Implementation of different control strategies on a quadruple tank system

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Abstract— **This paper implements various control strategies on a quadruple tank system, it also discusses about interacting behavior of tank systems. The system considered is a quadruple tank system with flow rates of two tanks considered as manipulated variable and liquid level in the tanks are considered as controlled variable. Process transfer function is obtained through open loop response and controller parameters have been calculated using various tuning methods to establish a closed loop control system. Response of various controllers have been compared. IMC controller and Optimal PID control have been implemented**

Keywords—closed loop system, PI, PID, decoupler, Ziegler Nicholas, Cohen Coon, direct synthesis, IMC, Optimal PID, IMC

I. Introduction

The Control of liquid level in tanks and flow between tanks are the basic problems in process industries. In process industry the liquid is pumped and stored in a tank and then pumped to another tank. Many times the liquid will be processed by chemical or mixing treatment in the tanks [1]. The liquid should be processed such that the level of fluid in the tanks must be controlled and flow between the tanks must be regulated. It is essential to understand how the flow rate is manipulated and desired level is maintained. The tank level system is classified into two parts, i.e. interacting and non-interacting. The system is called as non-interacting system, if the level of tank 1 is affecting the level of tank 2 but level of tank 1 is independent of level of tank 2. If the level of tank 1 affects level of tank 2 and level of tank 2 affects level of tank 1, it is called an interacting system. The main causes of interaction between the input/output loops are due to the effect of input variables over the output variables in muti-input multi-output (MIMO) systems. Hence, it is more difficult to control the multivariable systems as compared to the single input single output (SISO) system. Therefore, the degree of interaction plays an important role to quantify the proper input/output pairings that minimizes the impact of the interaction.

II. EXPERIMENTAL SETUP

The experimental setup of the four tank system is shown in Figure 1. The experimental setup consists of four connected process tanks. The four tanks in the four tank system (FTS) are of Acrylic type. The height of the tanks is of 60 cm. A Vi Data Acquisition System-01 data acquisition (DAQ) card of Vi Microsystems is used to interface the FTS with PC. There are four differential pressure transmitters (DPT) to sense the levels of each tank. Two control valves are mounted on the mechanical rigid frame to control the flow rate of the water. Two current to pressure (I/P) converter are connected to convert the DAQ output (current) to pressure in the range of 3 - 15 psi. It also contains the storage tank which has the capacity of 75 liters. Centrifugal pumps are provided to circulate the water from the storage tank into the process tanks. Four rotameters are connected in the inlet of the process tanks to visualize the flow rate which is (10 - 100) liters per hour (LPH). Two current to pressure (I/P) converters are connected to convert the DAQ output (current) to pressure in the range of 3 - 15 psi. The two manipulated variables are the pump speeds. The two controlled variables are the levels of tanks 1 and 2.

Fig 1. Quadruple Tank System

The mathematical transfer function of interacting two tank system shown in Figure 2 is calculated using mass balances and Bernoulli's law.

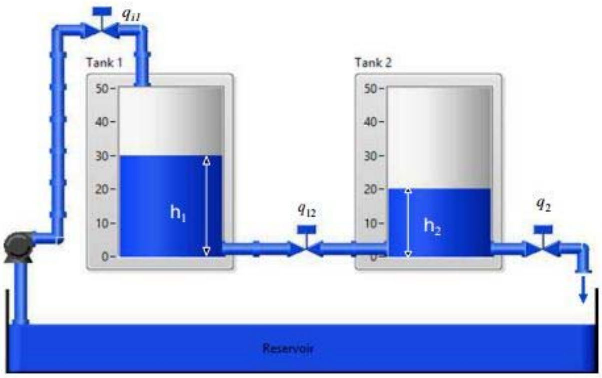

Fig 2 Interacting two tank system

In the Figure 2, q_{i1} is the inlet flow rate into the tank 1, q_{12} is the flow rate from tank 1 to tank 2 through the valve and q_2 is the flow rate at the outlet of tank 2 through a valve. The variables h_1 and h_2 represent liquid level of tank 1 and tank 2 respectively. The valve connecting the two tanks allows water to flow from tank 1 to tank 2 and the valve connected to the output of tank 2 allows water out to a reservoir. The control input to the process is the inlet flow rate and the level of liquid in tank 2 is the output of the process.

Applying mass balances we get Equ. 1 and 2,

$$\frac{dh_1(t)}{dt} = \frac{1}{A_1}\left(q_{i1}(t) - q_{12}(t)\right) \tag{1}$$

$$\frac{dh_2(t)}{dt} = \frac{1}{A_2}\left(q_{12}(t) - q_2(t)\right) \tag{2}$$

Applying Bernoulli's law we get Equ, 3 and 4,

$$q_{12}(t) = a_{12}\sqrt{2g(h_1(t) - h_2(t))} \tag{3}$$

$$q_2(t) = a_2\sqrt{2gh_2(t)} \tag{4}$$

Where A_1 and A_2 are cross sectional area of tank 1 and tank 2, a_{12} is the area of pipe connected between tank 1 and tank 2, a_2 is the area of cross section of the outlet pipe of tank 2 and g is gravitational constant. Substituting Eqn. 3 and 4 in Equ. 1 and 2 respectively, we get Equ. 5 and 6,

$$\frac{dh_1(t)}{dt} = \frac{q_{i1}(t)}{A_1} - \frac{a_{12}}{A_1}\sqrt{2g(h_1(t) - h_2(t))} \tag{5}$$

$$\frac{dh_2(t)}{dt} = \frac{a_{12}}{A_2}\sqrt{2g\left(h_1(t) - h_2(t)\right)} - \frac{a_2}{A_2}\sqrt{2gh_2(t)} \tag{6}$$

Applying linearization and calculating Laplace response we get

$$G_i(s) = \frac{M_1 M_2}{(sT_{i1}+1)(sT_{i2}+1) - M_2} \tag{7}$$

Where,

$T_{i1} = \frac{A_1}{C_{12}}$, $T_{i2} = \frac{A_1}{(C_{12}+C_2)}$, $M_1 = \frac{1}{C_{12}}$, $M_2 = \frac{C_{12}}{C_{12}+C_2}$,

$C_{12} = a_{11}\sqrt{g/2(h'_1 - h'_2)}$, $C_2 = a_2\sqrt{g/2h'_2}$

h'_1 and h'_2 are steady state value of level of fluid in tank 1 and tank 2.

Since FTS is a MIMO system with 2 inputs and 2 outputs, its system response is a square matrix of order 2 as shown in Equ. 8. The system model has been obtained using system identification toolbox in Matlab. Data acquisition has been done in labVIEW by providing a fixed flow rate as step input and plotting the level of tanks. The process transfer function is a matrix shown in Equ. 8.

$$\begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} = \begin{bmatrix} \frac{0.12}{25s+1} & \frac{0.2}{2547.77s^2+90.75s+1} \\ \frac{0.055}{259s^2+17.09s+1} & \frac{0.15}{120s+1} \end{bmatrix} \tag{8}$$

Where G_{11} is the transfer function when the input (manipulated) variable is the inlet flow rate of tank 1 and output (controlled) variable is liquid level of tank 1. G_{12} is the transfer function when the input (manipulated) variable is the inlet flow rate of tank 1 and output (controlled) variable is liquid level of tank 2. G_{21} is the transfer function when the input (manipulated) variable is the inlet flow rate of tank 2 and output (controlled) variable is liquid level of tank 1. G_{22} is the transfer function when the input (manipulated) variable is the inlet flow rate of tank 2 and output (controlled) variable is liquid level of tank 2.

III. Controller Design

A. *Proportional Integral Controller*

Since the system model is a first order transfer function, a simple controller which can be designed is a PI controller using direct synthesis approach [2]. Since the system is an interacting system, a decoupler [3] is required to reduce the interaction. Decoupler matrix is calculated using Equ. 9.

$$\begin{bmatrix} 1 & -G_{12}/G_{11} \\ -G_{21}/G_{22} & 1 \end{bmatrix} \tag{9}$$

Accordingly, the decoupler matrix is shown in Equ.10

$$\begin{bmatrix} 1 & \frac{-41.5s-1.66}{2547.77s^2+90.75s+1} \\ \frac{-44s-0.36}{259s^2+17.099s+1} & 1 \end{bmatrix} \tag{10}$$

The PI controller parameters for different τ ratios are shown in Table 1.

Table 1 PI controller parameters

τ ratio	G_{pi1}	G_{pi2}
1	8.31(1+1/25s)	6.66(1+1/120s)
2	4.16(1+1/25s)	3.33(1+1/120s)
3	2.77(1+1/25s)	2.22(1+1/120s)

The value of τ ratio to be used depends on the model confidence. For a model perfectly resembling the actual system, tau ratio can be taken as 1. The responses of system with PI control for different τ ratios are shown in Figure 3, 4 and 5.

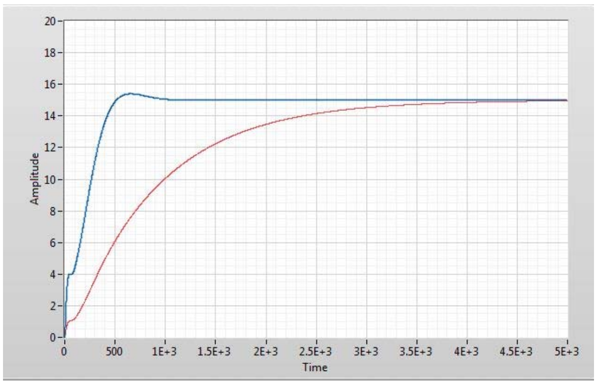

Fig 3. τ = 3

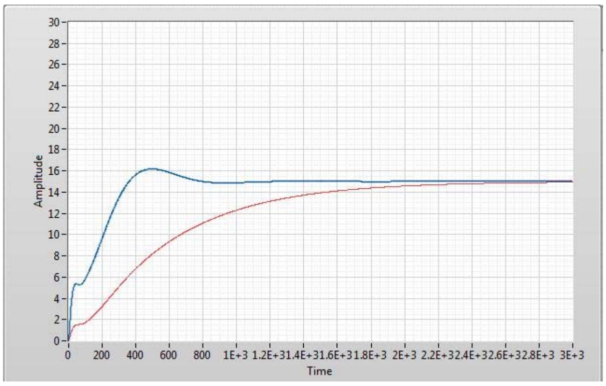

Fig 4. τ = 2

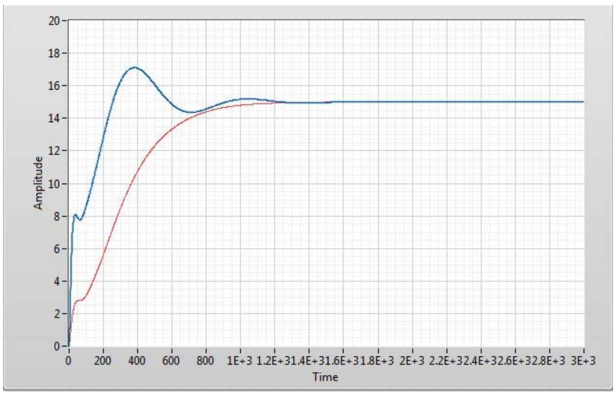

Fig 5. τ = 1

The responses shown in Figure 3, 4 and 5 consists of level of tank 1 and level of tank 2 in centimeters shown in blue and red curve respectively, plotted against time in seconds. The set point is fixed at 15 cm. As seen from the responses, the overshoot increases and settling time reduces with decrease in τ ratio. PI controller can also be designed using other approaches such as Ziegler Nicholas and Cohen coon tuning methods.

B. PID controller design

To reduce settling time and overshoot other tuning techniques are applied. To apply those techniques, the process model transfer function is converted from first order to first order plus dead time. The resulting first order plus dead time(FOPDT) transfer function is shown in Equ. 11.

$$\begin{bmatrix} \frac{0.124e^{-10.1s}}{15.15s+1} & \frac{0.2}{2547.77s^2+90.75s+1} \\ \frac{0.055}{259s^2+17.09s+1} & \frac{0.15e^{-44.75s}}{67.12s+1} \end{bmatrix} \quad (11)$$

Since the system is an interacting system, a decoupler is required to cancel out the effect of interaction between loops. Decoupler matrix is calculated using Equ. 9.
Accordingly, the decoupler matrix is shown in Equ. 12 after applying taylor's expansion,

$$\begin{bmatrix} 1 & \frac{-246.63s^2-40.703s-1.612}{2547.77s^2+90.75s+1} \\ \frac{-1081.30s^2-40.273s-0.36}{259s^2+17.099s+1} & 1 \end{bmatrix} \quad (12)$$

PID controller parameters for above FOPDT process [4] with decoupler are calculated using different tuning methods, Set point is fixed at 15 cm.

The PID parameters for Ziegler Nicholas tuning method are shown in Table 2.

Table 2. PID controller parameters (Ziegler Nicholas)

	K_P	τ_I	τ_D
G_{C1}	0.266	10	2.5
G_{C2}	0.342	18	4.5

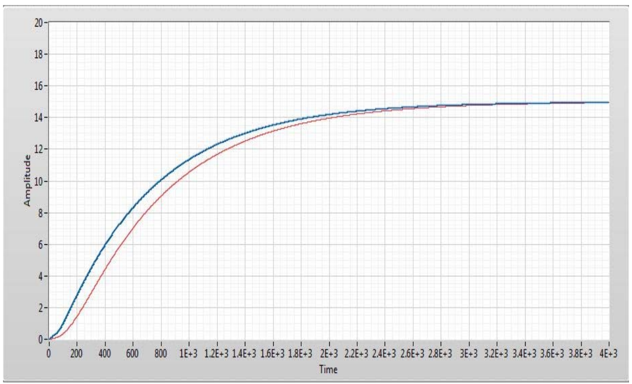

Fig 6. Ziegler Nicholas PID tuning response

the response shown in Figure 6 consist of level of tank 1 and level of tank 2 in centimeters shown in blue and red curve respectively, plotted against time in seconds. The set point is fixed at 15 cm. As seen from the response in Figure 6, there is no overshoot but settling time is very high. PID parameters calculated according to Cohen coon [5] tuning methodology are shown in Table 3.

Table 3. Cohen Coon PID tuning parameters

	K_P	τ_I	τ_D
G_{C1}	18.43	20.3	3.316
G_{C2}	15.12	90.13	14.69

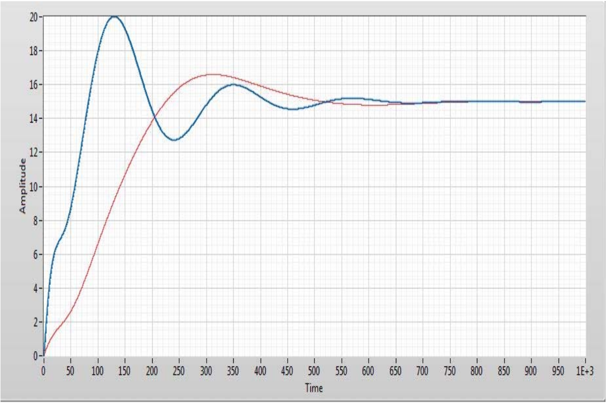

Fig 7. Cohen Coon PID tuning response

the response shown in figure 7 consist of level of tank 1 and level of tank 2 in centimeters shown in blue and red curve respectively, plotted against time in seconds. The set point is fixed at 15 cm. From the response in figure 7, it is observed that settling time is less in comparison to Ziegler Nicholas but there is high overshoot.

PID parameters calculated according to Wang Juang Chan tuning methodology [6] are shown in Table 4.

Table 4. PID tuning parameters (Wang Juang Chan)

	K_P	τ_I	τ_D
G_{C1}	9.838	20.2	3.785
G_{C2}	8.132	89.49	16.78

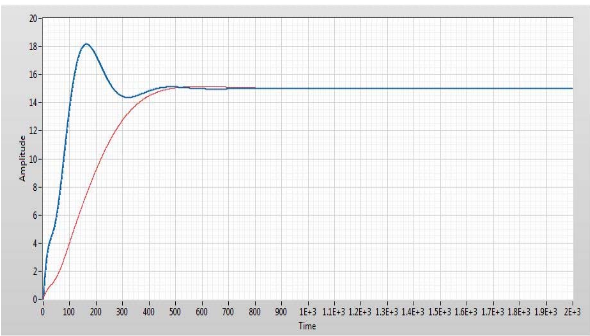

Fig 8. Wang Juang Chan PID tuning response

The response shown in figure 8 consist of level of tank 1 and level of tank 2 in centimeters shown in blue and red curve respectively, plotted against time in seconds. The set point is fixed at 15 cm. From the response in figure 8, it is observed that settling time is greater in comparison to Cohen coon and less in comparison to Ziegler Nicholas and overshoot is less in comparison to Cohen Coon method.

PID parameters calculated according to different error criterions such as ISE, ISTE, and IST^{2}E for optimum control are shown in Table 5.

Table 5. Optimum PID control

Controller	K_P	τ_I	τ_D
ISE G_{C1}	12.15	15.95	5.16
ISE G_{C2}	10.05	70.6	22.8
ISTE G_{C1}	12.08	18.28	4.03
ISTE G_{C2}	9.99	81.03	17.89
IST^{2}E G_{C1}	11.2	18.74	3.33
IST^{2}E G_{C2}	9.30	83.03	14.77

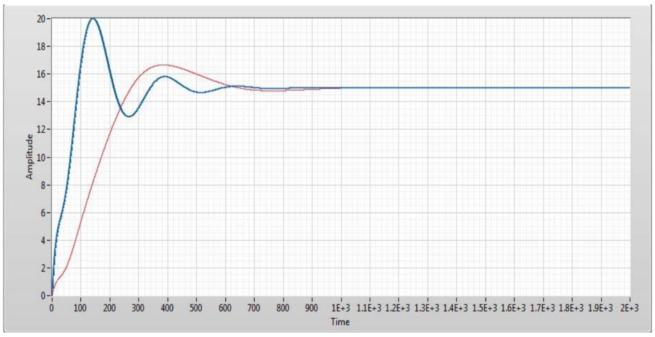

Fig 9. ISE criterion PID controller

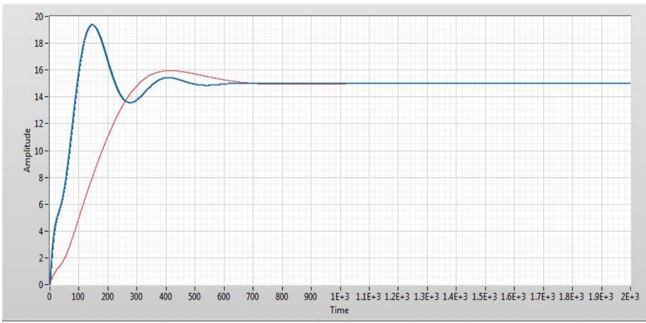

Fig 10. ISTE criterion PID controller

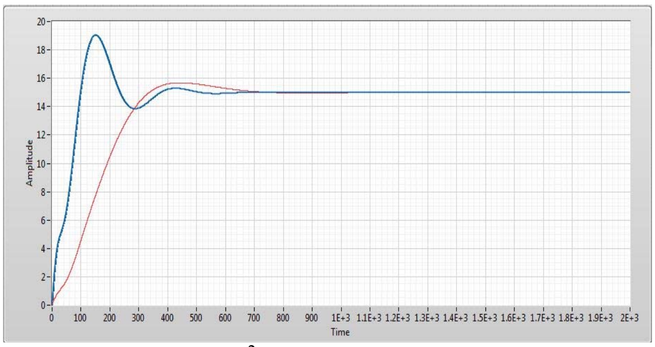

Fig 11. IST^{2}E criterion PID controller

The response shown in figure 9, 10 and 11 consist of level of tank 1 and level of tank 2 in centimeters shown in blue and red curve respectively, plotted against time in seconds. The set point is fixed at 15 cm. The performance of optimal PID control varies according to the error criterion used to optimize the controller. The response of optimized PID controller can be observed from Figure 9, 10 and 11. It can be observed that overshoot decreases for ISTE when compared to ISE and further reduces for IST^{2}E when compared to ISTE and ISE.

C. IMC controller

The block diagram of IMC controller is shown in figure 12. In IMC controller we have a stable internal model controller *Q(s)* and *G'(s)* is the model of the plant. *F(s)* is internal model controller filter selected to make *Q(s)* proper. *F(s)* proper by selecting an appropriate λ parameter value [7]. λ is taken as a value greater than *0.2T*. The Equivalent feedback controller is shown in Equ. 13

$$C(s) = \frac{(1+Ts)\left(1+\frac{L}{2}\right)}{Ks(L+\lambda)} \quad (13)$$

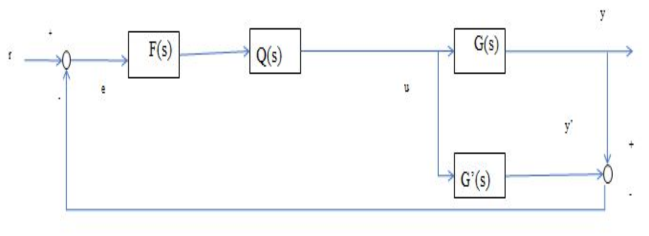

Fig 12. IMC controller

PID parameters for IMC controllers are shown in Table 6.

Table 6. IMC controller parameters

	K_P	τ_I	τ_D
G_{C1}	12.34	20.2	3.76
G_{C2}	10.25	89.48	16.78

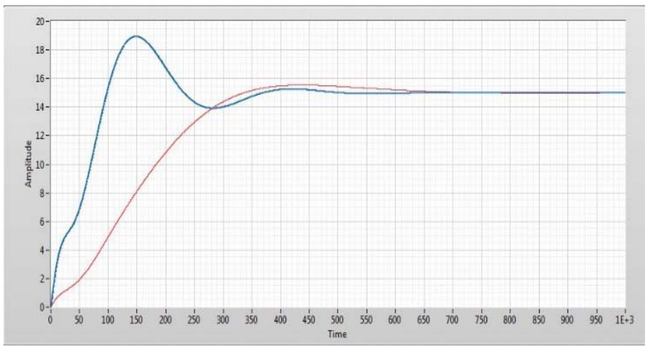

Fig 13. IMC controller response

The response shown in figure 13 consist of level of tank 1 and level of tank 2 in centimeters shown in blue and red curve respectively, plotted against time in seconds. The set point is fixed at 15 cm. From the response in figure 13, it is observed that IMC controller gives slightly less overshoot than optimal PID controller but higher overshoot in comparison to Wang Juan Chan method. The settling time of IMC controller is less than optimal PID controller and comparable to Wang Juang Chan method.

IV. Results and Discussion

The characteristics of controllers G_{c1} and G_{c2} are compared in Table 7. Different characteristics which are compared are rise time (t_r), peak time (t_p), settling time (t_s) in seconds and maximum overshoot (M_P) in percentage. For a first order system, PI controller gives high overshoot with increase in Tau ratio and requires more settling time. When converted to FOPDT system, different tuning methodologies give different results. Ziegler Nicholas offers the advantage of no overshoot but takes more settling time, Cohen Coon tuning approach gives fast response but also produces overshoot, Chan method produces less overshoot and also gives quick response. PID controllers provide quicker response but with higher overshoot in comparison to PI controllers as expected. IMC controller performance is better than optimal PID controller but less impressive in comparison to the PID controller with Wang Juang Chan method of tuning.

Table 7. Comparison of various methodologies

Controller	t_r(sec)		t_p(sec)		t_s(sec)		M_p(%)	
	G_{c1}	G_{c2}	G_{c1}	G_{c2}	G_{c1}	G_{c2}	G_{c1}	G_{c2}
PI(τ = 3)	384	1886	652	-	438	2600	2.66	-
PI(τ = 2)	294	1264	488	-	619	1690	7.86	-
PI(τ = 1)	211	605	380	-	523	758	14.06	-
PID(ZN)	1575	1685	-	-	2040	2220	-	-
PID(CC)	71.3	170	128	306	376	416	33.5	10.6
PID(Chan)	92.6	284	163	593	240	379	21.1	0.93
PID (ISE)	77.7	201	141	385	405	530	33.6	11.0
PID(ISTE)	81.3	216	145	413	321	492	29.2	6.40
PID(IST^{2}E)	84.2	229	150	443	327	298	27.0	4.53
IMC	83.6	228	148	437	318	294	26.33	3.6

V. Conclusion

After comparing the characteristics, it can be concluded that choice of tuning procedure depends on the preferred performance criterion. If less overshoot is the desired performance criterion with no constraint on settling time, PI controllers are an effective choice since PI controllers are easy to manufacture and give less overshoot. PID controller is useful when settling time is the performance criterion. PID controller with Wang Juang Chan tuning method gives a good combination of less overshoot and less settling time among the PID controllers.

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