Probabilistic models for shear bond strength of clay and fly ash bricks

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Abstract

In the current investigation, ninety fly ash and clay brick masonry triplets are prepared with three different grades of mortar mix. The shear bond strength of prepared samples is tested and analyzed on an extended statistical domain. The test data obtained from the experiments conducted was statistically analyzed for fifty-eight selected probability distribution functions with two, three and four parameters using the computer program *EasyFit 5.6* Professional. The two Goodness of Fit tests, namely Kolmogorov-Smirnov and Anderson-Darling, are carried out to choose the best-fit model out of it for the shear bond strength parameter of clay and fly ash brick masonry. Different Goodness of Fit tests suggest different statistic and rank for different distributions, but in all the case Johnson SB distribution is found best to portray the variability in shear bond strength.

1 Introduction

Masonry is considered as an oldest and vital construction component used globally because of its low cost, easy accessibility of raw materials, good strength, ease of construction, less supervision and good sound and heat insulation. Many past literatures are available on the various experimental studies of clay brick [1-8] and fly ash brick [3-5]. Most of the previous studies on brick masonry focussed on the compressive strength, shear bond strength, initial absorption rate, water absorption and constitutive relations including elastic modulus. A large variation in the mechanical properties of bricks can be seen due to the production of the brick with the available local raw materials. The variation in the raw materials leads to variation in strength values of the brick masonry.

The shear failure is one of the most common failure modes for brick masonry. The shear bond strength of brick masonry is affected by brick strength, mortar grade, water to cement ratio, and so on. Most of the existing literature reports the relationship between shear bond strength and masonry compressive strength for different types of mortar grades [4, 6 and 8]. However, the previous literature did not give due attention to report the variability in structural properties of brick masonry. The analytical models based on the assumption of uniformity in material properties without considering the variability are not only unrealistic but also unsatisfactory. This constitutes the underlying motivation of the present study and is an attempt to develop a description of the variability of the shear bond strength of fly ash brick (FAB) and clay brick (CB) masonry.

The shear bond strength is the most critical parameter that governs the resistance capacity of both load bearing and infilled masonry structures. The performance of masonry structures in a probabilistic framework can be expressed using a limit state function 'g' as (1).

$$g(R, Q) = R - Q \geq 0 \tag{1}$$

Where R is resistance capacity that directly depends on the shear bond strength and Q is the load effect. The limit state function 'g' corresponds to the boundary between desired and undesired performance and its value should be greater than or equal to zero for desired performance. Both R and Q are continuous random variables with associated probability-density function. The evaluation of safety margin of any structure depends on the probability distribution of R which in turn governed by shear bond strength. Therefore, in the present study, the shear bond strength is considered to express the probability distribution of R. It is obvious that for reliability-based calculation, the probabilistic models of R and Q, especially the former one, should be clearly known [9]. Therefore, the outcome of this research can be useful for reliability analysis of masonry structure Some of the past literature [10-15] focussed on finding the optimal fitting probability distribution of concrete materials by statistical analysis where this aspect is overlooked in case of brick. By exploring the variability of shear bond strength of brick masonry, the study emphasizes the importance of considering such variability in the design process.

2 Experimenal programme.

The experimental test is carried out on ninty brick triplets to determine the shear bond strength of clay brick and fly ash brick masonry with three different grades of mortar mix. This includes the preparation of the samples for the experimental work and testing of samples as discussed in the following sections.

2.1 Materials used

The materials used for the experimental purpose include mainly bricks, cement and sand. In this study, both clay and fly ash bricks are used for the preparation of brick triplets. The size of both CB and FAB is 230 × 110 × 75 mm. The brick units are bonded using three types of cement mortar mix (CM_1, CM_2 and CM_3). The mixing ratio of all the three types of mortar are presented in Table 1. The sand used here is locally available river sand of Zone-II as per Indian Standard. Portland slag cement is used as a binder. The cement mortar is prepared by varrying the cement-sand proportion and water to binder ratio for achieving good workability. Six stes of specimen ($CBCM_1$, $CBCM_2$, $CBCM_3$, $FABCM_1$, $FABCM_2$, and $FABCM_3$) with different brick and mortar mix are prepared. Total ninty samples are prepared considering fifteen from each set.

Table 1 Mortar-mix proportion used in sample preparation.

Mortar mix type	Mix Propertion	Water-cement ratio
CM_1	1:6	0.80
CM_2	1:4.5	0.55
CM_3	1:3	0.45

2.2 Preparation of test specimens

Specimens considered for assessing the shear bond strength in this study are of three brick stacking prisms or triplets joined with mortar. The thickness of the mortar-brick joints is maintained at 8 to 10 mm. The dimensions of the brick triplets are 230 mm × 110 mm × 245 mm. The ratio of height to the thickness of the three-brick high prism remains at 2.23, as described in Indian Standard IS 1905: 1987 [15], *i.e.*, in the range of 2 to 5. The masonry specimens prepared for the experimental study are shown in Fig. 1. After a day of casting the samples, these are covered with wet jute gunny bags and cured for 28 days.

Fig. 1 Typical brick triplets for shear bond strength.

2.3 Experimental tests

The experimental test is carried to obtain the bond shear strength of masonry assemblages. Under the direct axial force in UTM (Universal Testing Machine), the shear bond strength of the brick triplets is determined. For this purpose, the prisms are placed in a UTM, where loading is applied through a plunger on the central brick through a wooden slate on it to allow the load to transfer uniformly over the entire surface of the central brick. The load at which the middle brick detaches from masonry is the failure load. The shear bond strength is calculated by dividing the load with twice the surface area of brick. Fig. 2 shows the test set-up used. The results of the test are presented in Table. 2.

Fig. 2 Test set up for shear bond strength test with brick triplet specimens.

Table 2 Shear bond strength (MPa) of Clay and Fly ash Brick triplets

Sample No.	Shear Bond strength of CB			Shear Bond strength of FAB		
	CBCM$_1$	CBCM$_2$	CBCM$_3$	FABCM$_1$	FABCM$_2$	FABCM$_3$
1	0.02	0.07	0.14	0.04	0.06	0.02
2	0.07	0.06	0.27	0.06	0.12	0.09
3	0.11	0.11	0.28	0.07	0.08	0.19
4	0.07	0.04	0.25	0.08	0.09	0.11
5	0.05	0.04	0.27	0.005	0.05	0.19
6	0.03	0.03	0.22	0.08	0.13	0.15
7	0.03	0.07	0.21	0.03	0.09	0.09
8	0.02	0.02	0.28	0.04	0.09	0.4
9	0.04	0.05	0.29	0.05	0.12	0.16
10	0.09	0.05	0.25	0.03	0.14	0.18
11	0.10	0.1	0.24	0.04	0.13	0.39
12	0.06	0.09	0.12	0.07	0.12	0.08
13	0.05	0.04	0.29	0.08	0.04	0.14
14	0.09	0.07	0.19	0.08	0.1	0.13
15	0.09	0.08	0.08	0.09	0.11	0.007

3 Varriability study of shear strength of brick masonry

In the previous section, the shear bond strength of ninety brick triplets was determined. The variation in the test results can be seen from Table 2. It is required to check the regularity of the strength values to study its variability pattern. For this purpose, statistical techniques are adapted to determine the regularities in the obtained results and to propose the best probability distributions that can model the variability in the shear bond strength of bricks. The test results were statistically analyzed for fifty-eight selected probability distribution functions with two, three and four parameters using the computer program *EasyFit 5.6* Professional. The parameters of the considered distribution functions are calculated separetly for each set of specimens. Table 3 shows some (three out of fifty-eight) of the considered distribution function and its parameter for clay brick triplets while the rest are not presented here.

The two Goodness of Fit (GOF) tests, namely Kolmogorov-Smirnov (K-S) and Anderson- Darling (A-D), are carried out for the data set to obtain the best-fit distribution model for the shear bond strength of bricks. It is to be noted here that both K-S and A-D are non-parametric, distribution-free, and are applicable for continuous distributions. Each of the GOF tests suggests different statistic value and rank for each of the probability distribution. The lower the statistic value of a distribution, the higher is it's rank. Table 4 represents the statistic value and rank of all considered distributions obtained from K-S test for clay bricks. From the table, it can be seen that Johnson SB (JSB) distribution ranks first and Wakeby distribution ranks second in K-S test. The same procedure is also carried out for A-D test, and the best three distributions with their satistic value and rank are presented in Table 5. Similarly, Tables 6-7 present the statistical results of fly ash brick triplets for K-S and A-D tests respectively. Based on the analysis performed, it can be proposed that JSB distribution is best suited in every case for the shear bond strength of clay and fly ash brick masonry.

Table 3 Parameters of some selected distribution functions for shear bond strength (MPa) of clay brick specimens

Distribution	Distribution parameters for clay bricks		
	$CBCM_1$	$CBCM_2$	$CBCM_3$
Beta	$\alpha_1 = 0.57$ $\alpha_2 = 0.67$ a = 0.02 b = 0.11	$\alpha_1 = 0.89$ $\alpha_2 = 1.05$ a = 0.02 b = 0.11	$\alpha_1 = 0.80$ $\alpha_2 = 0.36$ a = 0.08 b = 0.29
Johnson SB	$\gamma = -0.58$ $\delta = 0.86$ $\lambda = 0.11$ $\xi = -0.01$	$\gamma = 0.48$ $\delta = 1.04$ $\lambda = 0.13$ $\xi = 0.007$	$\gamma = -0.93$ $\delta = 0.56$ $\lambda = 0.27$ $\xi = 0.02$
Burr	k = 932.85 $\alpha = 2.30$ $\beta = 1.36$	k = 292.21 $\alpha = 2.64$ $\beta = 0.59$	k = 2044.20 $\alpha = 4.58$ $\beta = 1.31$

Table 4 Statistic value and rank of shear bond strength of clay brick triplets for K-S test

Distribution	$CBCM_1$		$CBCM_2$		$CBCM_3$	
	Statistic	Rank	Statistic	Rank	Statistic	Rank
Beta	0.133	7	0.140	22	0.170	10
Burr	0.169	27	0.124	5	0.180	13
Burr (4P)	0.364	56	0.133	11	0.167	8
Cauchy	0.163	17	0.158	35	0.231	36
Dagum	0.134	8	0.135	16	0.253	45
Dagum (4P)	0.323	55	0.394	56	0.138	5
Erlang	0.195	44	0.217	48	0.299	49
Erlang (3P)	0.183	36	0.171	42	0.205	25
Error	0.110	5	0.123	4	0.193	16
Exponential (2P)	0.183	37	0.250	50	0.331	50
Fatigue Life	0.163	18	0.161	38	0.237	37
Fatigue Life (3P)	0.171	28	0.138	18	0.193	17
Frechet	0.206	47	0.194	45	0.287	48
Frechet (3P)	0.166	22	0.148	30	0.217	28
Gamma	0.174	33	0.143	23	0.224	32
Gamma (3P)	0.286	53	0.140	21	0.198	19

Gen. Extreme Value	0.149	10	0.123	3	0.115	2
Gen. Gamma	0.169	24	0.144	25	0.222	30
Gen. Gamma (4P)	0.270	49	0.230	49	0.205	24
Gen. Logistic	0.169	26	0.135	15	0.131	4
Gen. Pareto	0.108	3	0.139	20	0.163	7
Gumbel Max	0.182	35	0.159	37	0.256	46
Gumbel Min	0.186	40	0.191	44	0.138	6
Hypersecant	0.194	42	0.168	40	0.211	26
Inv. Gaussian	0.185	39	0.166	39	0.241	39
Inv. Gaussian (3P)	0.150	12	0.155	34	0.190	15
Johnson SB	**0.105**	**1**	**0.110**	**1**	**0.099**	**1**
Kumaraswamy	0.205	46	0.197	47	0.242	41
Laplace	0.204	45	0.196	46	0.239	38
Levy (2P)	0.321	54	0.378	55	0.475	55
Log-Logistic	0.160	15	0.174	43	0.243	42
Log-Logistic (3P)	0.164	19	0.143	24	0.176	11
Log-Pearson 3	0.150	13	0.126	7	0.199	20
Logistic	0.183	38	0.154	33	0.200	21
Lognormal	0.160	16	0.158	36	0.228	33
Lognormal (3P)	0.172	31	0.136	17	0.195	18
Nakagami	0.179	34	0.126	8	0.204	22
Normal	0.164	20	0.134	14	0.188	14
Pareto	0.272	50	0.358	54	0.386	54
Pareto 2	0.278	52	0.346	52	0.369	52
Pearson 5	0.169	25	0.170	41	0.245	44
Pearson 5 (3P)	0.173	32	0.134	13	0.204	23
Pearson 6	0.167	23	0.151	32	0.220	29
Pearson 6(4P)	0.172	30	0.133	12	0.348	51
Pert	0.136	9	0.124	6	0.177	12
Power Function	0.133	6	0.145	27	0.228	34
Rayleigh (2P)	0.171	29	0.131	9	0.243	43
Reciprocal	0.216	48	0.273	51	0.483	56
Rice	0.189	41	0.144	26	0.223	31
Triangular	0.195	43	0.146	29	0.231	35
Uniform	0.109	4	0.138	19	0.217	27
Wakeby	0.108	2	0.113	2	0.126	3
Weibull	0.154	14	0.145	28	0.242	40
Weibull (3P)	0.164	21	0.132	10	0.168	9

Table 5 Statistic value and rank of shear bond strength of clay brick triplets for A-D test

Distribution	$CBCM_1$		$CBCM_2$		$CBCM_3$	
	Statistic	Rank	Statistic	Rank	Statistic	Rank
Johnson SB	0.202	1	0.169	1	0.171	1
Wakeby	0.236	5	0.173	2	0.190	2
Gen. Extreme Value	0.298	6	0.188	4	0.225	3

Table 6 Statistic value and rank of shear bond strength of fly ash brick triplets for K-S test

Distribution	$FABCM_1$		$FABCM_2$		$FABCM_3$	
	Statistic	Rank	Statistic	Rank	Statistic	Rank
Johnson SB	0.356	1	0.103	1	0.105	1
Error	0.173	2	0.145	10	0.109	5
Dagum	0.154	4	0.1224	4	0.134	8

Table 7 Statistic value and rank of shear bond strength of fly ash brick triplets for A-D test

Distribution	$FABCM_1$		$FABCM_2$		$FABCM_3$	
	Statistic	Rank	Statistic	Rank	Statistic	Rank
Johnson SB	0.356	1	0.197	1	0.202	1
Error	0.448	5	0.371	12	0.217	2
Dagum	0.412	4	0.207	2	0.362	10

The probability distribution of the shear bond strength with the best fit distributions can be shown in various ways. Some of the most common ways of describing the variability of the property are the probability density function (PDF) and cumulative distribution function (CDF), which are shown in Fig. 3 (a) and (b) respectively for $CBCM_1$ specimens with JSB distribution.

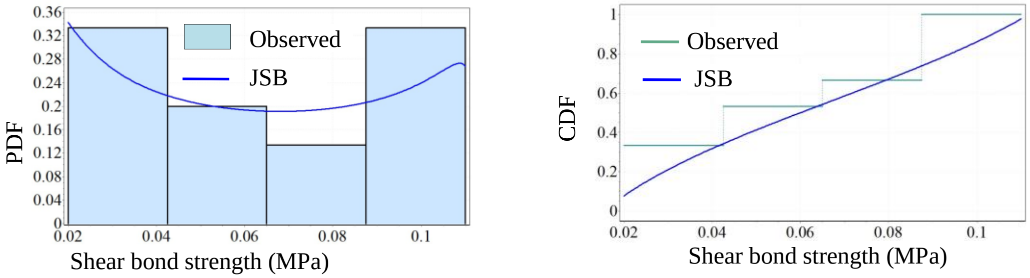
Hazard function, $h(x)$ and cumulative hazard function $H(x)$ are two parameters often used in the reliability analysis. Hazard function, for a given distribution, is defined as the instantaneous rate of occurrence of the event and can be expressed as in (2). The hazard function plot and cumulative hazard function plot for the shear bond strength of CBCM1 specimens considering JSB distribution are presented in the Fig. 4 (a) and (b) respectively.

$$h(x) = \frac{f(x)}{S(x)} = \frac{f(x)}{1 - F(x)} \tag{2}$$

A survival function plot represents the probability of exceeding the shear bond strength of a certain value and can be expressed as in (3). It is often used in reliability and related fields to find out the probability of the variate to take on a greater value than the specified value.The probability difference plot presents the difference between the observed CDF and the fitted CDF. Fig. 5 (a) and (b) respectively present the survival function plot and probability difference plot of the observed data with JSB function for CBCM1 specimens.

$$S(x) = P(X > x) = 1 - F(x) \tag{3}$$

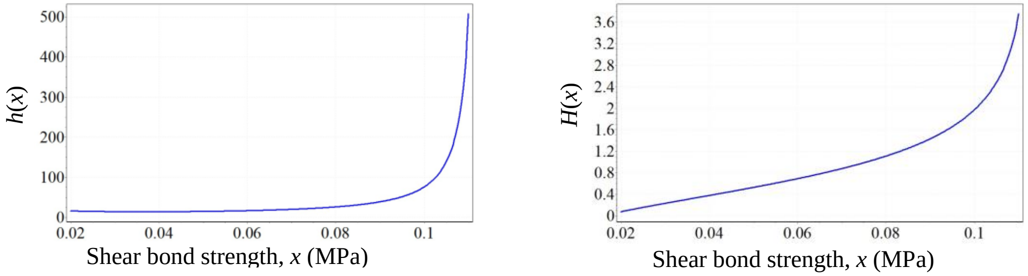
A probability-probability (P-P) plot is a graph of the observed CDF values plotted against the fitted CDF values. Similarly, quantile-quantile (Q-Q) plot is a graph of the observed data values plotted against the fitted distribution quantiles. Both of these two plots are used to determine how well a specific distribution fits the observed data. Fig. 6 (a) and (b) respectively present the P-P and Q-Q plot for CBCM1 specimens with JSB function. These plots may be used to evaluate the safety of CB and FAB masonry structures.



(a) Probability density function (PDF)

(b) Cumulative density function (CDF)

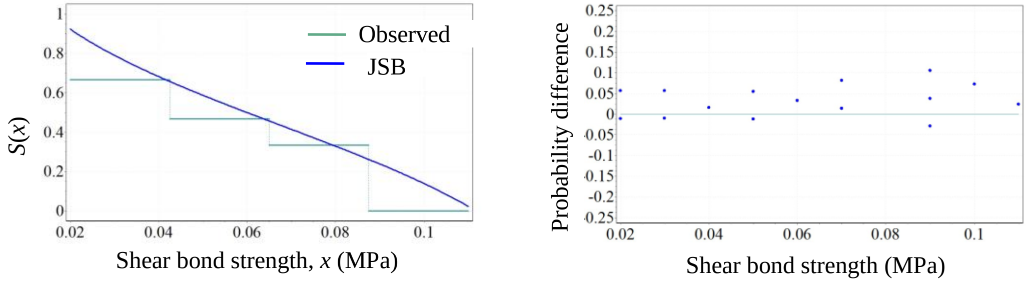
Fig. 3 PDF and CDF plot of CBCM$_1$ with JSB distribution.



(a) Hazard function

(b) Cumulative hazard function

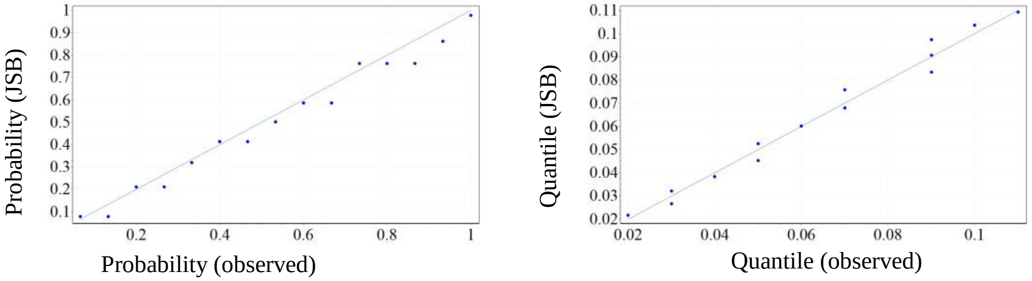
Fig. 4 Hazard function and Cumulative hazard function plot of CBCM$_1$ with JSB distribution.



(a) Survival function plot

(b) Probability difference plot

Fig. 5 Survival function and P robability difference plot of CBCM$_1$ with JSB distribution.



(a) P-P plot

(b) Q-Q plot

Fig. 6 P-P and Q-Q plot of CBCM$_1$ with JSB distribution.

4 Conclusion

Experimental tests were conducted on ninety specimens to determine the shear bond strength of brick triplets. The test samples are prepared with two brick variants (CB and FAB) and three grades of mortar (CM_1 CM_2 and CM_3). Fifty-eight probability distribution functions are considered which involves two, three and four parametric distributions to analyze the test results. The best-fit probability distribution model is assessed from two goodness-of-fit tests namely K-S, A-D test, respectively. JSB is found to be the best-fit distribution function representing the variation in the shear bond strength of clay brick and fly ash brick masonry. The variability plots of specimens $CBCM_1$ specimens with JSB distribution are also presented.

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