Large-Scale Antenna System Performance with Imperfect CSI in Cooperative Networks

Varun Kumar, Poonam Singh
Department of Electronics & Communication
National Institute of Technology, Rourkela, India
email: varun001986@hotmail.com, psingh@nitrkl.ac.in

Sarat Kumar Patra
Indian Institute of Information Technology, Vadodara
email:skpatra@iiitvadodara.ac.in

***Abstract*—In this paper, we have considered a relay assisted cooperative network, where relay station (RS) and base station (BS) have very large but finite number of antenna. The data detection is done by linear zero forcing (ZF) technique assuming BS and RS have imperfect channel state information (CSI). We derive a new analytical expression for the uplink rate in different channel imperfection scenario. In single hop signal transmission from mobile users (MU) to BS via RS, large number of RS and BS antennas play a vital role over the fixed channel error variance. For dealing with large MIMO, we have used the property of random matrix theory (specially Wishart Matrix decomposition). We have drawn the relation, where uplink rate in single hop signal transmission is the function of number of RS and BS antenna, both link channel error variance and other parameter also. Keeping other parameters constant the uplink rate vs number of RS and BS antenna has been numerically validated for large MIMO perspective under suitable simulation parameter.**

Keywords: Massive MIMO, zero forcing, cooperative network, imperfect CSI

I. Introduction

Demand for high data rate in mobile and fixed devices is increasing with rapid rate. Massive MIMO is the promising candidate for attaining this demand [1]. The beneficial feature of this emerging technology incorporate the extensive usage of low power RF components, reduced latency, robustness against intentional jamming e.t.c [2]. Large number of antenna across the BS support substantial multiplexing and diversity gain, but under perform in multi cellular scenario during pilot reuse causes pilot contamination [3]. Motivated by [3], the author of [4] developed a new multi-cell MMSE based precoding that mitigate this problem. In same problem, the author of [5] suggested that the base station (BS) coordination can mitigate the problem of channel imperfection arises during channel estimation in TDD based multi-cell scenario.

In single cell scenario, relay assisted cooperative network support better link reliability and larger coverage. If cooperative network empowered by massive MIMO technology then the objective of network densification [6] and spatial densification both can be achieved according to the current demand. In dual hop signal transmission, where data detection is done through ZF technique, the addition of multiple node can be beneficial or detrimental over the outage probability [7]. In [7], perfect CSI and small scale MIMO has been considered for system performance evaluation. Massive MIMO has also been implemented for relay based cooperative network, where relay is enabled with very large number of antenna [8]. In [8], wireless channel for two link i.e, form MU to RS and RS to destination nodes are considered to be perfect and system performance is examined and compared using MRT/MRC, ZF precoder/combiner technique. The author of [9] claims that the MMSE-SIC and Tabu search (TS) detectors, which both can operate very close to ML detector compare to ZF and MMSE. But in [8], this assumption is not viable in case of cellular application, because RS is the low power device and it has less flexibility in terms of large number antenna integration compare to BS. Due to random nature of channel, large mobility of mobile user (MU) and and presence of mobile environment causes channel imperfection also in single cell scenario. Massive MIMO in single cell and multi-cell scenario have been jointly addressed in case of perfect and imperfect CSI, where achievable rate and energy efficiency are the main performance metrics [10]. In recent work, massive MIMO system performance has been analyzed in single cell scenario with imperfect reciprocity and channel estimation error, where one BS assist multiple MU [11]. In single cell scenario, channel imperfection has not been addressed for a cooperative network where RS and BS both are enabled with large number of antenna, which may be one of the possible research direction.

Motivated by above research and case study, the major contribution of this paper can be summarized as follow

(i) Analyzing the impact of large MIMO based cooperative network in case of imperfect channel scenario. All analysis consider channel estimation error to show the compound effects on the uplink rate of the additive and multiplicative error.
(ii) Using random matrix theory, uplink rate is deterministically approximated, where system variables are number of RS and BS antenna, minimum detectable SNR (γ_{th}) and expected link imperfection factor.

The remainder of this paper is organized as follows. The system model is presented in Section II. In Section III numerical results of the derived mathematical model over uplink rate performance are analyzed while limitation and concluding remarks are presented in Section IV.

Notation: Superscript $(^H)$ stands for the conjugate transpose and I_K is $K \times K$ identity matrix. The expectation operation and the trace operator are denoted by $\mathbb{E}(.)$ and $Tr(.)$ respectively.

II. System Model

A relay assisted cooperative network having M_s and M_r number of antenna across the BS and RS, whereas K number of mobile user (MU) located at the cell edge boundary is shown in Fig.1. M_s and M_r are considered to be very large such that $M_s > M_r$, because there is a practical limitation for accommodating very large number of antenna across relay station (RS) compare to BS in cellular scenario. We have ignored the impact of direct path and only relay assisted path is our prime concern. From Fig.1, this system model comprises the uplink scenario, where end-to-end signal transmission occur in two time slot. In 1^{st} time slot, MU transmit signal to the RS, whereas in 2^{nd} time slot, received signal is amplified and is further transmitted to the BS using amplify and forward (AF) relaying technique.

So the received signal across RS in 1^{st} time slot can be expressed as

$$Y_r = \sqrt{p_u}\hat{H}_{ur}X + n_{ur} \tag{1}$$

where p_u, $\hat{H}_{ur}$ and X are the transmitted power by each user, $M_r \times K$ channel matrix and $K \times 1$ transmitted symbol vector such that $\mathbb{E}(X^H X) = I_K$ respectively. n_{ur} is the $M_r \times 1$ circularly symmetric complex Gaussian (CSCG) vector, where $\mathbb{E}(n_{ur}^H n_{ur}) = I_{M_r}$. Here $\hat{H}_{ur}$ is a perfect channel matrix and imperfectly estimated channel matrix is H_{ur}. Mathematically it can be expressed as

$$H_{ur} = \sqrt{1-\tau_1^2}\hat{H}_{ur} + \tau_1\bar{H}_{ur} \tag{2}$$

where τ_1 is the degree of channel accuracy and $\bar{H}_{ur}$ is the $M_r \times K$ independent uncorrelated channel matrix with respect to perfect channel matrix $\hat{H}_{ur}$. Due to imperfectly estimated channel matrix H_{ur}, the detected data symbol vector across the RS using zero forcing (ZF) detection technique can be expressed as

$$Y_{dr} = (H_{ur}^H H_{ur})^{-1} H_{ur}^H (\sqrt{p_u}\hat{H}_{ur}X + n_{ur}) \tag{3}$$

Expanding (3), the detected signal vector under imperfect channel condition can be expressed as

$$Y_{dr} = \underbrace{\sqrt{1-\tau_1^2}\sqrt{p_u}(H_{ur}^H H_{ur})^{-1}|\hat{H}_{ur}|^2 X}_{S_1} + \sqrt{1-\tau_1^2}\sqrt{p_u}\times \underbrace{(H_{ur}^H H_{ur})^{-1}\bar{H}_{ur}^H\hat{H}_{ur}X}_{I_1} + \underbrace{(H_{ur}^H H_{ur})^{-1}H_{ur}^H n_{ur}}_{N_1} \tag{4}$$

Detected symbol vector Y_{dr} is the sum of signal (S_1), interference (I_1), noise (N_1) component respectively, which are considered to be statistically independent. This detected symbol vector is further transmitted to BS using AF relaying technique. So the received signal in 2^{nd} time slot at BS end can be expressed as

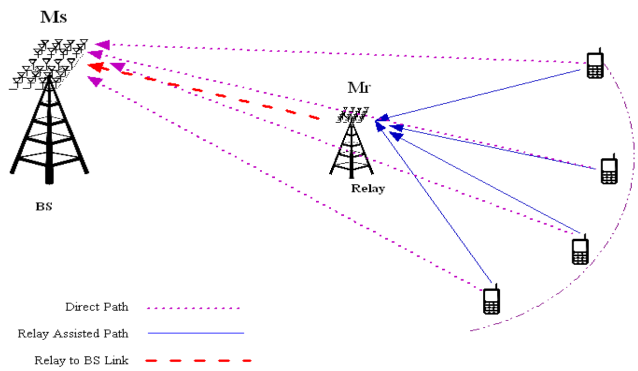

Fig. 1: Relay assisted cooperative network enabled with large antenna across relay and BS

$$Y_b = \frac{1}{\sqrt{M_r}}\zeta_{rb}\hat{H}_{rb}W_{rb}Y_{dr} + n_{rb} \tag{5}$$

$\hat{H}_{rb}$ and W_{rb} are the $M_s \times M_r$ channel matrix carry Gaussian input and $M_r \times K$ precoding matrix such that $\left(\frac{1}{M_r}W_{rb}^H W_{rb} = I_K\right)$ respectively. $n_{rb} \sim \mathcal{CN}(0,1)$ is the $M_s \times 1$ noise vector, where $\mathbb{E}(n_{rb}^H n_{rb}) = I_{M_s}$. ζ_{rb} is the amplification factor which can be represented as

$$\zeta_{rb} = \sqrt{\frac{p_r}{P_{S_1} + P_{I_1} + P_{N_1}}} \tag{6}$$

where p_r is the power transmitted by RS. P_{S_1}, P_{I_1} and P_{N_1} are the total power of signal, interference due to channel error and noise component respectively. Here $P_{S_1} = \mathbb{E}(S_1^H S_1)$, $P_{I_1} = \mathbb{E}(I_1^H I_1)$, $P_{N_1} = \mathbb{E}(N_1^H N_1)$, whereas expected cross power term are considered to zero. Using (4), these three power component can be expressed as

$$P_{S_1} = Tr\Big[p_u(1-\tau_1^2)\mathbb{E}\big\{\big(H_{ur}^H H_{ur}\big)^{-2}\big\}\mathbb{E}\big\{\big(\hat{H}_{ur}^H\hat{H}_{ur}\big)^2\big\}\Big] \tag{7}$$

$$P_{I_1} = Tr\Big[p_u(1-\tau_1^2)\mathbb{E}\big\{\big(H_{ur}^H H_{ur}\big)^{-2}\big\}\mathbb{E}\big\{\big(\hat{H}_{ur}^H\bar{H}_{ur}^H\bar{H}_{ur}\hat{H}_{ur}\big)\big\}\Big] \tag{8}$$

$$P_{N_1} = Tr\Big[\mathbb{E}\big\{\big(\hat{H}_{ur}^H\hat{H}_{ur}\big)^{-1}\sigma_{n_{ur}}^2\big\}\Big] \tag{9}$$

$\hat{H}_{rb}$ is the perfect channel matrix from RS to BS link. Imperfectly estimated channel matrix H_{rb} can be expressed as

$$H_{rb} = \sqrt{1-\tau_2^2}\hat{H}_{rb} + \tau_2\bar{H}_{rb} \tag{10}$$

where τ_2 is the degree of accuracy of the wireless link between RS and BS. $\bar{H}_{rb}$ is the $M_s \times M_r$ perfectly independent and statistically uncorrelated to the channel matrix $\hat{H}_{rb}$. In this scenario, it is assumed that BS knows the precoding matrix.

From (5), let $\hat{G}_{rb} = \hat{H}_{rb}W_{rb}$ and $G_{rb} = H_{rb}W_{rb}$ are the $M_s \times K$ equivalent channel matrix, where G_{rb} and $\hat{G}_{rb}$ are considered as imperfect and perfect equivalent. We apply the ZF detection technique considering K number of data symbol transmission from transmitter (Tx) end to receiver (Rx) end. Applying ZF detection on received signal

$$Y_{b_d} = (G_{rb}^H G_{rb})^{-1} G_{rb}^H Y_b$$
$$\Rightarrow \big(W_{rb}^H H_{rb}^H H_{rb} W_{rb}\big)^{-1} (W_{rb}^H H_{rb}) Y_b$$
$$\big(W_{rb}^H H_{rb}^H H_{rb} W_{rb}\big)^{-1} (W_{rb}^H H_{rb}) \Big(\frac{1}{\sqrt{M_r}} \zeta_{rb} \hat{H}_{rb} W_{rb} Y_{dr} + n_{rb} \Big) \tag{11}$$

From (11), the received signal vector Y_b consist signal, interference and noise component respectively. Expanding (11) using (4), (6) and (10), the signal, interference and noise component can be expressed as

$$S_2 = \frac{\zeta_{rb}}{\sqrt{M_r}} \big(W_{rb}^H H_{rb}^H H_{rb} W_{rb}\big)^{-1} (W_{rb}^H \sqrt{1-\tau_2^2} \hat{H}_{rb}^H) \times$$
$$\hat{H}_{rb} W_{rb} \Big(\sqrt{1-\tau_1^2} \sqrt{p_u} (H_{ur}^H H_{ur})^{-1} |\hat{H}_{ur}|^2 X \Big) \tag{12}$$

where S_2 is the $K \times 1$ detected symbol vector. Similarly interference generated in AF relaying scheme can be expressed as

$$I_2 = \frac{\zeta_{rb}}{\sqrt{M_r}} \big(W_{rb}^H H_{rb}^H H_{rb} W_{rb}\big)^{-1} \times (W_{rb}^H \tau_2 \bar{H}_{rb}) \hat{H}_{rb} W_{rb}$$
$$\Big(\sqrt{1-\tau_1^2} \sqrt{p_u} (H_{ur}^H H_{ur})^{-1} |\hat{H}_{ur}|^2 X \Big)$$
$$+ \frac{\zeta_{rb}}{\sqrt{M_r}} \big(W_{rb}^H H_{rb}^H H_{rb} W_{rb}\big)^{-1} (W_{rb}^H H_{rb}) \hat{H}_{rb} W_{rb}$$
$$\Big(\sqrt{1-\tau_1^2} \sqrt{p_u} (H_{ur}^H H_{ur})^{-1} \bar{H}_{ur}^H \hat{H}_{ur} X + (H_{ur}^H H_{ur})^{-1} H_{ur}^H n_{ur} \Big) \tag{13}$$

where I_2 is the interference component observed in 2^{nd} time slot. Similarly the detected noise vector can be expressed as

$$N_2 = \big(W_{rb}^H H_{rb}^H H_{rb} W_{rb}\big)^{-1} (W_{rb}^H H_{rb}) n_{rb} \tag{14}$$

The detected signal, interference and noise component are considered as statistically independent and their cross covariance is supposed to be null matrix. We deterministically approximated the auto covariance matrix of signal, interference and noise component using following **Theorem 1**.

Theorem 1: For a central Wishart matrix $W \sim \mathcal{W}_m(n, I)$ with $n > m$ [12]

$$\mathbb{E}[tr(W^2)] = mn(m+n) \tag{15}$$

$$\mathbb{E}[tr^2(W)] = mn(mn+1) \tag{16}$$

whereas central Wishart matrix $W \sim \mathcal{W}_m(n, I)$ with $n > m+1$

$$\mathbb{E}[tr(W^{-1})] = \frac{m}{(n-m)} \tag{17}$$

$$\mathbb{E}[tr(W^{-2})] = \frac{mn}{(n-m)^3 - (n-m)} \tag{18}$$

Based on the closed-form expression of the Wishart matrix, similar intuition have been drawn for the matrix Z which has been given in *Proposition 1*.

Proposition 1: Given $U \sim \mathcal{CN}(0,1)^{M \times N}$ and $V \sim \mathcal{CN}(0,1)^{N \times P}$ are two random matrix and Z is another matrix of size $M \times P$ such that $Z = UV$ and also $M > N > P$. If above property is applied for the product matrix Z, the closed-form expression can be write as

$$\mathbb{E}[Tr\{(Z^H Z)^{-1}\}] = \frac{P}{(M-P)(N-P)} \tag{19}$$

[1]**Note:** $W = A^H A$ and $A \sim \mathcal{CN}(0,1)$ be the $n \times m$ matrix.

Similarly in case of inverse square scenario ie

$$\mathbb{E}[Tr((Z^H Z)^{-2})] = \frac{MNP}{\{(M-P)(N-P)\}^3 - (M-P)(N-P)} \tag{20}$$

On the other side product square can be expressed as

$$\mathbb{E}[Tr\{(Z^H Z)^2\}] = a_0 (MN)^2 P \tag{21}$$

where $a_0 = \sqrt{\frac{\pi}{2}}$ is the scaling factor. From (12), (16), (18) and (20) the auto covariance matrix, which shows the expected signal power covariance matrix can be expressed as

$$P_s = \mathbb{E}(S_2^H S_2)$$
$$\Rightarrow \sqrt{\frac{\pi}{2}} \frac{\zeta_{rb}^2}{M_r} \frac{p_u (1-\tau_1^2)(1-\tau_2^2)(M_s M_r)^3}{(M_s - K)(M_r - K)\}^3 - (M_s - K)(M_r - K)} \times$$
$$\frac{M_r^2 (M_r + K)}{(M_r - K)^3 - (M_r - K)} I_K \tag{22}$$

From (13), (15), (17) and (20) the auto covariance matrix, which shows the expected interference power covariance matrix. Mathematically it can be expressed as

$$P_I = \mathbb{E}(I_2^H I_2)$$
$$\Rightarrow \zeta_{rb}^2 \frac{p_u (1-\tau_1^2) \tau_2^2 M_s^2 M_r^4 (M_r + K)}{(M_s - K)(M_b - K)\}^3 - (M_s - K)(M_r - K)} \times$$
$$\frac{1}{(M_r - K)^3 - (M_r - K)} I_K + \zeta_{rb}^2 (1-\tau_2^2) M_s^3 M_r^2 \times$$
$$\frac{1}{\{(M_s - K)(M_r - K)\}^3 - (M_s - K)(M_r - K)} \times$$
$$\Big(\frac{p_u (1 - \tau_1^2 M_r^2 K)}{(M_r - K)^3 - (M_r - K)} + 1 \Big) I_K \tag{23}$$

Similarly from (14) and (19) the auto covariance matrix, which shows the expected noise power covariance matrix can be expressed as

$$P_N = \mathbb{E}(N_2^H N_2) = \frac{1}{(M_s - K)(M_r - K)} I_K \tag{24}$$

So observed covariance matrix for SINR at the BS end can be expressed as

$$\Upsilon = \Big(\frac{P_s}{P_I + P_N} \Big) = P_s (P_I + P_N)^{-1}$$
$$\Rightarrow \begin{bmatrix} \gamma_1 & \epsilon_{12} & \epsilon_{13} & \cdots & \epsilon_{1K} \\ \epsilon_{21} & \gamma_2 & \epsilon_{23} & \cdots & \epsilon_{2K} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \epsilon_{K1} & \epsilon_{K2} & & \cdots & \gamma_K \end{bmatrix} \tag{25}$$

TABLE I: Simulation Parameter

Distance between user and relay	500 m
Distance between relay and base station	500 m
Path loss exponent	2.87
Detector used across relay and base station	ZF
Number of user	5

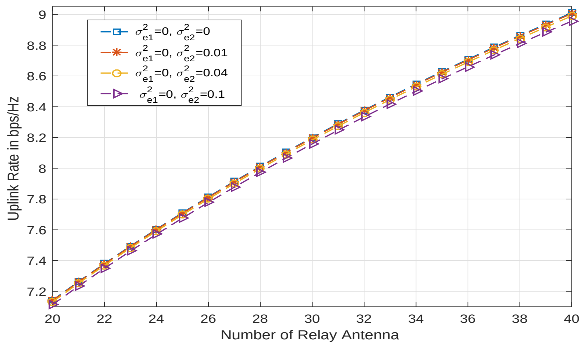

Fig. 2: Uplink rate vs M_r, when MU$\rightarrow$ RS link have perfect channel and RS $\rightarrow$ BS link have imperfect CSI

From (25), $\Upsilon_{ij_{i \neq j}} = \epsilon_{ij} \rightarrow 0$ and $\Upsilon_{ij_{i=j}} = \gamma_i$ In such a scenario BS observe the capacity due to k^{th} user signal transmission can be expressed as

$$C = \frac{1}{2} \log_2 \left(det \left| I_K + \Upsilon \right| \right) \tag{26}$$

III. Result Discussion

In this section analytical results have been numerically validated using suitable simulation parameter which has been shown in Table I. Expected channel error variance between single MU to per antenna terminal of relay link is $\sigma^2_{e1} = \tau^2_1$. Similarly $\sigma^2_{e2} = \tau^2_2$ is the expected channel error variance between each antenna of relay terminal to the each antenna of BS. We have considered five MU at the cell edge boundary ie K=5. For doing simulation 1000 times channel realization have taken and it is similar to the derived analytical expression. Relay is placed in between the MU to BS. Large antenna across RS and BS are our system variable, whereas channel impairment is our performance constraint. Since large number of antennas have been considered across RS and BS, so impact of relay antenna under different channel impairment constraints have been shown in Fig.2, Fig.4 and Fig.6, whereas impact of BS antenna under same channel impairment constraints have been shown in Fig.3, Fig.5 and Fig.7. Based on channel impairment constraints, we observe our analytical formulation through three case study.

A. *Case 1: $\tau_1 = 0$ and $\tau_2 \neq 0$*

In this case it is assumed that there is no channel error between MU to RS or in another sense relay knows the channel information for uplink signal transmission, whereas RS to BS link suffers from channel impairment. Due to increment of M_r keeping $M_s = 50$ to be constant, the uplink rate under different channel error variance condition ie (1%, 4% and

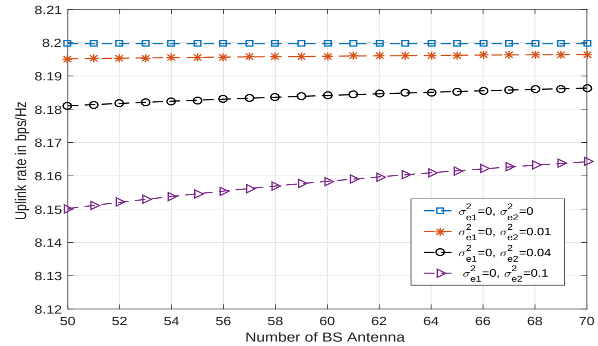

Fig. 3: Capacity per user vs M_s, when MU$\rightarrow$ RS link have perfect channel and RS $\rightarrow$ BS link have imperfect CSI

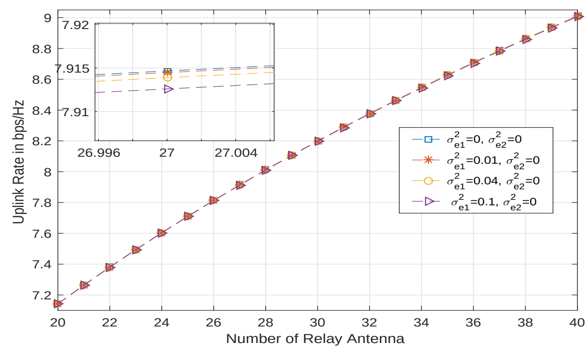

Fig. 4: Uplink rate vs M_r, when RS $\rightarrow$ BS link have perfect channel and MU $\rightarrow$ RS link have imperfect CSI

10%) has been shown in Fig.2. From Fig.2 we can observe two thing.

(i) Capacity increases with increase of M_r.
(ii) Capacity reduction with increasing the channel variance.

On another side, if M_s increases keeping M_r fixed, the capacity performance gives the flat response, when perfect channel is considered for (MU$\rightarrow$RS) and (RS$\rightarrow$BS) link. If channel impairment comes into RS$\rightarrow$ BS link then capacity increase by increasing the M_s with minimal rate. For large channel error variance let 10%, uplink rate increases more by increasing the M_s, which can be easily visualized by Fig.3.

B. *Case 2: $\tau_1 \neq 0$ and $\tau_2 = 0$*

In this case, channel impairment is considered across MU $\rightarrow$ RS link, whereas RS$\rightarrow$ BS link have perfect channel information. If M_r is increased keeping M_s fixed, the capacity performance increases with rapid rate. Unlike Fig.2, in such scenario channel error impact does play a vital role. From Fig.4 it can be easily observed that if MU$\rightarrow$ RS link suffers nearly 1%, 4% and 10% channel error variance, the uplink rate is nearly same as the error free channel. On another side if M_s is increased keeping M_r fixed, the capacity performance nearly flat. By increasing the channel error variance, capacity decreases but response remain flat, which has been shown in

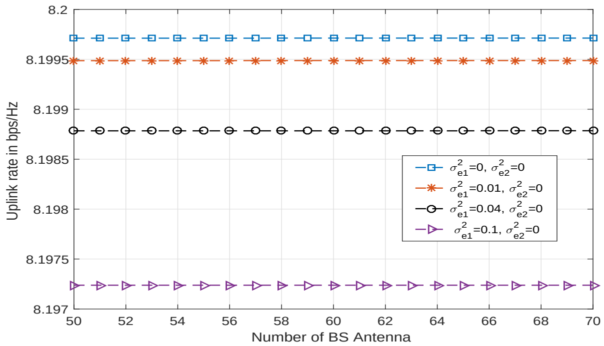

Fig. 5: Capacity per user vs M_s, when RS $\rightarrow$ BS link have perfect channel and MU$\rightarrow$ RS link have imperfect CSI

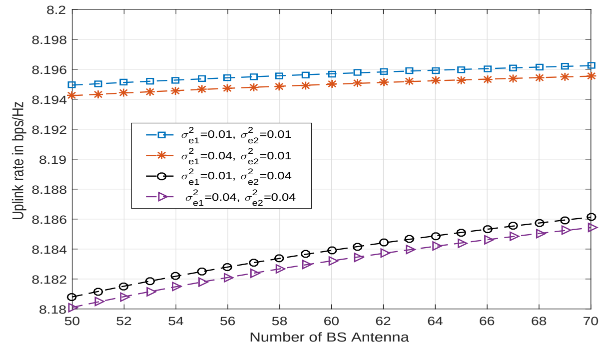

Fig. 7: Capacity per user vs M_s, when MU$\rightarrow$ relay link and relay $\rightarrow$ BS link both have imperfect CSI

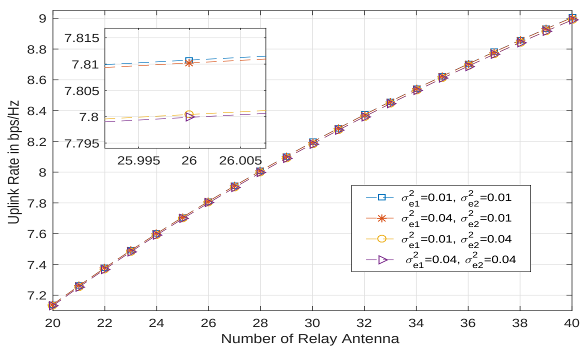

Fig. 6: Uplink rate vs M_r, when MU$\rightarrow$ RS link and RS $\rightarrow$ BS link both have imperfect CSI

Fig.5. Unlike Fig.3, capacity does not increases by increasing M_s.

C. Case 3: $\tau_1 \neq 0$ and $\tau_2 \neq 0$

In this case, channel impairments are considered across both the wireless link ie (MU$\rightarrow$ RS and RS $\rightarrow$ BS). From Fig.6, the uplink rate has been compared and analyzed, when M_r is increased keeping M_s fixed. Let channel error variance be 1% and 4% for the MU$\rightarrow$RS and RS$\rightarrow$BS link also 4% and 1% channel error variance for MU$\rightarrow$RS and RS$\rightarrow$BS link then the performance results is not symmetric. From Fig.6 it can be visualized that 1% and 4% error variance provide nearly equal performance like 4% and 4% error variance across the MU$\rightarrow$RS and RS$\rightarrow$BS link. From Fig.6 it is also clear that 4%, 1% channel error variance across MU$\rightarrow$RS and RS$\rightarrow$BS link have nearly equal performance compare to 1% and 1% channel error variance. On the other side if M_s is increased keeping M_r fixed then by increasing the channel error variance capacity performance decreases. From Fig.7 it can be easily observed that channel error for MU $\rightarrow$ RS have less impact over capacity compare to RS$\rightarrow$BS link when both the links are not perfectly estimated.

IV. Conclusion

In this paper we have investigated the impact of channel impairment on large scale cooperative MIMO network. Relay plays a vital role, if it is enabled with very large antenna. Larger antenna across relay have greater impact on capacity compare to BS side. In another sense, AF forward relaying causes no significant diversity gain improvement due to increment of BS antenna. Accommodating large number of antenna across RS is not viable for cellular application compare to BS. In case of channel impairment condition BS antenna increment provides better capacity.

References

[1] T. L. Marzetta, “Massive MIMO: an introduction,” *Bell Labs Technical Journal*, vol. 20, pp. 11–22, 2015.

[2] E. G. Larsson, O. Edfors, F. Tufvesson, and T. L. Marzetta, “Massive MIMO for next generation wireless systems,” *IEEE Communications Magazine*, vol. 52, no. 2, pp. 186–195, 2014.

[3] T. L. Marzetta, “Noncooperative cellular wireless with unlimited numbers of base station antennas,” *Wireless Communications, IEEE Transactions on*, vol. 9, no. 11, pp. 3590–3600, 2010.

[4] J. Jose, A. Ashikhmin, T. L. Marzetta, and S. Vishwanath, “Pilot contamination and precoding in multi-cell TDD systems,” *IEEE Transactions on Wireless Communications*, vol. 10, no. 8, pp. 2640–2651, 2011.

[5] H. Yin, D. Gesbert, M. Filippou, and Y. Liu, “A coordinated approach to channel estimation in large-scale multiple-antenna systems,” *IEEE Journal on Selected Areas in Communications*, vol. 31, no. 2, pp. 264–273, 2013.

[6] N. Bhushan, J. Li, D. Malladi, R. Gilmore, D. Brenner, A. Damnjanovic, R. Sukhavasi, C. Patel, and S. Geirhofer, “Network densification: the dominant theme for wireless evolution into 5G,” *IEEE Communications Magazine*, vol. 52, no. 2, pp. 82–89, 2014.

[7] R. H. Louie, Y. Li, and B. Vucetic, “Zero forcing in general two-hop relay networks,” *IEEE Transactions on Vehicular Technology*, vol. 59, no. 1, pp. 191–202, 2010.

[8] H. A. Suraweera, H. Q. Ngo, T. Q. Duong, C. Yuen, and E. G. Larsson, “Multi-pair amplify-and-forward relaying with very large antenna arrays,” in *Communications (ICC), 2013 IEEE international conference on*. IEEE, 2013, pp. 4635–4640.

[9] F. Rusek, D. Persson, B. K. Lau, E. G. Larsson, T. L. Marzetta, O. Edfors, and F. Tufvesson, “Scaling up MIMO: Opportunities and challenges with very large arrays,” *IEEE Signal Processing Magazine*, vol. 30, no. 1, pp. 40–60, 2013.

[10] H. Q. Ngo, E. G. Larsson, and T. L. Marzetta, “Energy and spectral efficiency of very large multiuser MIMO systems,” *Communications, IEEE Transactions on*, vol. 61, no. 4, pp. 1436–1449, 2013.

[11] D. Mi, M. Dianati, L. Zhang, S. Muhaidat, and R. Tafazolli, “Massive MIMO performance with imperfect channel reciprocity and channel estimation error,” *IEEE Transactions on Communications*, 2017.

[12] A. M. Tulino and S. Verdú, *Random matrix theory and wireless communications*. Now Publishers Inc, 2004, vol. 1.