A Level Set Approach for Dynamics of Flow in Accelerating Rectangular Container

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Abstract. The level set approach is a numerical technique to capture the interface for a system of two immiscible fluids separated by a sharp interface. Since the interface is captured by an implicit function using this approach, this is a Eulerian formulation of the evolution of interface. The different fluids must be identified by using a marker function, called the level set function that takes different values in the various fluids. The present work uses a level set approach for modeling the dynamics of flow and the shape of the liquid free surface in a partly filled rectangular tank which is subjected to an impulsive motion from rest. The governing equations are written about a non-inertial frame attached to the container in motion. Extremely high viscosity and density ratios of the two immiscible fluids make the problem more challenging. The surface tension forces are ignored in this study. The signed distance property of the level set function is retained by using the reinitialization algorithm. In the present study, we have compared the computed shapes of equilibrium free surface with theoretically predicted shapes at equilibrium.
Keywords: level set method; implicit function; signed distance property; reinitialization

1. Introduction

A partially filled fluid tank undergoes the phenomenon of sloshing when subjected to an external perturbation. Studies related to sloshing waves have been carried out for the past several decades. A comprehensive literature review on this topic can be found in the research literature (Ibrahim et al. [1]; Ibrahim [2]; Lin [3]). Graham and Rodriguez [4] and Lewison [5] studied mechanical models of the slosh dynamics. Von Kerczek [6] analyzed some preliminary numerical models of the Rayleigh-Taylor instability, which is a typical sloshing problem.

Faltinsen [7] elucidated the potential flow problem in liquid sloshing. He solved it with intricate care of boundary conditions on the free surface. Lee [8] investigated on Glimm's method [9] as applied to water sloshing and impacting is carried out numerically. Frandsen [10] proposed an entirely non-linear finite difference technique taking an ideal fluid flow in a two-dimensional container into consideration. Celebi and Akyildiz [11] used the VoF (volume of fluid) approach for free surface tracking by the use of the finite difference approximation. Sames et al. [12] used a commercially available VoF approach to cylindrical and rectangular containers. Shao et al. [13] used a refined Smoothed-particle hydrodynamics (SPH) approach for solving sloshing problems.

2. Mathematical modelling

This problem consists of an upright rectangular container containing two fluids of different densities. The container is impetuously made to move from rest with a uniform linear acceleration along the horizontal direction, and the motion of the interface is recorded till a steady state configuration is reached. The interface will evolve transiently, and at steady state, it assumes a flat surface whose slope depends on the relative magnitude of gravitational acceleration and the imposed acceleration in the horizontal direction. The problem is assumed to be two-dimensional at the beginning stage to avoid unnecessary complexity in the problem. The problem is shown in schematic form in figure 1.

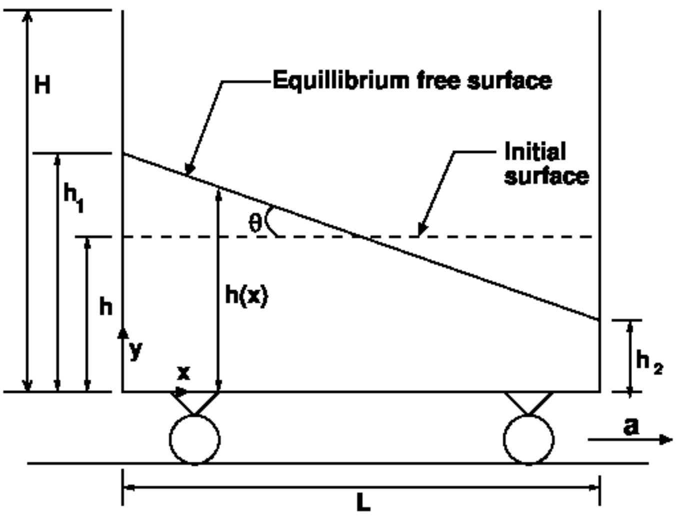

Figure 1: Schematic diagram: Rectangular tank under constant linear acceleration.

2.1. Some Important dimensionless parameters

The pertinent dimensionless numbers of the problem under consideration are the Reynolds number (Re), the Froude number (Fr), and the Acceleration number (Ac_x). Here, the characteristic length is the breadth of the tank (L). The characteristic velocity and the characteristic time can be defined as $U = \sqrt{gL}$ (and $t = L/g$ respectively. Hence, we have $Re = \rho^2 gL^3/\mu^2$, $Fr = 1$, and $Ac_x = g/a_x$, where ρ is the density of fluid-1, μ is the viscosity of fluid-1, g is the gravitational acceleration, and a_x is the linear acceleration of the tank in the x-direction.

2.2. Governing equation

The governing differential equations in non-dimensional form with reference to a non-inertial frame attached to the moving container are given by,

$$\nabla \cdot \mathbf{u} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} = -(\mathbf{u} \cdot \nabla)u - \frac{\nabla p}{\rho_\epsilon} + \left(\frac{1}{Re}\right)\left(\frac{1}{\rho_\epsilon}\right)\nabla.[(\nabla u + (\nabla u)^T)] - \frac{1}{Ac_x} \tag{2}$$

$$\frac{\partial v}{\partial t} = -(\mathbf{v} \cdot \nabla)v - \frac{\nabla p}{\rho_\epsilon} + \left(\frac{1}{Re}\right)\left(\frac{1}{\rho_\epsilon}\right)\nabla.[(\nabla u + (\nabla u)^T)] - \frac{1}{Fr} \tag{3}$$

$$\frac{\partial \phi}{\partial t} + \mathbf{u} \cdot \nabla \phi = 0 \tag{4}$$

2.3. Partial slip boundary condition

The meeting point of the solid surface and liquid interface is called a contact point (contact line in three-dimension). Ideally, classical no-slip boundary condition fails at the contact point because a finite force is needed to cause the contact point to move as shown by Huh and Scriven (1971) [14]. For moving contact line problems slip occurs in the neighborhood of the contact line. Chung (2002) [15] introduced the normal derivative of velocity with slip is expressed as, $\left(\frac{\partial u}{\partial n}\right)_{slip} = (1-\eta)\left(\frac{\partial u}{\partial n}\right)_{no\ slip}$, where η is a slip coefficient. The range of η is 0 to 1. A value of $\eta = 0$ indicates no-slip and 1 indicates free-slip. The partial-slip can be imposed by taking the value of η between 0 and 1. The value of η has been taken as 0.8.

3. Numerical method

The governing partial differential equations are discretized with the finite volume discretization technique by using a staggered grid. The Navier-Stokes equation is solved by using the Chorin's projection method. The projection method is at least second order accurate in time. For discretization of the convective terms, second-order accurate ENO (essentially non-oscillatory) scheme is used. And for diffusive terms, central difference scheme is used. The pressure equation has been solved using a preconditioned conjugate gradient (PCG) method which uses incomplete Cholesky preconditioner.

3.1. Level set equation

The level set equation, $(\frac{\partial \phi}{\partial t} + \mathbf{u} \cdot \boldsymbol{\nabla}\phi = 0$ is a hyperbolic equation of Hamilton-Jacobi type. Its solution is based on well-established high-resolution schemes for hyperbolic conservation laws, such as ENO scheme proposed by Hartel *et al*. (1987) [16] or WENO (weighted ENO) scheme. A smooth scalar function called the level set fuction ϕ is defined in the domain Ω.

3.2. Level set representation of fluid properties

The level set function ϕ in the domain Ω is given by,

$$\phi(x,t) < 0 \ for\ x \in \Omega^-; \qquad \phi(x,t) > 0 \ for\ x \in \Omega^+; \qquad \phi(x,t) = 0 \ for\ x \in \Gamma \tag{5}$$

With each fluid, the fluid property is assumed to be constant. The region Ω^- is occupied by fluid 1 and the region Ω^+ is occupied by fluid 2. Thus, If ρ_1 and ρ_2 are constant densities in Ω^- and Ω^+ respectively, and μ_1 and μ_2 are the constant viscosities in Ω^- and Ω^+respectively, the effective density, and viscosity in Ω can be given by

$$\rho_\epsilon(\phi) = \rho_1 + (\rho_2 - \rho_1)H_\epsilon(\phi) \tag{6}$$

$$\mu_\epsilon(\phi) = \mu_1 + (\mu_2 - \mu_1)H_\epsilon(\phi) \tag{7}$$

where, $H_\epsilon(\phi)$ is the smeared-out Heaviside function and can be represented as follows:

$$H_\epsilon(\phi) = \begin{cases} 0, & if\ \phi < -\varepsilon \\ \dfrac{\phi+\varepsilon}{2\varepsilon} + \dfrac{1}{2\pi}\sin\left(\dfrac{\pi\phi}{\varepsilon}\right), & if\ |\phi| \le \varepsilon \\ 1, & if\ \phi > \varepsilon \end{cases} \tag{8}$$

Smoothing of the density and viscosity at the interface has been carried out, to prevent undesirable instabilities.

3.3. Construction of initial level set function

The level set function ϕ should be initialized at time $t = 0$, by defining ϕ at each grid point in the computational domain. For this purpose, a variety of functions satisfying the condition of zero level set at the interface can be employed. Depending upon the nature of the interface and its interaction with the boundaries one can choose a vertical distance function, a horizontal distance function, a normal distance function or some special kind of distance function such as minimum distance function.

	Free surface height at left wall (h_1)	% error	Free surface height at right wall(h_2)	% error	Angle of inclination, θ (in degree)	% error
$Ac_x = 4$						
Analytical	0.445		0.195		-14.036	
80 × 64	0.468	5.17	0.218	11.79	-14.01	0.18
120 × 96	0.452	1.57	0.201	3.08	-14.106	0.49
160 × 128	0.446	0.22	0.194	0.51	-14.139	0.73
$Ac_x = 2$						
Analytical	0.570		0.070		-26.565	
80 × 64	0.602	5.61	0.102	45.71	-26.560	0.02
120 × 96	0.580	1.75	0.080	14.28	-26.540	0.09
160 × 128	0.573	0.52	0.074	5.71	-26.522	0.16

Table 1: Equilibrium free surface-rectangular tank: Summary of result.

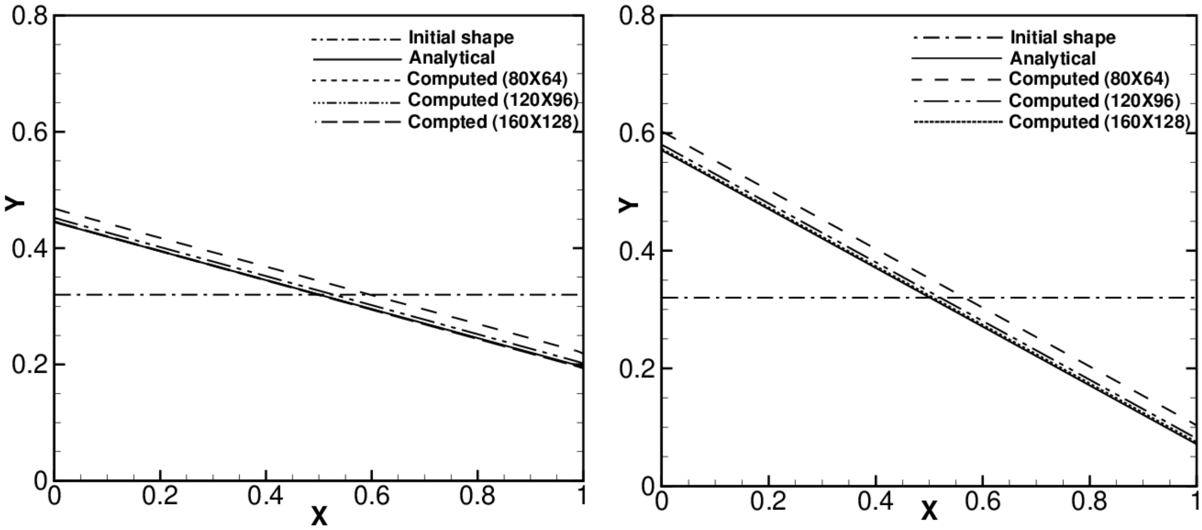

Figure 2: Exact and computed equilibrium free surface shapes: for $Ac_x = 4$ (left) and $Ac_x = 2$ (right).

3.4. *Reinitialization of level set function*

The smoothing of fluid properties will work only when the level set function continues to exist a distance function. Unfortunately, for flows that encounter substantial topological changes, the level set function immediately stops to act a distance function as noted by Sussman *et al.* [17]. If the process of reinitialization is not carried out, then the magnitude of the gradient of the level set function, $|\nabla\phi|$, turns out to be extremely small or large nearby the zero level set of ϕ. Chopp [18] introduced the idea that the level set function should be reinitialized periodically throughout the computation to minimize numerical errors. Reinitialization algorithms maintain the signed distance property of the level set function.

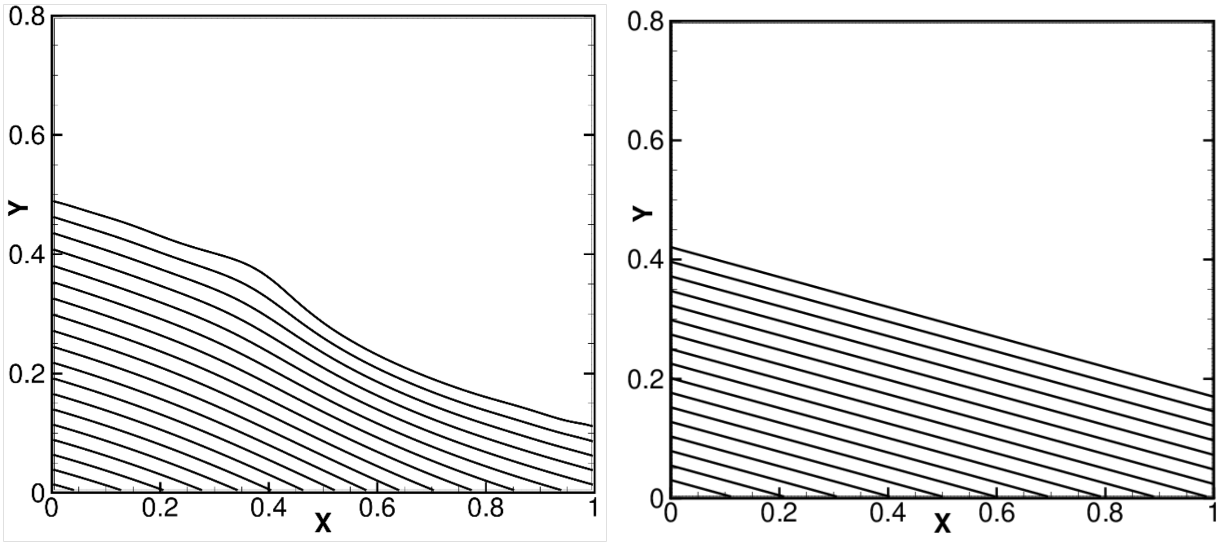

Figure 3: Pressure contours: Comparison between intermediate and steady state result.

4. Results and Discussion

The computations were carried out by employing the following input data. The rectangular tank is having a breadth of $L = 1.0$ and a height of $H = 0.8$. The two immiscible fluids are taken as water and air. For the two fluid system, the density ratio λ is taken as 820 and the viscosity ratio η as 55.5. The acceleration due to gravity $g = 9.81$. The initial undisturbed free surface height of water h is 0.32. Reinitialization is done at every alternate time step. On all the walls, the no slip condition is implemented. The condition of no-slip or Navier slip is applied at the contact points (the meeting point of the walls and the interface). The Reynolds number, $Re = 313.2$, and the Acceleration number, $Ac_x = 4,\ 2$ are used for the computation. Different sets of grids; namely, 80×64, 120×96, 160×128 are chosen to perform the computations. The non-dimensional Δt has been taken as 0.001. The calculations were permitted to continue till a non-dimensional time, $t = 100$ which needs 1.0×10^5 time steps and $t = 200$ which requires 2.0×10^5. The results obtained are summarized in table 1.

Figure 2 exhibits the equilibrium shape of the free surface, computed on different grids, along with the theoretical shape. The dimensionless form of the theoretical shape is calculated using, $y = h - \frac{Fr}{Ac_x}\left(x - \frac{1}{2}\right)$. The exact shape of the interface is a straight line with an inclination, $\theta = \tan^{-1}(Fr/Ac_x)$with respect to the x-axis. Both the analytical and computed values are shown in table

1 for comparison. In spite of the longtime computation and high density and viscosity ratios, the computed surface shape is seen to agree extremely well with the exact solution given by the equation. This demonstrates that accurate steady-state solution can be achieved with the level set formulation.

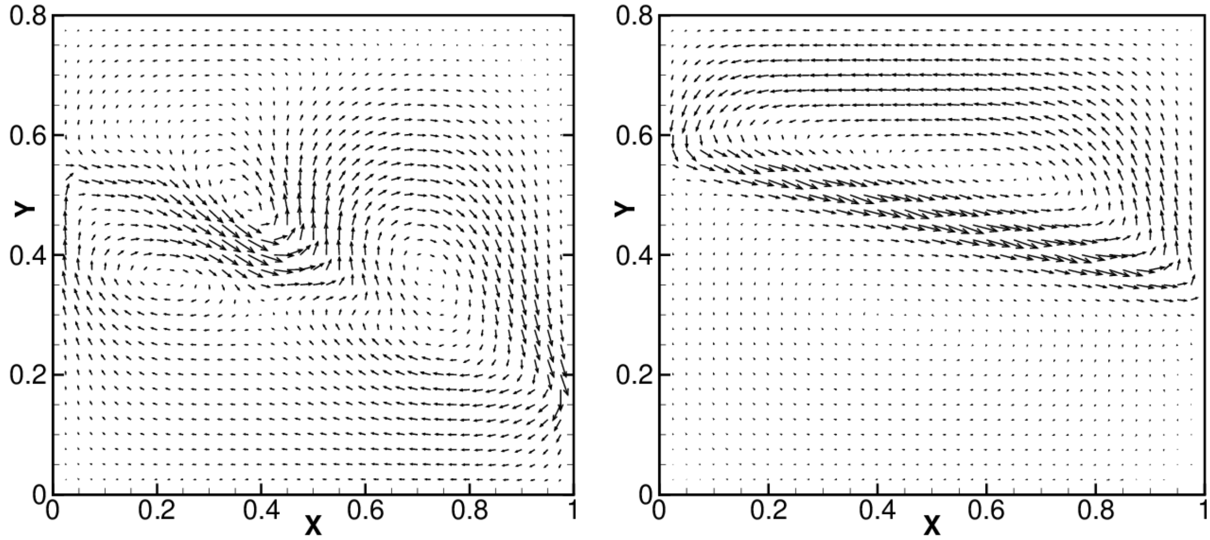

Figure 4: Vector fields: Comparison between intermediate and steady state result.

Figure 3 shows the intermediate velocity field for $Ac_x = 4$ at time $t = 1, 2, 3$. From these plots, it is clear that the impulsive acceleration of the tank gives rise to sloshing of liquid in the tank as expected. The amplitude of sloshing quickly gets damped because of the relatively low Reynolds number for which simulation has been performed. Figure 4 shows the comparison between intermediate and steady state result representing the vector field. Figure 5 shows transient behavior of height of the free surface at both left and right walls of the container. Figure 6 shows the local velocity at a particular point in the domain with respect time.

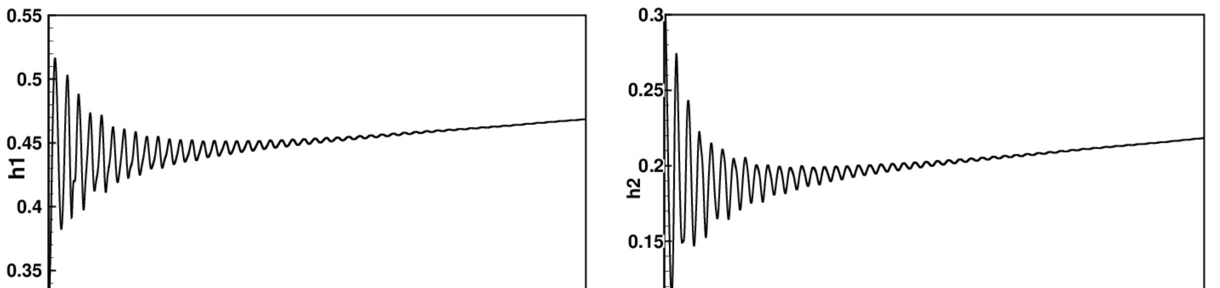

Figure 5: Free surface height at the wall verses Time.

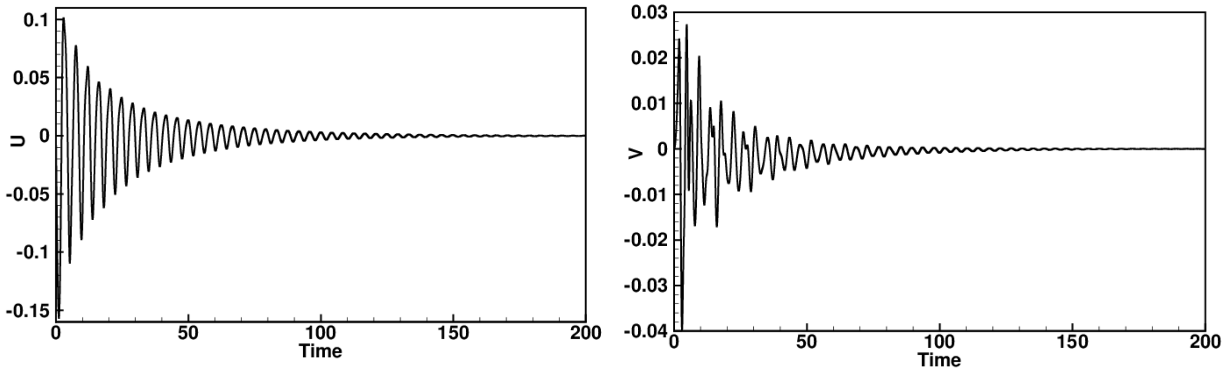

Figure 6: Variation of velocity with time.

5. Conclusion

The liquid sloshing of a two-dimensional tank under constant linear acceleration has been studied numerically using the level set approach for tracking free surface. In contrast to other interface capturing or tracking methods the level set method does not demand any special care to describe the free surface here. The numerical results match extremely well with the analytical results. The percentage errors in computing free surface height at left (h_1) and right (h_2) walls and angle of inclination (θ) is calculated for different grid numbers. It is observed that the percentage error remains within 1% of analytical value with fine grid arrangement (160×128 grids), except that of h_2 for Acceleration number 2 as the value of h_2 is very small (0.070) in this case.

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