In-Network Distributed Identification of Wiener and Volterra-Laguerre Models for Nonlinear Systems

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Objective

- To identify nonlinear systems using Wiener and Volterra-Laguerre models in a distributed recursive manner.

Introduction

- Generally, all the real-time systems have nonlinear nature hence nonlinear modeling is preferred .
- Data-based system modeling is a key issue for a many engineering applications such as pH-neutralization, two tank system control.
- Higher order Volterra kernels can represent these systems but with high parameter complexity.
- So block-structured models are employed but they can model some specific nonlinearities.
- Expanding the nonlinear Volterra kernels with orthogonal Laguerre functions can relegate the above limitations [1].
- A distributed alternating direction method of multipliers (ADMM) based recursive algorithm for the identification of above-mentioned nonlinear models is designed.

Traditional Wiener Model

$$G(q) = \sum_{i=1}^{n_\alpha} \alpha_i g_i(q), \tag{1}$$

where α_i are the parameters to be estimated and $g_i(q)$ $(i = 1, ..., n_\alpha)$ are the known basis functions, can take any of the generalized basis functions.

$$d(t) = G(q)\,a(t) + v(t) = \sum_{i=1}^{n_\alpha} \alpha_i g_i(q) a(t) + v(t). \tag{2}$$

Assuming nonlinearity is invertible,

$$d(t) = F^{-1}(\gamma(t)) = \sum_{j=1}^{n_\beta} \beta_j f_j(\gamma(t)), \tag{3}$$

where $\beta_j \in \mathbb{R}\,(j = 1, ...n_\beta)$ are the unknown parameters associated to the nonlinear basis functions $f_j(\cdot) : \mathbb{R} \to \mathbb{R}\,(j = 1, ...n_\beta)$,

$$\gamma(t) = \bar{s}^T \bar{\zeta}(t) + v(t), \tag{4}$$

where $\bar{s} = \left[\alpha_1, ...\alpha_{n_\alpha}, \beta_2, ...\beta_{n_\beta}\right]^T$, $\bar{\zeta}(t) = [(g_1(q)\,a(t))^T, ..., (g_{n_\alpha}(q)\,a(t))^T, -f_2^T(\gamma(t)), ..., -f_{n_\beta}^T(\gamma(t))]^T$.

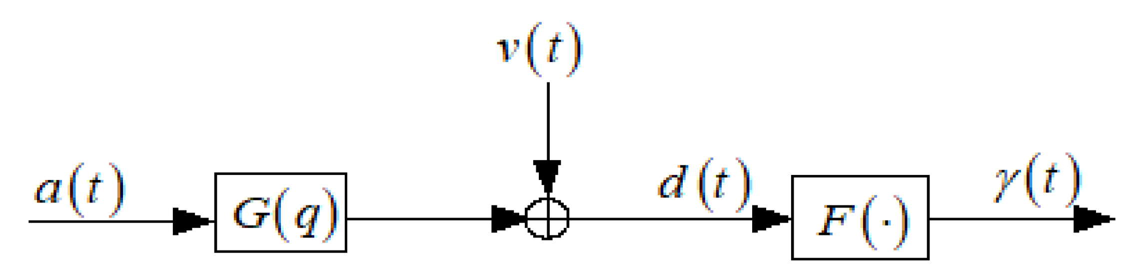


Figure: Wiener nonlinear system

Traditional Volterra-Laguerre (V-L) Model

Consider a fading memory causal nonlinear system

$$\gamma(t) = F_d\{a(\tau)\} + v(t), \tag{5}$$

The finite order R, discrete-time Volterra model with fading memory M can be given as [2]

$$\gamma(t) = \sum_{n=1}^{R} \sum_{\tau_1=0}^{M-1} \cdots \sum_{\tau_n=0}^{M-1} h_n(\tau_1, \ldots, \tau_n) \prod_{i=1}^{n} a(t-\tau_i) + v(t), \tag{6}$$

The n^{th}-order Volterra kernel h_n can be approximated using r-dimensional Laguerre function as

$$h_n(\cdot) = \sum_{k_1=1}^{r} \cdots \sum_{k_n=1}^{r} L^{(n)}_{k_1 \ldots k_n} \prod_{i=1}^{n} \phi_{k_i}(\tau_i), \tag{7}$$

$$\gamma(t) = \bar{S}^T \bar{\Phi}(t) + v(t),$$

where

$$\bar{S} = \left[\bar{S}^{(1)}, ..., \bar{S}^{(R)}\right]^T \in \mathbb{R}^{(r+\ldots+r^R)\times 1}, \tag{8}$$

$$\bar{\Phi}(t) = \left[\bar{\Phi}^{(1)}(t), ..., \bar{\Phi}^{(R)}(t)\right]^T \in \mathbb{R}^{(r+\ldots+r^R)}, \tag{9}$$

with

$$\bar{S}^{(n)} = \left[L^{(n)}_{1\ldots 1} \cdots L^{(n)}_{r\ldots r}\right] \in \mathbb{R}^{1\times r^n},\; l^{(n)}_{k_1\ldots k_n}(t) = \prod_{i=1}^{n} l_{k_i}(t).$$

$$\bar{\Phi}^{(n)}(t) = \left[l^{(n)}_{1\ldots 1}(t) \cdots l^{(n)}_{r\ldots r}(t)\right] \in \mathbb{R}^{1\times r^n}$$

The objective is to estimate the parameter vector $\bar{s}$ and $\bar{S}$ in a distributed manner.

Distributed V-L Modeling and Distributed Wiener modeling

- Consider an ad-hoc WSN with P number of spatially dispersed sensors.
- At any time, node j measures the output $\gamma_j(t)$ corresponding to input $\{a_j(\tau) | \tau = (t - M + 1), .., t\}$.
- The scalar measurements of all the nodes are stacked into a global vector $\bar{\Gamma}(t) = [\gamma_1(t), ..., \gamma_P(t)]^T \in \mathbb{R}^{P\times 1}$ with their corresponding regressors stacked in global matrix $\boldsymbol{U}(t) = \left[\bar{\Phi}_1(t), ..., \bar{\Phi}_P(t)\right] \in \mathbb{R}^{(r+\ldots+r^R)\times P}$.
- Then estimate the vector $\bar{S}$ by minimization of

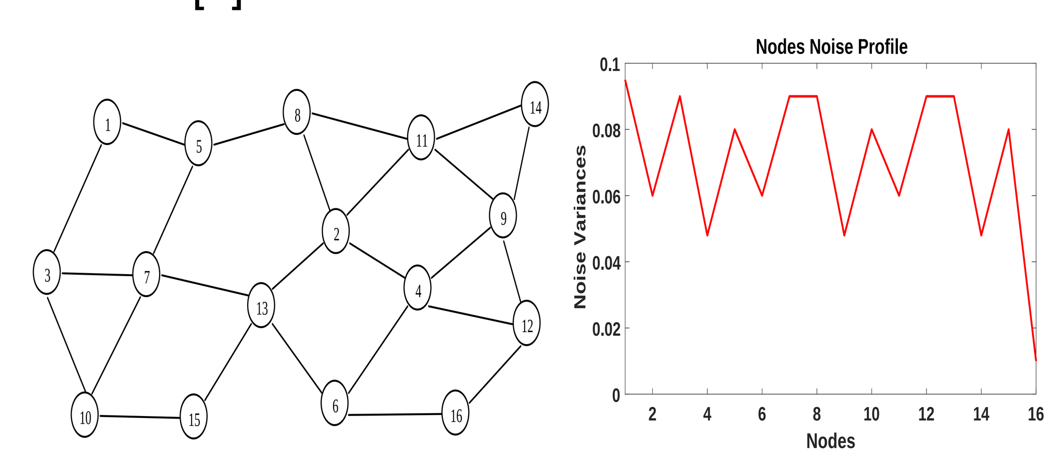
$$\hat{\bar{S}} = \arg\min_{\bar{S}} E\left\|\bar{\Gamma}(t) - \boldsymbol{U}^T(t)\,\bar{S}\right\|^2, \tag{10}$$

- To facilitate the distributed estimation of $\bar{S}$, auxiliary variables $\left\{\bar{S}_j\right\}_{j=1}^{P}$ are introduced to represent the local estimates at each nodes.
- The optimization problem in (10) can be re-expressed as

$$\left\{\hat{\bar{S}}_j(t)\right\}_{j=1}^{P} = \arg\min_{\{\bar{S}_j\}_{j=1}^{P}} \sum_{m=0}^{t} \sum_{j=1}^{P} \lambda^{t-m}\left[\gamma_j(m) - \bar{\Phi}_j^T(m)\bar{S}_j\right]^2 + P^{-1}\lambda^t \sum_{j=1}^{P} \bar{S}_j^T \boldsymbol{\Psi}_0 \bar{S}_j,$$

$$\text{s.t. } \bar{S}_j = \bar{S}_{j'},\; j \in [1, .., P],\; j' \in \mathcal{N}_j, \tag{11}$$

where λ is the forgetting factor and $\boldsymbol{\Psi}_0$ is the positive definite matrix used for regularization.
- ADMM is employed to optimize (11) in a distributed fashion [3].



(a) Sixteen node ad-hoc WSN (b) Noise-variance at nodes

Conclusion

A distributed identification of Volterra-Laguerre model and Wiener model is designed.

- More responsive and robust performance.
- Simulations are plotted under noisy environment.
- Results are compared with the non-cooperative estimation.

References

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Simulation Results

- Consider an infinite-order, finite length $(M = 5)$ Wiener type nonlinear dynamical system

$$d(t) = 2.5a(t) + 2a(t-1) + 0.5a(t-2) + 0.1a(t-3) + 0.05a(t-4)$$
$$\gamma(t) = \frac{1}{1+e^{-d(t)}}. \tag{12}$$

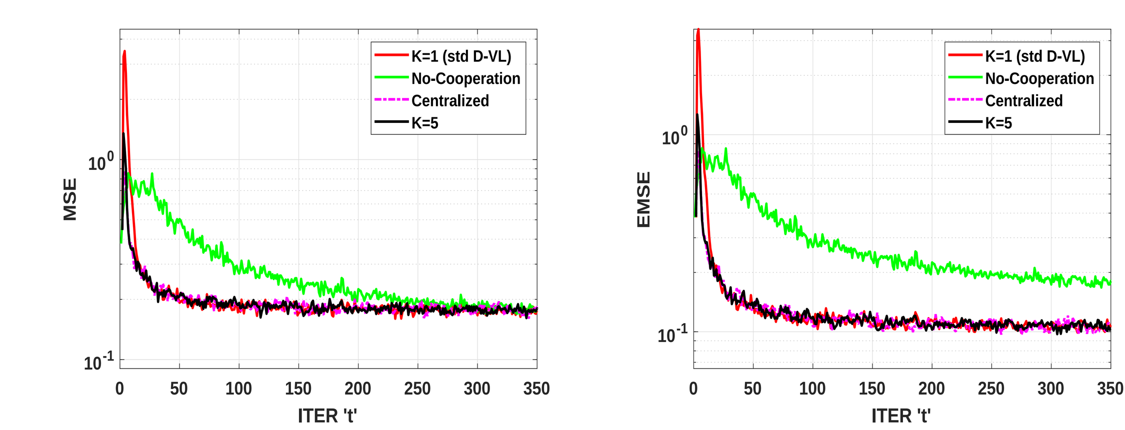

Figure: Network performance of the proposed distributed Volterra-Laguerre model for the nonlinear system (12).

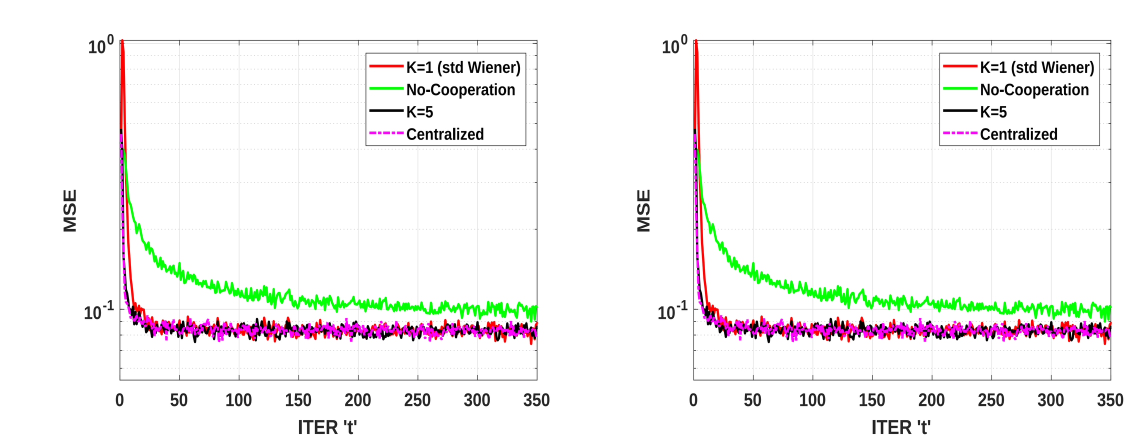

Figure: Network performance of the proposed distributed Wiener model for the nonlinear system (12).

- Next, let us consider a 2^{nd}-order nonlinear system for which Wiener model does not exists,

$$d(t) = 2.5a(t) + 2a(t-1)\,0.5a(t-2) + + 0.1a(t-3) + 0.05a(t-4)$$
$$\gamma(t) = 10d(t) - 2(d(t))^2. \tag{13}$$

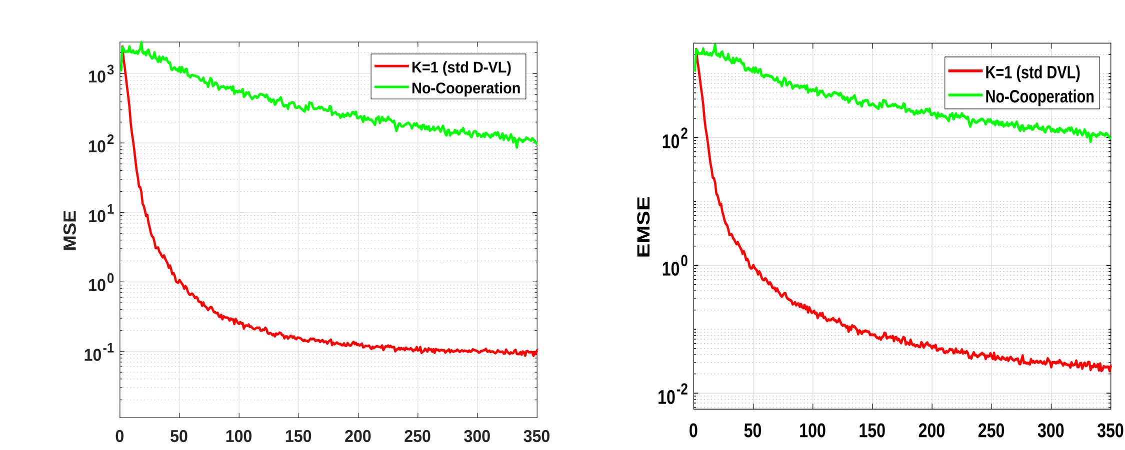

Figure: Network performance of the 2^{nd}-order distributed Volterra-Laguerre model for the nonlinear system (13).

- Steady-state values of the performance curves are significantly less hence the proposed modeling can be effectively use to model the nonlinear systems.