

BENCHMARK ANALYSIS OF A HELMHOLTZ RESONATOR FOR ESTIMATING ACOUSTIC METAMATERIAL PROPERTIES

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Acoustic metamaterial gets significant attention due to possibility in control, direct and manipulate sound waves. Various metamaterial models have been proposed mostly for air medium, however applicable to water medium for cloaking purpose. Control of the various forms of sound waves is possible with a negative **refractive index material**, mostly accomplished through bulk modulus and density of the material. However, in case of acoustic metamaterial, the shapes and structures play vital role in accomplishing same. Present research focuses in analysing most known acoustic structure, Helmholtz resonator, to estimate the metamaterial properties such as effective mass density and effective bulk modulus. The **transfer matrix** of Helmholtz resonator is used to extract the **scattering matrix**, which is subsequently used to estimate the effective bulk modulus and effective mass density. Next, a finite element analysis (FEA) has been carried out using two load boundary condition to estimate the transfer matrix, validated against experimental results. In a similar manner, the **effective mass density** and **effective bulk modulus** have been extracted and validated against analytical result. Moreover, two Helmholtz resonators separated with a known duct have been analysed to evaluate the applicability of transfer matrix method in estimating acoustic metamaterial properties. All analytical results have been validated against numerical results for air medium.

Keywords: [Acoustic Metamaterial](#), [Transfer Matrix](#), [Scattering matrix](#), [Effective mass density](#), [effective bulk modulus](#), [Negative refractive index](#)

1. Introduction

Metamaterial is a recent trends in material science, in general, is not available in nature due to its uncommon material properties in an effect can manipulate the conventional law of physics. If we talk about specifically acoustic metamaterial, we can say that using such materials the sound wave gets manipulated as an outcome bending and twisting phenomena can be introduced. Metamaterials can be classified with zero or negative refractive index. The refractive index can be controlled by controlling the effective mass density or effective bulk modulus or both in case of acoustic metamaterial. Various acoustic metamaterial patterns have been introduced and reported till date to demonstrate the possibilities of such phenomena which is really uncommon in nature [1-10].

In present work, the possibilities of estimating acoustic metamaterial properties from electro-acoustic modelling is investigated [1]. In line with the objective, the most classical acoustic filter such as a Helmholtz resonator has been taken for analysis. In first the analytical technique has been used to estimate the transfer matrix followed by extraction of effective mass density and effective bulk modulus [1,4]. Subsequently, an array of Helmholtz resonator, specifically, two different Helmholtz resonators separated by a duct has been analysed. The report demonstrates the potential of analytical modelling, potential of numerical modelling of the experiment defined by ASTM E2611 to extract the transfer matrix which is subsequently used in estimating the metamaterial properties. However, it can be noticed that the acoustic metamaterial properties such as negative bulk modulus and negative mass density estimated by using transfer matrix, which is very essential to extract transmission and reflection coefficient.

1.1. Research Purpose

The present research aims to extract the reflection and transmission coefficients of a Helmholtz resonator using analytical or transfer matrix method. The estimated reflection and transmission coefficients will be used to extract the effective mass density and effective bulk modulus. The estimated results will be judged against the numerical results.

1.2. Research Scope

At this study, we consider the following observations of the problem:

- Benchmarking of Helmholtz resonator.
- Extracting the metamaterial properties such as effective mass density and effective bulk modulus.
- A Helmholtz resonator and two Helmholtz resonators separated by a duct have been investigated by analytically, numerically and experimentally.

2. Analytical Modelling

Helmholtz resonator is a kind of classical acoustic filter, more specifically a passive acoustic band stop filter, which used to be modelled analytically with electro-acoustic modelling theory using the empirical model to estimate the lumped acoustic impedance of the resonator [1]. In general, it is essential to get transfer matrix of any acoustic filter to estimate corresponding acoustic properties. As the focus of the work is to understand the behaviour of a Helmholtz resonator from metamaterial prospective, we first try to model the resonator of a given dimension. The analytically extracted transfer matrix is subsequently used to estimate the acoustic transmission loss of the given resonator and two resonators separated by a duct. Mathematically, the corresponding transfer matrix for resonators, shown in, Fig.3. (c) and (b), can be written as:

$$\begin{cases} TM^1 = TM_{duct-1} \times TM_{HR-1} \times TM_{duct-2} \\ TM^2 = TM_{duct-1} \times TM_{HR-1} \times TM_{duct-2} \times TM_{HR-2} \times TM_{duct-3} \end{cases} \quad (1)$$

The transfer matrix of any acoustic duct (TM_{duct}) in liner acoustics can be modelled using duct radius (d_r), and duct length (l_r). In a similar fashion, the lumped system can also be modelled for any Helmholtz resonator, TM_{HR} , using the geometrical parameters such as cavity volume (V_c), neck radius (r_n), and length (l_n). Mathematically it can be written as [1-3]:

$$TM_{duct} = \begin{bmatrix} \cos(kl_r) & -jY_r \sin(kl_r) \\ -j/Y_r \sin(kl_r) & \cos(kl_r) \end{bmatrix}, \text{ where } Y_r = c / \pi d_r^2, \quad (2)$$

$$\text{and } TM_{HR} = \begin{bmatrix} 1 & 0 \\ Z_{HR}^{-1} & 1 \end{bmatrix}, \text{ where } \begin{cases} Z_{HR} = j \left\{ w \frac{l_{eq}}{S_n} - \frac{c^2}{wV_c} \right\} + \frac{w^2}{\pi c^2} \\ l_{eq} = l_n + 1.7r_n, \text{ (neglecting thickness)} \end{cases}. \quad (3)$$

In above expressions, w is frequency in radian, c is the speed of sound and k is the wave number.

3. Extraction of Metamaterial Properties

Now, using aforementioned electro-acoustic modelling, the transfer matrix of any acoustic filter can be presented as mentioned below. The upper subscript '1' and '2' stands for two different acoustic filter configurations, shown in Fig.3. (c) and (b).

$$TM^1 = \begin{bmatrix} TM_{11}^1 & TM_{12}^1 \\ TM_{21}^1 & TM_{22}^1 \end{bmatrix}, \text{ and } TM^2 = \begin{bmatrix} TM_{11}^2 & TM_{12}^2 \\ TM_{21}^2 & TM_{22}^2 \end{bmatrix} \quad (4)$$

The effective acoustic metamaterial properties such as acoustic impedance (Z_{eff}) and refractive index (n_{eff}) can be extracted using reflection coefficient of the acoustic filter (R) and transmission coefficient of the filter (T) as mentioned below [4].

$$\begin{cases} K_{eff} / K_0 = n_{eff} / Z_{eff}, \text{ and } \rho_{eff} / \rho_0 = n_{eff} Z_{eff} \\ Z_{eff} = \frac{r}{1 - 2R + R^2 - T^2}, \text{ and } n_{eff} = \frac{-j \log x + 2\pi m}{kd}, \\ \text{where } r = \mp \sqrt{(R^2 - T^2 - 1)^2 - 4T^2}, x = \frac{1 - R^2 + T^2 + r}{2T} \end{cases} \quad (5)$$

In order to extract the aforementioned properties, the reflection coefficient (R) and transmission coefficient (T) are essential which can be extracted from the corresponding transfer matrix using below mathematical relations [5].

$$R = \frac{T_{11} - \rho c T_{21}}{T_{11} + \rho c T_{21}}, \text{ and } T = \frac{2e^{ikd}}{T_{11} + T_{12} / \rho c + \rho c T_{21} + T_{22}} \quad (6)$$

The transmission loss can be estimated as

$$TL = 20 \log_{10} \left(\frac{1}{T} \right) \quad (7)$$

4. Numerical Modelling

From design prospective numerical modelling is essential, particularly, from industrial prospective. In line with this, first a Helmholtz resonator configuration is taken from literature for analysis as shown in Fig.1 (a) [6]. The modelling and analysis has been carried out in COMSOL platform which is a boundary element based (BEM) based software. The analyses, mimicking two source experiment ASTM E2611, have been carried. The sound pressure from four different microphone positions (m_1 to m_4 mentioned in Fig.1 have been used to extract the corresponding transfer matrix following ASTM E2611 standard. The perfectly matched layer (PML) has been used at outlet to simulate the non-reflecting boundary condition)

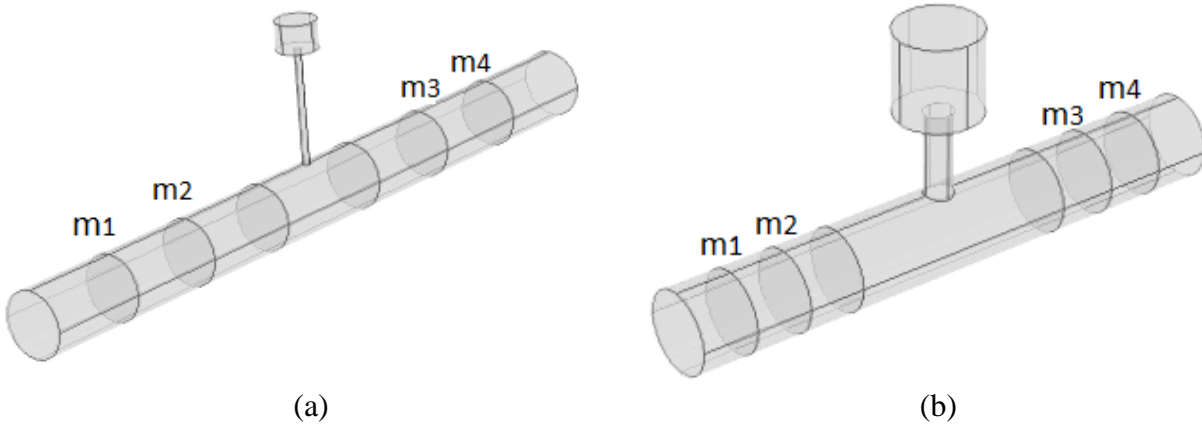


Fig.1. Numerical model of Helmholtz resonators; (a) Helmholtz resonator from literature [6], and (b) experimental Helmholtz resonator

The corresponding results such as real part of effective mass density ratio and effective compressibility ratio extracted using aforementioned equations are shown in Fig.2. It can be noted that if outlet is modelled by applying characteristic acoustic impedance ($Z_{air} = \rho c$) as boundary conditions, same fact also is observed. Analysing results, it can be concluded that the analytical result agrees adequately with numerical result. So now we can do further analyses using above said technique to understand the response of a single Helmholtz resonator and two Helmholtz resonators separated by a duct in detail from acoustic metamaterial point of view

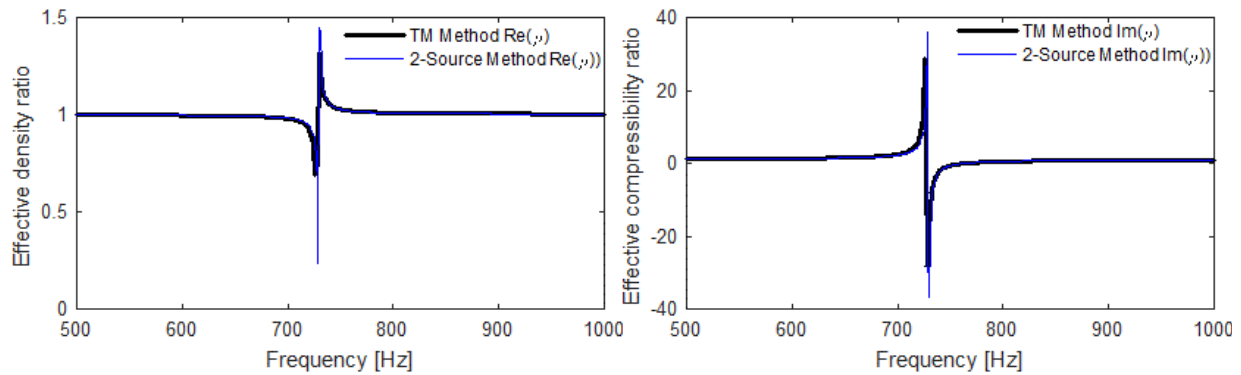


Fig.2. (a) Effective mass density ratio, and (b) Effective compressibility ratio of Helmholtz resonator [6].

5. Experiment

So as to authenticate the analytical modelling, subsequently, a Helmholtz resonator and two Helmholtz resonators separated by a duct have been fabricated using 3D printing for experimental investigation as shown in below Fig.3. The neck radius, neck length, and cavity radius of Helmholtz resonators are 5mm, 30mm, and 15mm respectively. However, the cavity length of smaller one is 30mm and the larger one is 37.5mm and the duct length attached to each resonator is 80mm where the resonator is mounted on the mid of the duct. The experiments have been conducted with a B&K[®] make transmission loss tube and corresponding results against analytical results have been shown in Fig.4. From experiments, the potential of transfer matrix method again gets re-established through adequate agreement with experimental results, which motivate us for further investigation.

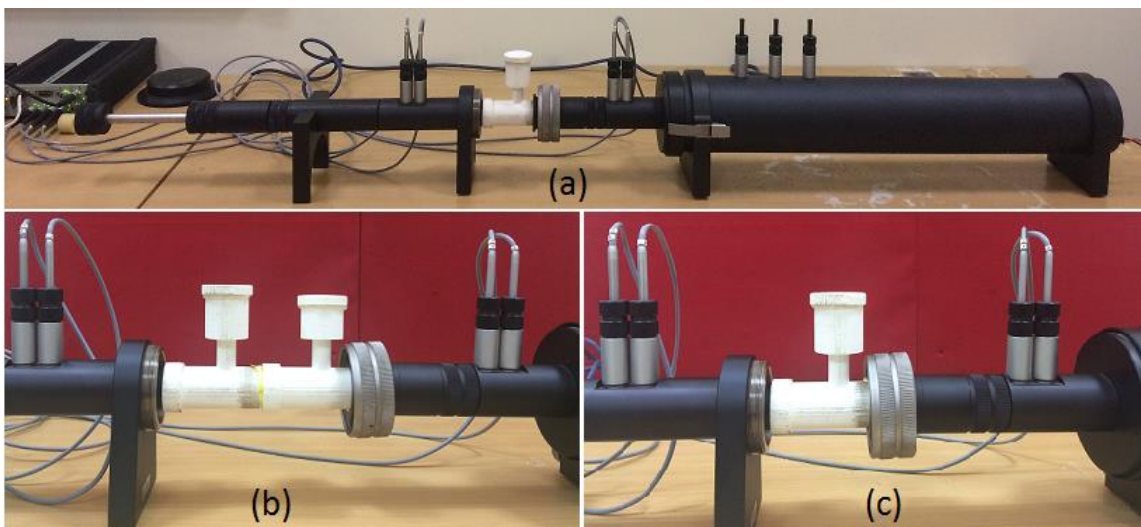


Fig.3. Experimental test setup; (a) B&K[®] make transmission loss tube, (b) two Helmholtz resonators separated by a duct, and (c) single Helmholtz resonator fitted in test setup (the experiments have been carried out at IIT Bhubaneswar).

6. Results and Discussion

Now, the objective is to understand the possibilities of estimating effective material properties of a Helmholtz resonator and an array using linear electro-acoustic modelling. Accordingly, first the transfer matrix of the single Helmholtz resonator has been estimated. The transmission and reflection coefficients have been extracted using aforementioned Equation (6).

Subsequently, those coefficients have been used in Equation (5) for estimating the effective material properties. In a similar fashion, the numerical model as discussed earlier has been analysed to estimate the transfer matrix using two source conditions. The numerically extracted transfer matrix has also been processed using aforesaid equations to estimate the effective material properties. The corresponding results have been shown in Fig.5.

In a similar fashion, the array of Helmholtz resonators has also been analysed. The corresponding analytical and numerical results have been shown in Fig.6. From above analyses, it can be noticed that in case of single Helmholtz resonator, the real part of compressibility only gets negative in resonant frequency. However, in case of array the real part of both effective mass and compressibility get negative. The effect may be justified with the phenomena of resonance considering the array as a single meta-atom.

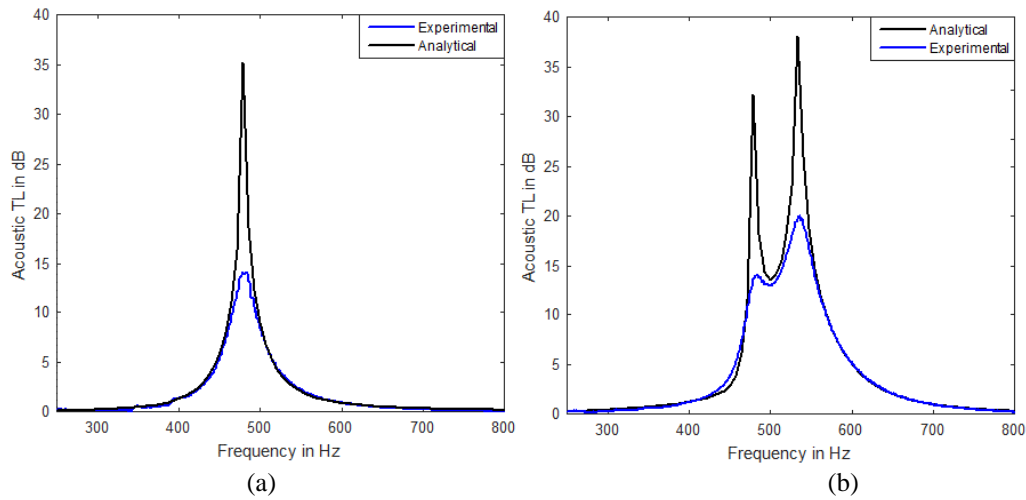


Fig.4. Acoustic transmission loss of Helmholtz resonators; (a) single Helmholtz resonator, (b) two Helmholtz resonators separated by a duct.

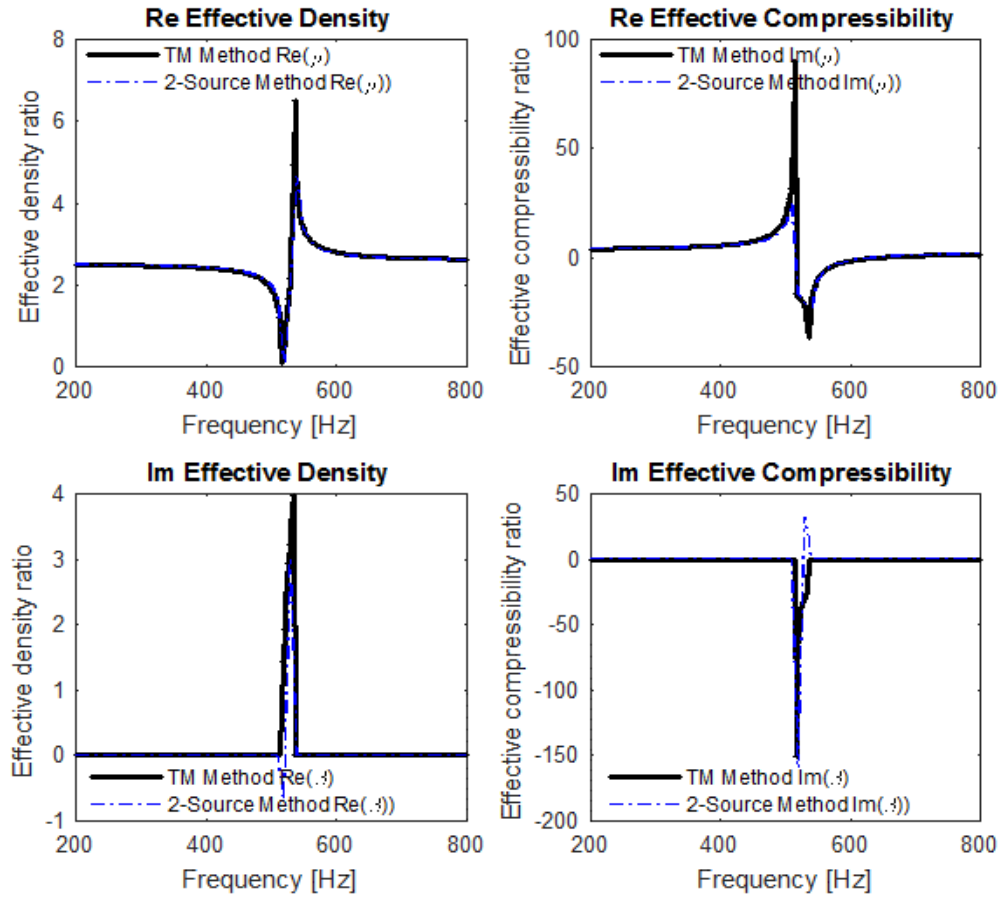


Fig.5. Effective metamaterial properties of a single Helmholtz resonator (“Re” stands for real part and “Im” stands for imaginary part).

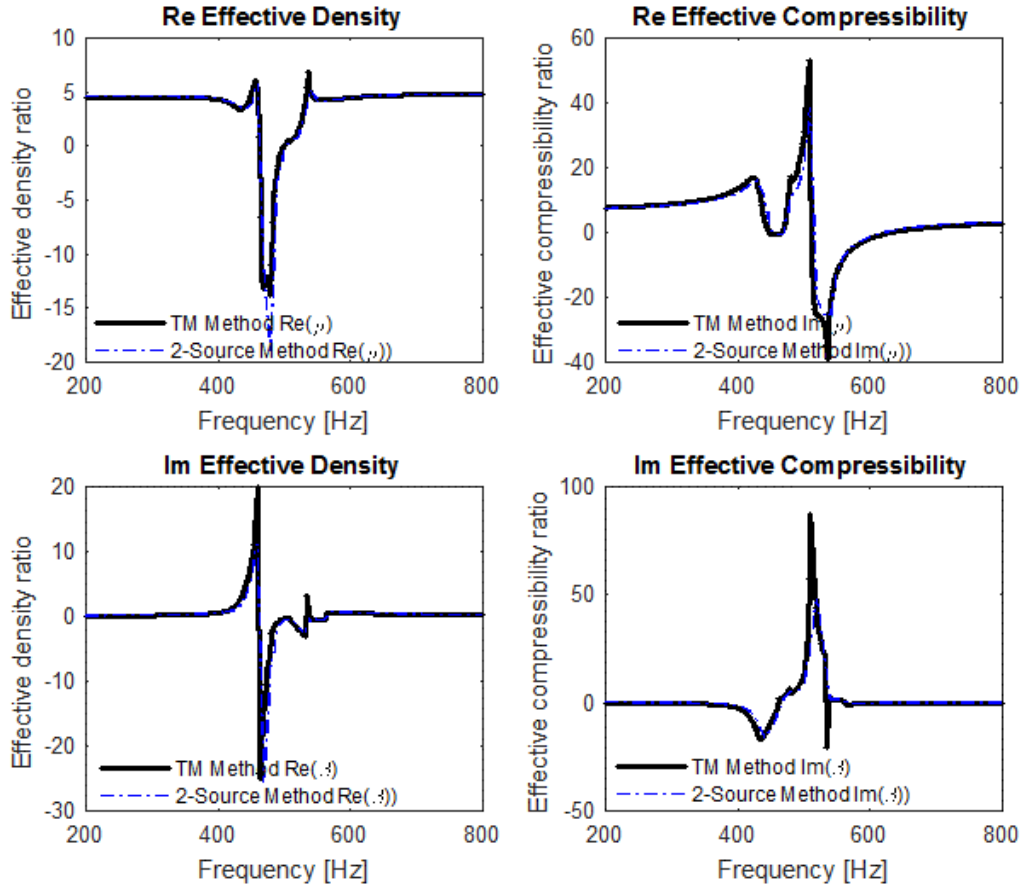


Fig.6. Effective metamaterial properties of two Helmholtz resonators separated by a duct (“Re” stands for real part and “Im” stands for imaginary part).

7. Conclusion

From experimental analyses, it is re-established that the linear acoustic modelling can be used to extract almost accurate transfer matrix of an acoustic filter. Moreover, it can be noticed that the corresponding transfer matrix can be used to extract the transmission and reflection coefficient to estimate the effective acoustic metamaterial properties such as effective mass density and effective compressibility or bulk modulus. The numerical and analytical results agree to each other adequately. The above report also may be perceived that the layers of acoustic metamaterial is possible to model using corresponding transfer matrix, based on linear electro-acoustic modelling theory, if the impedance of the meta-atom is known. Last but not the least, the potential of numerical technique, mimicking experiment of two source method defined by ASTM E2611, has been demonstrated to be effective in estimating the effective metamaterial properties in case of unknown metamaterial which is essential from industrial design prospective.

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