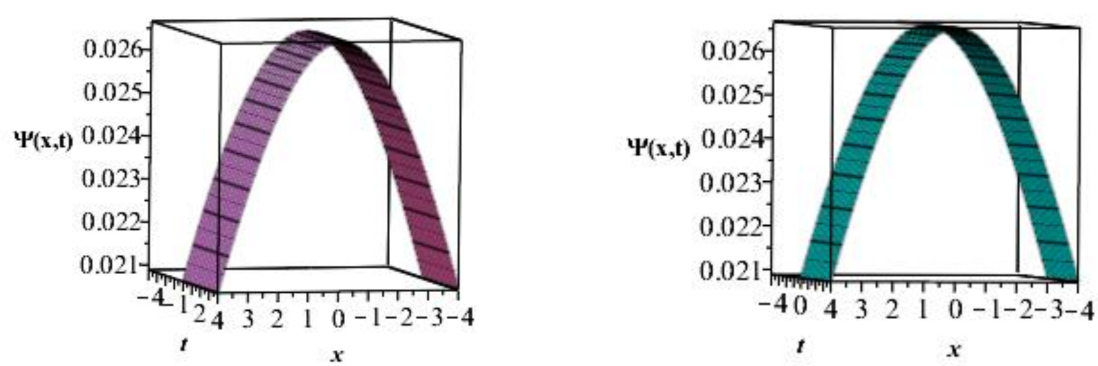
Table 2. Comparison of the present solution with the exact solution of Eq. (3) and their absolute error at $a = 0.1$ and $\alpha = 1$.

t	*x*	*Present solution*	*Exact solution [36]*	*Absolute Error*
-4	-2	0.2509759131E-1	0.2511356173E-1	-0.15970338E-4
	0	0.2666057464E-1	0.2666666201E-1	-0.6087390863E-5
	2	0.2512389335E-1	0.2510320234E-1	0.20690918E-4
-2	-2	0.2510030777E-1	0.2511097485E-1	-0.10667079E-4
	0	0.2666514366E-1	0.2666666550E-1	-0.1521847716E-5
	2	0.2511764239E-1	0.2510579515E-1	0.11847225E-4
0	-2	0.2510838599E-1	0.2510838599E-1	0
	0	0.2666666667E-1	0.2666666667E-1	0
	2	0.2510838599E-1	0.2510838599E-1	0
2	-2	0.2511764239E-1	0.2510579515E-1	0.11847225E-4
	0	0.2666514366E-1	0.2666666550E-1	-0.1521847716E-5
	2	0.2510030777E-1	0.2511097485E-1	-0.10667079E-4
4	-2	0.2512389335E-1	0.2510320234E-1	0.20690918E-4
	0	0.2666057464E-1	0.2666666201E-1	-0.6087390863E-5
	2	0.2509759131E-1	0.2511356173E-1	-0.15970338E-4

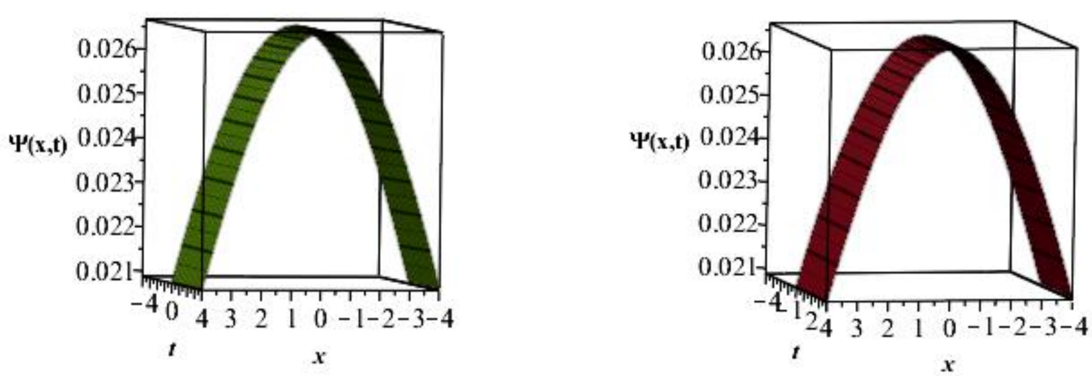


(a) (b)

Fig.1. Comparison plots of the (a) third-order present solution with (b) the exact solution of Eq. (3) at $a = 0.1$ and $\alpha = 1$.



(a) (b)



(c) (d)

Fig.2. Plots of the present solution (n=3) of Eq. (3) at (a) $\alpha = 0.2$ (b) $\alpha = 0.5$ (c) $\alpha = 0.7$ and (d) $\alpha = 0.9$.

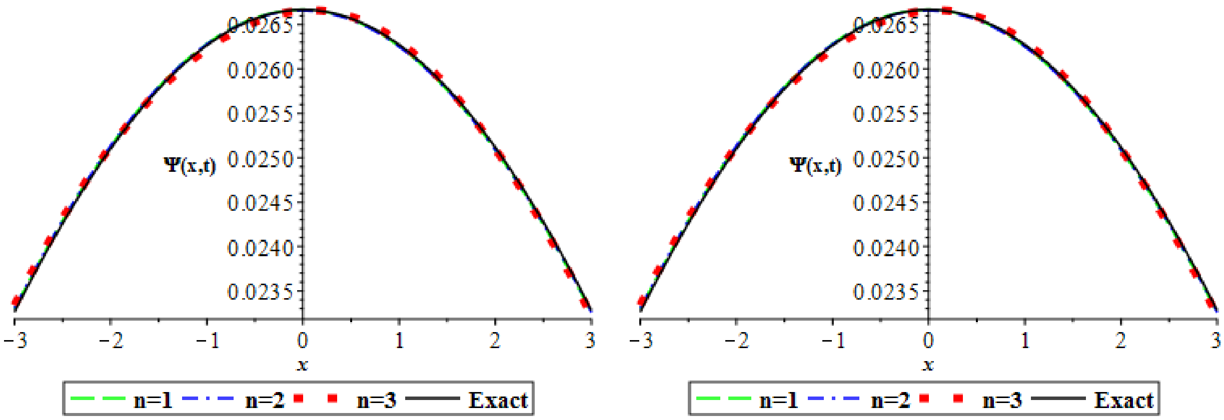


Fig.3. Solution plots of Eq. (3) at $a = 0.1$ and $t = 5$ when (a) $\alpha = 1$ (b) $\alpha = 0.5$ for different values of n.

7. Conclusion

In this research, approximate solutions of time-fractional seventh-order Sawada–Kotera–Ito equation is obtained with the help of an efficient method, namely FRDTM. Obtained outcomes are compared with the existing results at a particular value of $\alpha = 1$ and are found to be in precise agreement. The main benefit of applying this method is that it does not require any assumption, perturbation, and discretization for solving the governing time-fractional seventh-order SK–Ito equation. Also, computation time is less compared to other techniques. The computational outcomes assure that the present technique is very accurate, fast converges, and effective. It is also a very easily implementable mathematical tool for solving real-life problems arising in different areas of engineering and sciences.

Acknowledgment

The first-named author acknowledges the Department of Science and Technology of the Government of India for providing INSPIRE Fellowship (IF170207) in order to carry out the present research.

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