SPLINTS OF ROOT SYSTEMS OF BASIC LIE SUPERALGEBRAS

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1. Abstract

Splints[11] of root systems of simple Lie algebras appear naturally on the studies of embedding of reductive subalgebras. A splint can be used to construct branching rules, as implementation of this idea simplifies calculation of branching coefficients. We extend the concept of splints to basic Lie superalgebras case as these algebras have wide applications in physics. In this context we have determined the splints of root system of all basic Lie superalgebras and hope to contribute towards a small step in the direction of representation of these algebras.

2. Introduction

In this section we review some terminology on Lie superalgebra and recall notations used in the paper. A *superaglebra*[1,2,3] is a $\mathbb{Z}_2$-graded algebra $A = A_{\overline{0}} \oplus A_{\overline{1}}$ (that is, if $a \in A_\alpha$, $b \in A_\beta$, $\alpha, \beta \in Z_2$, then $ab \in A_{\alpha+\beta}$) . A *Lie superalgebra* is a superalgebra $G = G_{\overline{0}} \oplus G_{\overline{1}}$ with the operation [.,.] satisfying the following axiom:

(1) $[a, b] = -(-1)^{deg(a)deg(b)}[b, a]$ for $a \in G_\alpha$ and $b \in G_\beta$
(2) $[a, [b, c]] = [[a, b], c] + (-1)^{deg(a)deg(b)}[b, [a, c]]$ for $a \in G_\alpha$ and $b \in G_\beta$

Where $deg(a)$ is 0 if $a \in G_{\overline{0}}$ and $deg(a)$ is 1 if $a \in G_{\overline{1}}$. For a Lie superalgebra $G = G_{\overline{0}} \oplus G_{\overline{1}}$, the even part $G_{\overline{0}}$ is a Lie algebra and $G_{\overline{1}}$ is a $G_{\overline{0}}$-module. Let H be a cartan subalgebra of $G_{\overline{0}}$. The dimension of the cartan subalgebra H is the rank of the Lie superalgebra. Let us Denote Δ_0(respectively Δ_1) be the set of all even(respectively odd) root of G. If Δ is the set of all roots of the Lie superalgebra G, then $\Delta = \Delta_0 \cup \Delta_1$. A root α is called degenerate if $(\alpha, \alpha) = 0$ and an degenerate root is necessarily an odd root. For each basic Lie superalgebra, there exists a simple root system for which the number of odd simple roots is smallest one. Such a simple root system is called the distinguished simple root system. We denote $\Delta(G)$ to be the set of all positive roots of the basic Lie superalgebra G.

Let Δ and Δ' be positive root systems of two different basic Lie superalgebras with $\Delta = \Delta_0 + \Delta_1$ and $\Delta' = \Delta'_0 + \Delta'_1$, where Δ_0 (Δ'_0)and Δ_1 (Δ'_1) are even and odd roots of Δ (Δ') respectively. The map $\iota : \Delta \hookrightarrow \Delta'$ is an *embedding* if

(1) ι is a injective map and $\iota(\gamma) = \iota(\alpha) + \iota(\beta)$ for all $\alpha, \beta, \gamma \in \Delta$ such that $\gamma = \alpha + \beta$
(2) $\iota(\Delta_0) \subseteq \Delta'_0$ and $\iota(\Delta_1) \subseteq \Delta'_1$.

A root system Δ *splinters* as (Δ_1, Δ_2) if there are two embedding $\iota_1 : \Delta_1 \hookrightarrow \Delta$ and $\iota_2 : \Delta_2 \hookrightarrow \Delta$ where, Δ is the disjoint union of the images of ι_1 and ι_2 and neither the rank of Δ_1 nor the rank of Δ_2 exceeds the rank of Δ.

In this paper we use $A_n,\ B_n,\ C_n,\ D_n$ etc. for classical Lie algebras in cartan notation, whereas $A(m, n),\ B(m, n),\ C(n + 1),\ D(m, n),\ F(4),\ G(3),\ D(2, 1; \alpha)$ stand for basic Lie superalgebras [12, 4].

Key words and phrases. Lie Superalgebras, splints, embedding .

For a Lie superalgebra G, we can write $\Delta(G) = \Delta_0 + \Delta_1$. If (Δ_0, Δ_1) is a splint of $\Delta(G)$, then it is called the *trivial* splint of $\Delta(G)$. Except $A(m,n)$, all the basic Lie superalgebras $B(m,n)$, $C(n+1)$, $D(m,n)$, $F(4)$, $G(3)$ and $D(2,\ 1;\alpha)$ always have a trivial splint.

Suppose $\iota : \Delta \hookrightarrow \Delta'$ is an embedding and suppose that $(,)_0$ and $(,)_1$ are normalization of Δ and Δ' respectively. Then the embedding ι is *metric* if there is a non-zero integer scalar λ such that $(\alpha,\beta)_0 = \lambda(\iota(\alpha),\iota(\beta))_1$ for $\alpha,\beta \in \Delta$ and *non metric* otherwise.
We can observe from the definition of embedding that if we restrict the embedding $\iota : \Delta \hookrightarrow \Delta'$ to the even root system Δ_0 then we will find an embedding of Lie algebra associated to each Lie superalgebras, i.e

$$\iota\mid_{\Delta_0} : \Delta_0 \hookrightarrow \Delta'_0.$$

In fact if ι is metric then $\iota\mid_{\Delta_0}$ is also metric. But the converse is not true in general, for example $C_n \hookrightarrow C_n$ but $B(0,n) \not\hookrightarrow D(m,n)$. In this paper we will try to answer partially the converse part.

In this paper we have found all the splints, up to equivalence with Weyl group W (Weyl reflections are with respect to even roots only). If Δ is a distinguished simple root system then the splints (Δ_1,Δ_2) and (Δ'_1,Δ'_2) of Δ are equivalent, if there exists $\sigma \in W$ such that $\sigma.(((\Delta_1 \cup (-\Delta_1))|_{\Delta_0}, (\Delta_2 \cup (-\Delta_2))|_{\Delta_0}) = ((\Delta'_1 \cup (-\Delta'_1))|_{\Delta_0}, (\Delta'_2 \cup (-\Delta'_2))|_{\Delta_0})$ and similar restrictions hold good for odd roots of also. Here we like to mention that Lie superalgebras have Weyl reflections with respect to both degenerate and non-degenerate odd roots. However, in that case we get non-equivalent classes, as grading is not respected.

3. Main results

3.1. $A(m-1,n-1) = sl(m|n)$, $m,n \geq 1$.

Lemma 3.1. *$\Delta(F(4))$ and $\Delta(G(3))$ are not embedded in $\Delta(A(m-1,n-1))$, $\Delta(B(m,n))$, $\Delta(C(n+1))$ and $\Delta(D(m,n))$. Also $\Delta(D(2,\ 1;\alpha))$ is not embedded in $\Delta(A(m-1,n-1))$ and $\Delta(C(n+1))$.*

Proof. Because even roots of $A(m-1,n-1)$, $B(m,n)$, $C(n+1)$ and $D(m,n)$ are linear combination of distinguished positive simple roots with coefficient one. □

Lemma 3.2. *If Δ is a distinguished positive simple root system of a basic Lie superalgebra and $\Delta \hookrightarrow \Delta(A(m-1,n-1))$, then $\Delta \cong \Delta(A(r,s))$ for some $r \leq m-1,\ s \leq n-1$.*

Proof. Because the longest distinguished root of $A(m-1,n-1)$ is a linear combination of distinguished simple roots, with coefficients is equal to 1 . □

Lemma 3.3. *$\Delta(A(r,0))$, $\Delta(A(0,s))$ and $\Delta(A(r,s))$ are metrically embedded $\Delta(A(m-1,n-1))$.*

Proof. It is enough to prove for $\Delta(A(r,0))$ case only. As A_r is metrically embedded in A_m, thus $\Delta(A(r,0))$ is also metrically embedded in $\Delta(A(m-1,n-1))$. □

Lemma 3.4. *If $\Delta(A(r_1,\ s_1)) \hookrightarrow \Delta(A(m-1,n-1))$ and $\Delta(A(r_2,\ s_2)) \hookrightarrow \Delta(A(m-1,n-1))$ are embeddings with disjoint images, then $r_1 + r_2 \leq m,\ s_1 + s_2 \leq n$.*

Proof. As $A_l \hookrightarrow A_n$ and $A_k \hookrightarrow A_n$ are embeddings with disjoint images, then $l + k \leq n$. □

Lemma 3.5. *Suppose $m \geq 3$, $n \geq 3$ and either $r \geq 3,\ s \geq 2$ or $r \geq 2,\ s \geq 3$, and $\Delta(A(m-1,n-1))$ has a splint where $\Delta(A(r-1,s-1))$ is a component, then $\Delta(A(m-2,n-2))$ has a splint having $\Delta(A(r-2,s-2))$ as a component.*

Proof. Suppose $(\Delta_1,\ \Delta_2)$ is a splint of $\Delta(A(m-1,n-1))$ with $\iota : \Delta(A(r-1,\ s-1)) \hookrightarrow \Delta_1$ as a component. Without loss of generality, one may assume that the roots in the image of ι have the form $\{\varepsilon_i \pm \varepsilon_j, \delta_k \pm \delta_l, \delta_k \pm \varepsilon_i\}$ where $1 \leqslant i \neq j \leqslant r, 1 \leqslant k \neq l \leqslant s$. If we are restricting the splint to the embedding $\iota_1 : \Delta(A(m-2, n-2)) \hookrightarrow \Delta(A(m-1, n-1))$ and all the components are embedded metrically, this yields a splint of $\Delta(A(m-2, n-2))$ having $\Delta(A(r-2, s-2))$ as a component. $\square$

Proposition 3.6. *Assume $m, n \geq 6$ and if $(\Delta_1,\ \Delta_2)$ is a splint of $A(m-1, n-1)$ having $A(r, s)$ as a component, then $r \in \{0,\ 1,\ m-1,\ m-2\}$ and $s \in \{0,\ 1,\ n-1,\ n-2\}$*

Proof. We can argue by preceding results and from the table of splints of the root system of $A(m-1, n-1)$. $\square$

If $(\Delta_1,\ \Delta_2)$ is a splint of $\Delta(A(m-1, n-1))$ with $\Delta_1 \cap \Delta(A(m-1, 0)) \neq \phi$ and $\Delta_2 \cap \Delta(A(m-1, 0)) \neq \phi$, then we find a splint of $\Delta(A(m-1, 0))$, if we restrict Δ_1 and Δ_2 to $\Delta(A(m-1, 0))$. As $A(0, 0)$ has only one odd root, so $A(0, 0)$ does not splint. Now all the splints of $\Delta(A(m, n))$ which are given in the table of $\Delta(A(m-1, n-1))$ are explicitly described below.

(1) The splint $(\Delta(A(2, n-1)) + A_2,\ 2D_2 + 2n\Delta(A(0, 0)))$ of $\Delta(A(4, n-1))$ is given by

$$\Delta_1 = \{\varepsilon_i - \varepsilon_j, \delta_k - \delta_l, \delta_k - \varepsilon_j : 1 \leq i \neq j \leq 3, 1 \leq k \neq l \leq n\},$$
$$\Delta_2 = \{\varepsilon_1 - \varepsilon_j, \varepsilon_2 - \varepsilon_j, \delta_k - \varepsilon_j : 4 \leq j \leq 5, 1 \leq k \leq n\}.$$

(2) For $n \neq 0$, the splint $(\Delta(A(m-1, 0)) + A_{n-1},\ (mn-m)\Delta(A(0, 0)))$ of $\Delta(A(m-1, n-1))$ is given by

$$\Delta_1 = \{\varepsilon_i - \varepsilon_j, \delta_1 - \varepsilon_j, \delta_k - \delta_l : 1 \leq i \neq j \leq m, 1 \leq k \neq l \leq n\},$$
$$\Delta_2 = \{\delta_k - \varepsilon_l : 2 \leq k \leq n, 1 \leq l \leq m\}.$$

(3) For $m \neq 0$, the splint $(\Delta(A(0, n-1)) + A_{m-1},\ (mn-n)\Delta(A(0, 0)))$ of $\Delta(A(m-1, n-1)$ is given by

$$\Delta_1 = \{\delta_i - \delta_j, \delta_j - \varepsilon_1, \varepsilon_k - \varepsilon_l : 1 \leq i \neq j \leq n, 1 \leq k \neq l \leq m\},$$
$$\Delta_2 = \{\delta_k - \varepsilon_l : 1 \leq k \leq m, 2 \leq l \leq n\}.$$

(4) If $m - n = 1$,then the splint $(\Delta(A(n-1, n-1))\ ,\ nA_1 + n\Delta(A(0, 0)))$ of $\Delta(A(m-1, n-1))$ is given by

$$\Delta_1 = \{\varepsilon_i - \varepsilon_j, \delta_i - \delta_j, \delta_i - \varepsilon_j : 1 \leq i \neq j \leq n\},$$
$$\Delta_2 = \{\varepsilon_i - \varepsilon_m, \delta_i - \varepsilon_m : 1 \leq i \leq n\}.$$

(5) If $m \geq 2$, the splint $(\Delta(A(1, n-1)) + A_{m-2}\ ,\ (m-2)A_1 + n(m-2)\Delta(A(0, 0)))$ of $\Delta(A(m-1, n-1))$ is given by

$$\Delta_1 = \{\varepsilon_1 - \varepsilon_2, \delta_k - \delta_l, \delta_k - \varepsilon_1, \delta_k - \varepsilon_2 : 1 \leq k \neq l \leq n\} \cup \{\varepsilon_i - \varepsilon_j : 2 \leq i \neq j \leq m\},$$
$$\Delta_2 = \{\varepsilon_1 - \varepsilon_j, \delta_i - \varepsilon_j : 3 \leq j \leq m, 1 \leq i \leq n\}.$$

(6) If $n \geq 2$, the splint $(\Delta(A(m-1, 1)) + A_{n-2},\ (n-2)A_1 + m(n-2)\Delta(A(0, 0)))$ of $\Delta(A(m-1, n-1))$ is given by

$$\Delta_1 = \{\varepsilon_k - \varepsilon_l, \delta_1 - \delta_2, \delta_1 - \varepsilon_k, \delta_2 - \varepsilon_k : 1 \leq k \neq l \leq m\} \cup \{\delta_i - \delta_j : 2 \leq i \neq j \leq n\},$$
$$\Delta_2 = \{\delta_1 - \delta_j, \delta_j - \varepsilon_k : 3 \leq j \leq n, 1 \leq k \leq m\}.$$

(7) If $m, n \geq 2$, the splint $(\Delta(A(m-2, n-2)) + A(1, 0),\ (m+n-3)A_1 + (m+n-3)\Delta(A(0, 0)))$ of $\Delta(A(m-1, n-1))$ is given by

$$\Delta_1 = \{\varepsilon_i - \varepsilon_j, \delta_k - \delta_l, \delta_k - \varepsilon_i : 2 \leq i \neq j \leq m, 2 \leq k \neq l \leq n\} \cup \{\varepsilon_1 - \varepsilon_2, \delta_1 - \varepsilon_1, \delta_1 - \varepsilon_2\},$$
$$\Delta_2 = \{\varepsilon_1 - \varepsilon_i, \delta_1 - \delta_k, \delta_1 - \varepsilon_i, \delta_k - \varepsilon_1 : 3 \leq i \neq m, 2 \leq k \leq n\}.$$

(8) If $m, n \geq 2$, the splint $(\Delta(A(m-1,n-2)),\ (n-1)A_1 + m\Delta(A(0,0)))$ of $\Delta(A(m-1,n-1))$ is given by

$$\Delta_1 = \{\varepsilon_i - \varepsilon_j, \delta_k - \delta_l, \delta_k - \varepsilon_i : 1 \leq i \neq j \leq m, 1 \leq k \neq l \leq n-1\},$$
$$\Delta_2 = \{\delta_k - \delta_n, \delta_n - \varepsilon_i : 1 \leq k \leq n-1, 1 \leq i \leq m\}.$$

(9) If $m, n \geq 2$, the splint $(\Delta(A(m-2,n-1)),\ (m-1)A_1 + n\Delta(A(0,0)))$ of $\Delta(A(m-1,n-1))$ is given by

$$\Delta_1 = \{\varepsilon_i - \varepsilon_j, \delta_k - \delta_l, \delta_k - \varepsilon_i : 1 \leq i \neq j \leq m-1, 1 \leq k \neq l \leq n\},$$
$$\Delta_2 = \{\varepsilon_j - \varepsilon_m, \delta_i - \varepsilon_m : 1 \leq j \leq m-1, 1 \leq i \leq n\}.$$

(10) If $m = n$, the splint $(\Delta(A(m-1,m-2)),\ (m-1)A_1 + m\Delta(A(0,0)))$ of $\Delta(A(m-1,n-1))$ is given by

$$\Delta_1 = \{\varepsilon_i - \varepsilon_j, \delta_k - \delta_l, \delta_k - \varepsilon_i : 1 \leq i \neq j \leq m, 1 \leq k \neq l \leq m-1\},$$
$$\Delta_2 = \{\delta_i - \delta_m, \delta_m - \varepsilon_j : 1 \leq i \leq m-1, 1 \leq j \leq m\}.$$

(11) If $m = n$, the splint $(\Delta(A(m-2,m-1)),\ (m-1)A_1 + m\Delta(A(0,0)))$ of $\Delta(A(m-1,n-1))$ is given by

$$\Delta_1 = \{\varepsilon_i - \varepsilon_j, \delta_k - \delta_l, \delta_k - \varepsilon_i : 1 \leq i \neq j \leq m-1, 1 \leq k \neq l \leq m\},$$
$$\Delta_2 = \{\varepsilon_i - \varepsilon_m, \delta_m - \varepsilon_j : 1 \leq i \leq m-1, 1 \leq j \leq m\}.$$

(12) If $m = n$, the splint $(\Delta(A(m-2,m-2)),\ 2(m-1)A_1 + (2m-1)\Delta(A(0,0)))$ of $\Delta(A(m-1,n-1))$ is given by

$$\Delta_1 = \{\varepsilon_i - \varepsilon_j, \delta_i - \delta_j, \delta_i - \varepsilon_j : 1 \leq i \neq j \leq m-1\},$$
$$\Delta_2 = \{\varepsilon_i - \varepsilon_m, \delta_i - \delta_m, \delta_i - \varepsilon_m, \delta_m - \varepsilon_j : 1 \leq i \leq m-1, 1 \leq j \leq m\}.$$

Table 1. $\Delta(A(m,n))$

Δ	Δ_1	Δ_2
$A(1,0)$	A_1	$2A(0,0)$
$A(1,1)$	$A(0,\ 1)$ $2A_1$	$A_1 + 2A(0,0)$ $4A(0,\ 0)$
$A(1,2)$	$A(0,2)$ $A(1,1)$ $A_2 + 2A(0,0)$ $A_1 + A_2$	$A_1 + 3A(0,0)$ $2A_1 + 2A(0,0)$ $A(1,0) + 2A(0,0)$ $6A(0,0)$
$A(2,2)$	$A(2,1)$ $A(1,1) + A_1$ $A_2 + A_2$	$2A_1 + 3A(0,0)$ $A(1,0) + 2A_1 + 3A(0,0)$ $9A(0,0)$
$A(0,2)$	$A_1 + A(0,1)$ A_2	$A_1 + A(0,0)$ $3A(0,0)$
$A(4,4)$	$A(2,4) + A_2$	$2D_2 + 10A(0,0)$
$A(4,n)$ $A(m-1,n-1)$	$A(2,n) + A_2$ $A(m-1,0) + A_{n-1}$ $A(0,n-1) + A_{m-1}$	$2D_2 + 2nA(0,0)$ $(mn-m)A(0,0)$ $(mn-n)A(0,0)$
$A(m-1,n-1)$	$A(1,n) + A_{m-2}$ $A(m,1) + A_{n-2}$	$(m-2)A_1 + n(m-2)A(0,0)$ $(n-2)A_1 + m(n-2)A(0,0)$

if $m-n=1$, $A(m-1,n-1)$	$A(n-1,n-1)$	$nA_1+nA(0,0)$
$A(m-1,m-1)$	$A(m-1,m-2)$ $A(m-2,m-1)$ $A(m-2,m-2)$	$(m-1)A_1+mA(0,0)$ $(m-1)A_1+mA(0,0)$ $2(m-1)A_1+(2m-1)A(0,0)$
$A(m-1,n-1)$ for $m,n\geqslant 2$	$A(m-2,n-2)+A(1,0)$ $A(m-2,n-1)$ $A(m-1,n-2)$	$(m+n-3)A_1+(m+n-3)A(0,0)$ $(m-1)A_1+nA(0,0)$ $(n-1)A_1+mA(0,0)$

3.2. $B(m,n)=osp(2m+1|2n)$ **and** $B(0,n)=osp(1|2n)$**.**

Lemma 3.7. *$\Delta(C(n+1))$ is not embedded in $\Delta(B(m,n))$ for $m>3$, $n\geq 2$.*

Proof. Suppose $\Delta(C(n+1))\hookrightarrow\Delta(B(m,n))$. As the even roots of C_3 does not embed in B_m for $m\geq 2$, hence the image of even part of $\Delta(C(n+1))$ under the map ι is $\{\delta_k\pm\delta_l,2\delta_k\}$ where $1\leq k\neq l\leq n$. Now without loss of generality, the distinguished simple root system of $C(n+1)$ under the map ι is $\{\alpha_1,\ \alpha_2,\cdots\alpha_{n-1},\ 2\alpha_n+2\alpha_{n+1}+\cdots+2\alpha_{n-1}\}\cup\{\beta\}$, where β is an odd root of $\Delta(B(m,n))$ and $\alpha_1=\delta_1-\delta_2$, $\alpha_2=\delta_2-\delta_3,\cdots$, $\alpha_{n-1}=\delta_{n-1}-\delta_n$ which belong to distinguished simple roots of even part of $\Delta(B(m,\ n))$. $\Delta(C(n+1))$ has an odd root $\beta+\alpha_1+\alpha_2+\cdots+\alpha_{n-1}+2\alpha_n+2\alpha_{n+1}+\cdots+2\alpha_{n-1}$ but $\Delta(B(m,n))$ has no such odd root. □

Now we can describe all the splints of $\Delta(B(m,n))$ in the following way,

(1) The splint $(\Delta(B(0,1))+\Delta(A(0,1)),A_1+\Delta(A(0,0)))$ of $\Delta(B(1,1))$ is given by
$$\Delta_1=\{\delta_1+\varepsilon_1\}\cup\{\delta_1,2\delta_1\}$$
$$\Delta_2=\{\delta_1-\varepsilon_1,\varepsilon_1\}.$$

(2) The splint $(\Delta(A(0,1))+A_1,2\Delta(B(0,1))+A_1+2\Delta(A(0,0)))$ of $\Delta(B(1,2))$ is given by
$$\Delta_1=\{\delta_1-\delta_2,\delta_1-\varepsilon_1,\delta_2-\varepsilon_1\}\cup\{\varepsilon_1\},$$
$$\Delta_2=\{\delta_1+\delta_2\}\cup\{2\delta_1,2\delta_2,\delta_1,\delta_2\}\cup\{\delta_2+\varepsilon_1,\delta_1+\varepsilon_1\}.$$
Another splint of $\Delta(B(1,2))$ is $(\Delta(B(0,2)),A_1+4\Delta(A(0,0)))$ which is given by
$$\Delta_1=\{\delta_1\pm\delta_2,2\delta_1,2\delta_2,\delta_1,\delta_2\},$$
$$\Delta_2=\{\varepsilon_1,\delta_1\pm\varepsilon_1,\delta_2\pm\varepsilon_1\}.$$

(3) The splint $(\Delta(A(1,1))+2A_1,2\Delta(B(0,1))+2A_1+4\Delta(A(0,0)))$ of $\Delta(B(2,2))$ is given by
$$\Delta_1=\{\varepsilon_1-\varepsilon_2,\delta_1-\delta_2,\delta_1-\varepsilon_1,\delta_1-\varepsilon_2,\delta_2-\varepsilon_1,\delta_2-\varepsilon_2\}\cup\{\varepsilon_1+\varepsilon_2,\delta_1+\delta_2\},$$
$$\Delta_2=\{2\delta_1,2\delta_2,\delta_1,\delta_2\}\cup\{\varepsilon_1,\varepsilon_2\}\cup\{\delta_1+\varepsilon_1,\delta_1+\varepsilon_2,\delta_2+\varepsilon_1,\delta_2+\varepsilon_2,\}.$$

(4) $\Delta(B(0,2))$ has two additional splints. The first one $(A_2,\Delta(B(0,1))+\Delta(A(0,0)))$ is given by
$$\Delta_1=\{\delta_1\pm\delta_2,2\delta_2\},$$
$$\Delta_2=\{2\delta_1,\delta_1\}\cup\{\delta_2\}.$$
and the second one $(2A_1+\Delta(A(0,0)),2A_1+\Delta(A(0,0)))$ is given by
$$\Delta_1=\{\delta_1+\delta_2,2\delta_2\}\cup\{\delta_1\},$$
$$\Delta_2=\{\delta_1-\delta_2,2\delta_1\}\cup\{\delta_2\}.$$

(5) The splint $(D_n, n\Delta(B(0,1)))$ of $\Delta(B(0,n))$ is given by

$$\Delta_1 = \{\delta_k \pm \delta_l : 1 \le k \ne l \le n\},$$
$$\Delta_2 = \{2\delta_k, \delta_k : 1 \le k \le n\}.$$

(6) The splint $(C_n, n\Delta(A(0,0)))$ of $\Delta(B(0,n))$ is given by

$$\Delta_1 = \{\delta_k \pm \delta_l, 2\delta_k : 1 \le k \ne l \le n\},$$
$$\Delta_2 = \{\delta_k : 1 \le k \le n\}.$$

(7) The splint $(\Delta(B(0,n)) + B_m,\ 2mn\Delta(A(0,0)))$ and $(B_m + C_n,\ (2mn+n)\Delta(A(0,0)))$ of $\Delta(B(m,n))$ are equivalent because when we restrict to even roots both the splinter are same. Hence we consider only one splint $(\Delta(B(0,n)) + B_m,\ 2mn\Delta(A(0,0)))$ which is given by

$$\Delta_1 = \{\delta_k \pm \delta_l, 2\delta_k, \delta_k\} \cup \{\varepsilon_j\} \cup \{\varepsilon_i \pm \varepsilon_j\},\ \textit{where}\ 1 \le i \ne j \le m, 1 \le k \ne l \le n$$
$$\Delta_2 = \{\delta_i \pm \varepsilon_k : 1 \le i \ne j \le m, 1 \le k \ne l \le n\}.$$

(8) For $m \geqslant 2,\ n \geqslant 1$, the splint $(\Delta(D(m,n)),\ mA_1 + n\Delta(A(0,0)))$ of $\Delta(B(m,n))$ is given by

$$\Delta_1 = \{\delta_k \pm \delta_l, 2\delta_k, \varepsilon_i \pm \varepsilon_j, \delta_k \pm \varepsilon_i : 1 \le i \ne j \le m, 1 \le k \ne l \le n\}$$
$$\Delta_2 = \{\delta_i, \varepsilon_k : 1 \le i \le m, 1 \le k \le n\}.$$

(9) For $n \le m+2$, the splint $(\Delta(B(m,n-1)),\ (n-1)D_2 + (n-1)\Delta(A(0,0)))$ of $\Delta(B(m,n))$ is given by

$$\Delta_1 = \{\varepsilon_i \pm \varepsilon_j, \varepsilon_i, \delta_k \pm \delta_l, 2\delta_k, \delta_k \pm \varepsilon_i, \delta_k\} \cup \{2\delta_n, \delta_n\} : 1 \le i \ne j \le m, 1 \le k \ne l \le n-1\}$$
$$\Delta_2 = \{\delta_k \pm \delta_n, \delta_k \pm \varepsilon_m : 1 \le k \le n-1\}.$$

TABLE 2. $\Delta(B(m,n))$

Δ	Δ_1	Δ_2
$B(0,1)$	A_1	$A(0,0)$
$B(1,1)$	$B(0,1) + A(0,1)$	$A_1 + A(0,0)$
$B(1,2)$	$B(0,2)$ $A(0,1) + A_1$	$A_1 + 4A(0,0)$ $2B(0,1) + A_1 + 2A(0,0)$
$B(2,1)$	$A(1,0) + A_1$	$3A_1 + 3A(0,0)$
$B(2,2)$	$A(1,1) + 2A_1$	$2B(0,1) + 2A_1 + 4A(0,0)$
$B(0,2)$	A_2 $2A_1 + A(0,0)$	$B(0,1) + A(0,0)$ $2A_1 + A(0,0)$
$B(0,n)$	D_n	$nB(0,1)$
$B(0,n)$	C_n	$nA(0,0)$
$B(m,n)$	$B(0,n) + B_m$	$2mnA(0,0)$
$B(m,n)$ for $m \geqslant 2, n \geqslant 1$	$D(m,n)$	$mA_1 + nA(0,0)$
$B(m,n)$ for $n \le m+2$	$\Delta(B(m,n-1))$	$(n-1)D_2 + (n-1)\Delta(A(0,0))$

3.3. $C(n+1) = osp(2|2n)$.

Lemma 3.8. *$C(n)$ is not a component of $\Delta(C(n+1))$ for $n \geqslant 3$.*

Proof. Suppose (Δ_1, Δ_2) is a splint of $\Delta(C(n+1))$ and $C(n) \hookrightarrow \Delta_1$. Then the other components of Δ_1 are isomorphic to either A_1 or $\Delta(A(0,0))$ and components of Δ_2 are isomorphic to either A_1 or D_2, which implies rank of Δ_2 is greater than $n+1$. Hence a contradiction. □

Lemma 3.9. *$\Delta(B(m,n))$ is not embedded in $\Delta(C(n+1))$.*

Proof. Consider an even root α and the odd root β in the distinguished simple root system of $\Delta(B(m,n))$. Then $\alpha + 2\beta$ is a odd root, but $\Delta(C(n+1))$ has no such roots. □

Lemma 3.10. *$\Delta(B(0,n))$ is not embedded in $\Delta(C(n+1))$.*

Proof. $\Delta(B(0,n))$ has an odd root β such that 2β is an even root, but $\Delta(C(n+1))$ has no such roots. □

Lemma 3.11. *$\Delta(D(r,s))$ is not embedded in $\Delta(C(n+1))$ for $r, s \leq n$*

Proof. We can observe from the properties of $\Delta(D(r,s))$ that there are even roots which are linear combination of even as well as odd roots. But $\Delta(C(n+1))$ does not have such type of even roots. □

We can describe all the splints of $\Delta(C(n+1))$ in the following way,

(1) For $n \geq 1$, $\Delta(C(n+1))$ has a splint $(C_n, 2n\Delta(A(0,0)))$ which is given by

$$\Delta_1 = \{\delta_k \pm \delta_l, 2\delta_k : 1 \leq k \neq l \leq n\},$$
$$\Delta_2 = \{\varepsilon \pm \delta_k : 1 \leq k \neq l \leq n\}.$$

(2) $\Delta(C(3))$ has two additional splints $(A_2, \Delta(C(2)) + 2\Delta(A(0,0)))$ which are given by

$$\Delta_1 = \{\delta_1 \pm \delta_2, 2\delta_1\},$$
$$\Delta_2 = \{2\delta_2\varepsilon \pm \delta_2\} \cup \{\varepsilon \pm \delta_1\}$$

and $(\Delta(C(2)) + A_1, \Delta(C(2)) + A_1)$ given by

$$\Delta_1 = \{2\delta_1, \varepsilon \pm \delta_1, \} \cup \{\delta_1 - \delta_2\},$$
$$\Delta_2 = \{2\delta_2, \varepsilon \pm \delta_2, \} \cup \{\delta_1 + \delta_2\}.$$

(3) $\Delta(C(4))$ has two additonal splints which are $(A_1 + B_2 + 2\Delta(A(0,0)), A_1 + A_2 + 4\Delta(A(0,0)))$ given by

$$\Delta_1 = \{\delta_2 - \delta_3\} \cup \{2\delta_1, 2\delta_2, \delta_1 \pm \delta_2\} \cup \{\varepsilon \pm \delta_3\},$$
$$\Delta_2 = \{\delta_2 + \delta_3\} \cup \{2\delta_3, \delta_1 \pm \delta_3\} \cup \{\varepsilon \pm \delta_1, \varepsilon \pm \delta_2\}$$

and $(\Delta(C(3)) + A_1, 2D_2 + 2\Delta(A(0,0)))$ given by

$$\Delta_1 = \{\delta_1 \pm \delta_2, 2\delta_1, 2\delta_2, \varepsilon \pm \delta_1, \varepsilon \pm \delta_2\} \cup \{2\delta_3\},$$
$$\Delta_2 = \{\delta_1 \pm \delta_3\} \cup \{\delta_2 \pm \delta_3\} \cup \{\varepsilon \pm \delta_3\}.$$

Table 3. $\Delta(C(n+1))$

Δ	Δ_1	Δ_2
$C(2)$	$A(1)$	$2A(0,0)$
$C(3)$	A_2 $C(2)+A_1$	$C(2)+2A(0,0)$ $C(2)+A_1$
$C(4)$	$A_1+B_2+2A(0,0)$ $C(3)+A_1$	$A_1+A_2+4A(0,0)$ $2D_2+2A(0,0)$
$C(n)$	C_n	$2nA(0,0)$
$C(n)$	C_n	$2nA(0,0)$

3.4. $D(m,n) = osp(2m|2n)$.

Lemma 3.12. *Suppose $m \geqslant 2$, $n \geqslant 3$ and $r \geqslant 2$. If $\Delta(D(m,n))$ has a splint where $\Delta(D(m,r))$ is a component, then $\Delta(D(m,n-1))$ has a splint having $\Delta(D(m,r-1))$ as a component.*

Proof. Suppose (Δ_1,Δ_2) is a splint of $\Delta(D(m,n))$ having $\iota : \Delta(D(m,r)) \hookrightarrow \Delta_1$ as a component. Without loss of generality, one can assume that the roots in the image of ι have the form $\{\varepsilon_i \pm \varepsilon_j, \delta_k \pm \delta_l, 2\delta_k, \delta_k \pm \varepsilon_i\}$ with $1 \leqslant i \neq j \leqslant m, t+1 \leqslant k \neq l \leqslant n$,when $r = n-t$ for some $t \in \mathbb{Z}$. Consider restricting the splint to the embedding $\iota_1 : \Delta(D(m,n-1)) \hookrightarrow \Delta(D(m,n))$, where the image of ι_1 consists of roots of the form $\{\varepsilon_i \pm \varepsilon_j, \delta_k \pm \delta_l, 2\delta_k, \delta_k \pm \varepsilon_i\}$ with $1 \leqslant i \neq j \leqslant m, 2 \leqslant k \neq l \leqslant n$. Since ι_1 is metric and all components of Δ_1 and Δ_2 are embedded metrically, this yields a splint of $\Delta(D(m,n-1))$ having $\Delta(D(m,r-1))$ as a component. □

Lemma 3.13. *For $m \geqslant 4$, $n \geqslant 1$ or $m \geqslant 2$, $n > 3$ and either $n-2 \leqslant m$, $m \geqslant n$ or $m-2 \leqslant n, n \geqslant m$. If (Δ_1,Δ_2) is a splinter of $\Delta(D(m,n))$ having $\Delta(D(r,s))$ as a component, then $r,s \in \{1, n-1, m-1\}$.*

Proof. One may argue by contradiction using previous lemma and table of $\Delta(D(m,n))$. □

Lemma 3.14. $\Delta(A(m-1,n-1))$ *is not a component of* $\Delta(D(m,n))$ *for either* $m \geqslant 4$, $n \geqslant 1$ *or* $m \geqslant 2$, $n > 3$ *and not a component of* $\Delta(C(n+1))$ *for* $n \geq 4$.

Proof. Suppose (Δ_1,Δ_2) is a splint of $\Delta(D(m,n))$ and $\Delta(A(m-1,n-1)) \hookrightarrow \Delta_1$. Then other components of Δ_1 are isomorphic to A_1 or $\Delta(A(0,0))$. Also all components of Δ_2 are isomorphic to A_1 or $A(0,0)$. Then rank of Δ_2 is greater than $m+n$, which is a contradiction. Similar argument for $\Delta(C(n+1))$. □

Lemma 3.15. $\Delta(B(m,n))$*and* $\Delta(B(0,n))$ *are not embedded in* $\Delta(D(m,n))$.

Proof. Because $\Delta(B(m,n))$ and $\Delta(B(0,n))$ has an odd root β such that 2β is an even root, but $\Delta(D(m,n))$ has no such root. □

Lemma 3.16. $\Delta(C(n+1))$ *is not embedded in* $\Delta(D(m,n))$.

Proof. As $\Delta(D(m,n))$ is embedded in $\Delta(B(m,n))$ and $\Delta(C(n+1))$ is not embedded in $\Delta(B(m,n))$. □

We can describe all the splints of $\Delta(D(m,n))$ in the following way,

(1) The splint $(\Delta(A(1,0)), 2A_1 + 2\Delta(A(0,0)))$ of $\Delta(D(2,1))$ is given by

$$\Delta_1 = \{\varepsilon_1 - \varepsilon_2, \delta_1 - \varepsilon_1, \delta_1 - \varepsilon_2\},$$
$$\Delta_2 = \{2\delta_1, \varepsilon_1 + \varepsilon_2\} \cup \{\delta_1 + \varepsilon_1, \delta_1 + \varepsilon_2\}.$$

(2) The splint $(\Delta(A(1,1)), 4A_1 + \Delta(A(0,0)))$ of $\Delta(D(2,2))$ is given by

$$\Delta_1 = \{\varepsilon_1 - \varepsilon_2, \delta_1 - \delta_2, \delta_1 - \varepsilon_1, \delta_1 - \varepsilon_2, \delta_2 - \varepsilon_1, \delta_2 - \varepsilon_2\},$$
$$\Delta_2 = \{\varepsilon_1 + \varepsilon_2, \delta_1 + \delta_2, 2\delta_1, 2\delta_2\} \cup \{\delta_1 + \varepsilon_1, \delta_1 + \varepsilon_2, \delta_2 + \varepsilon_1, \delta_2 + \varepsilon_2\}.$$

(3) The splint $(\Delta(A(2,0)), 4A_1 + 3\Delta(A(0,0)))$ of $\Delta(D(3,1))$ is given by

$$\Delta_1 = \{\varepsilon_1 - \varepsilon_2, \varepsilon_1 - \varepsilon_3, \varepsilon_2 - \varepsilon_3, \delta_1 - \varepsilon_1, \delta_1 - \varepsilon_2, \delta_1 - \varepsilon_3\},$$
$$\Delta_2 = \{\varepsilon_1 + \varepsilon_2, \varepsilon_1 + \varepsilon_3, \varepsilon_2 + \varepsilon_3, 2\delta_1\} \cup \{\delta_1 + \varepsilon_1, \delta_1 + \varepsilon_2, \delta_1 + \varepsilon_3\}.$$

(4) The splint$(\Delta(A(2,1)) + A_1, 5A_1 + 6\Delta(A(0,0)))$of $\Delta(D(3,2))$ is given by

$$\Delta_1 = \{\varepsilon_1 - \varepsilon_2, \varepsilon_1 - \varepsilon_3, \varepsilon_2 - \varepsilon_3, \delta_1 - \delta_2, \delta_1 - \varepsilon_1, \delta_1 - \varepsilon_2, \delta_1 - \varepsilon_3, \delta_2 - \varepsilon_1, \delta_2 - \varepsilon_2, \delta_2 - \varepsilon_3\} \cup \{2\delta_2\},$$
$$\Delta_2 = \{\varepsilon_1 + \varepsilon_2, \varepsilon_1 + \varepsilon_3, \varepsilon_2 + \varepsilon_3, \delta_1 + \delta_2, 2\delta_1\} \cup \{\delta_1 + \varepsilon_1, \delta_1 + \varepsilon_2, \delta_1 + \varepsilon_3, \delta_2 + \varepsilon_1, \delta_2 + \varepsilon_2, \delta_2 + \varepsilon_3\}.$$

(5) The splint $(\Delta(A(1,2)) + 2A_1, 5A_1 + 6\Delta(A(0,0)))$of $\Delta(D(2,3))$ is given by

$$\Delta_1 = \{\varepsilon_1 - \varepsilon_2, \delta_1 - \delta_2, \delta_1 - \delta_3, \delta_2 - \delta_3, \delta_1 - \varepsilon_1, \delta_2 - \varepsilon_1, \delta_3 - \varepsilon_1, \delta_1 - \varepsilon_2, \delta_2 - \varepsilon_2, \delta_3 - \varepsilon_2\} \cup \{2\delta_1, 2\delta_2\},$$
$$\Delta_1 = \{\varepsilon_1 + \varepsilon_2, \delta_1 + \delta_2, \delta_1 + \delta_3, \delta_2 + \delta_3, \delta_1 + \varepsilon_1, \delta_2 + \varepsilon_1, \delta_3 + \varepsilon_1, \delta_1 + \varepsilon_2, \delta_2 + \varepsilon_2, \delta_3 + \varepsilon_2\} \cup \{2\delta_3\}.$$

(6) For either $n-2 \leqslant m$ or $m \geqslant n$ the splint $(\Delta(D(m,n-1)) + A_1, (2n-2)A_1 + 2m\Delta(A(0,0)))$ of $\Delta(D(m,n))$ is given by

$$\Delta_1 = \{\varepsilon_i \pm \varepsilon_j, \delta_k \pm \delta_l, 2\delta_k, \delta_k \pm \varepsilon_i : 1 \le i \neq j \le m, 2 \le k \neq l \le n\} \cup \{2\delta_1\},$$
$$\Delta_2 = \{\delta_1 \pm \delta_l, \delta_1 \pm \varepsilon_i : 1 \le i \le m, 2 \le l \le n\}.$$

Similarly, for either $m-2 \leqslant n$ or $n \geqslant m$ the splint $(\Delta(D(m-1,n)), (2m-2)A_1 + 2n\Delta(A(0,0)))$ of $\Delta(D(m,n))$ is given by

$$\Delta_1 = \{\varepsilon_i \pm \varepsilon_j, \delta_k \pm \delta_l, 2\delta_k, \delta_k \pm \varepsilon_i : 2 \le i \neq j \le m, 1 \le k \neq l \le n\},$$
$$\Delta_2 = \{\varepsilon_1 \pm \varepsilon_i, \delta_1 \pm \varepsilon_k : 2 \le i \le m, 1 \le l \le n\}.$$

(7) The splint $(D_m + Cn, 2mn\Delta(A(0,0)))$ of $\Delta(D(m,n))$ is given by

$$\Delta_1 = \{\varepsilon_i \pm \varepsilon_j, \delta_k \pm \delta_l, 2\delta_k : 1 \le i \neq j \le m, 1 \le k \neq l \le n\},$$
$$\Delta_2 = \{\delta_i \pm \varepsilon_k : 1 \le i \le m, 1 \le k \le n\}.$$

TABLE 4. $\Delta(D(m,n))$

Δ	Δ_1	Δ_2
$D(2,1)$	$A(1,0)$	$2A_1 + 2A(0,0)$
$D(2,2)$	$A(1,1)$	$4A_1 + A(0,0)$
$D(3,1)$	$A(2,0)$	$4A_1 + 3A(0,0)$
$D(2,3)$	$A(1,2) + 2A_1$	$5A_1 + 6A(0,0)$
$D(3,2)$	$A(2,1) + A_1$	$5A_1 + 6A(0,0)$
$D(m,n)$	$D_m + C_n$	$2mnA(0,0)$
$D(m,n)$ for either $n-2 \leqslant m$ or $m \geqslant n$	$D(m,n-1) + A_1$	$(2n-2)A_1 + 2mA(0,0)$
$D(m,n)$ for either $m-2 \leqslant n$ or $n \geqslant m$	$D(m-1,n)$	$(2m-2)A_1 + 2nA(0,0)$

3.5. $G(3), F(4), D(2,1;\alpha)$.

(1) The $\Delta(G(3))$ has two splints and the splints are $(A_2 + 3\Delta(A(0,0)), A_2 + A_1 + 4\Delta(A(0,0)))$ and $(B_2 + A_1 + 3\Delta(A(0,0)), 2A_1 + 4\Delta(A(0,0)))$, these are given by

$$\Delta_1 = \{\alpha_2, \alpha_2 + \alpha_3, 2\alpha_2 + \alpha_3\} \cup \{\alpha_1 + \alpha_2, \alpha_1 + \alpha_2 + \alpha_3, \alpha_1 + 3\alpha_2 + \alpha_3\},$$
$$\Delta_2 = \{\alpha_3, 3\alpha_2 + \alpha_3, 3\alpha_2 + 2\alpha_3\} \cup \{\alpha_1, \alpha_1 + 2\alpha_2 + \alpha_3, \alpha_1 + 3\alpha_2 + 2\alpha_3, \alpha_1 + 4\alpha_2 + 2\alpha_3\} \cup \{2\alpha_1 + 4\alpha_2 + 2\alpha_3\}.$$

And another one is

$$\Delta_1 = \{\alpha_2, \alpha_3, \alpha_2 + \alpha_3, 2\alpha_2 + \alpha_3\} \cup \{2\alpha_1 + 4\alpha_2 + 2\alpha_3\} \cup \{\alpha_1, \alpha_1 + \alpha_2, \alpha_1 + \alpha_2 + \alpha_3\},$$
$$\Delta_2 = \{3\alpha_2 + \alpha_3, 3\alpha_2 + 2\alpha_3\} \cup \{\alpha_1 + 2\alpha_2 + \alpha_3, \alpha_1 + 3\alpha_2 + 2\alpha_3, \alpha_1 + 4\alpha_2 + 2\alpha_3, \alpha_1 + 3\alpha_2 + \alpha_3\}.$$

(2) From the positive root system of $F(4)$ we can observe that the root systems $\Delta(C(4))$,$\Delta(B(0,3))$,$\Delta(B(0,2))$ are not embedded in $\Delta(F(4))$. Only the root system of $\Delta(D(2,1))$ is embedded in $\Delta(F(4))$ which is identified as

$$\{\alpha_2+\alpha_3, 2\alpha_2+\alpha_3+\alpha_4, 2\alpha_1+3\alpha_2+2\alpha_3+\alpha_4\} \cup \{\alpha_1, \alpha_1+\alpha_2+\alpha_3, \alpha_1+2\alpha_2+\alpha_3+\alpha_4, \alpha_1+3\alpha_2+2\alpha_3+\alpha_4\}.$$

So $\Delta(A(2,1))$ is also embedded in $\Delta(F(4))$. Hence the root system $\Delta(F(4))$ has only one splint

$$(A_1 + B_3, 8A(0,0)),$$

where Δ_1 and Δ_2 are all even roots and odd roots respectively.

(3) The root system $D(2,1;\alpha)$ has two splints and these are $(\Delta(A(1,0)) + A_1, A_1 + 2\Delta(A(0,0)))$ and $(3A_1, 4\Delta(A(0,0)))$ given by

$$\Delta_1 = \{\alpha_1, \alpha_2, \alpha_1 + \alpha_2\} \cup \{\alpha_3\},$$
$$\Delta_2 = \{2\alpha_1 + \alpha_2 + \alpha_3\} \cup \{\alpha_1 + \alpha_3, \alpha_1 + \alpha_2 + \alpha_3\}$$

and

$$\Delta_1 = \{\alpha_1, \alpha_3, 2\alpha_1 + \alpha_2 + \alpha_3\},$$
$$\Delta_2 = \{\alpha_1, \alpha_1 + \alpha_2, \alpha_1 + \alpha_3, \alpha_1 + \alpha_2 + \alpha_3\}$$

respectively.

TABLE 5. $\Delta(F(4))$, $\Delta(G(3))$, $\Delta(D(2,1;\alpha))$

Δ	Δ_1	Δ_2
$G(3)$	$A_2 + 3A(0,0)$ $B_2 + A_1 + 3A(0,0)$	$A_2 + A_1 + 4A(0,0)$ $2A_1 + 4A(0,0)$
$F(4)$	$A_1 + B_3$	$8A(0,0)$
$D(2,1;\alpha)$	$A(1,0) + A_1$ $3A_1$	$A_1 + 2A(0,0)$ $4A(0,0)$

4. Concluding remarks

In this paper we have determined splints of all basic Lie superalgebras up to equivalence with Weyl group of the corresponding algebra. We hope results of this paper can help us to some extent in determining the branching coefficient. We want to delve in to this aspect of research in future.

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