DYNAMIC RESPONSE AND CONTROL NONLINEAR COUPLED ROLL-PITCH (2DOF) MOTION OF SHIP UNDER HARMONIC WAVES

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ABSTRACT

To control vibration and stability of different linear and nonlinear system using various analytical, semi analytical, numerical method, and the time delayed feedback control has been used in the recent past. The present study deals with the complex dynamic behavior of 2DOF coupled roll-pitch system via Numerical Integration based methods (NI). By choice of appropriate of displacement and velocity feedback gain and delay values which may be introduced in the feedback path, the NI based techniques are well suited in stabilizing motion by elimination of undesirable subharmonic, quasiperiodic and chaotic solutions. Therefore, in the present study, a comprehensive numerical schemes based on NI method is developed to analyze the control of nonlinear 2DOF motion under non-linear time-delayed feedback. Time delay feedback control of nonlinear 2DOF roll-pitch coupled motion under harmonic excitations thoroughly investigated by Numerical Integration method. The periodic solutions are obtained by the NI method and compared with the solutions obtained by without time-delay (i.e. uncontrolled) and with time-delay (i.e. controlled) of the system responses. The objective of this paper is to study the efficiency and performance of Numerical Integration Technique for analysis of time delay feedback control of 2DOF coupled roll-pitch motion under harmonic excitations and also study its efficiency in obtaining suppression of various fundamental by choice of appropriate displacement and velocity feedback gain values as well as proper choice of delay.

Key Words: *Time-delayed feedback, Numerical integration (NI), 2DOF, Velocity feedback.*

INTRODUCTION

Ships are being very useful in the industries which are sea depended or function in areas enclosed by waters. It is one of the most affordable ways to import and export goods through sea routes. All this ships are very useful and effective mode of transportation. So, engineers modelled numerically and analyzed the ship whether the ship is good or bad. Rolling motion indicates side to side of the ship. Pitch motion involves lifting at the bow and lowering at the stern and vice versa. Nowadays, many researchers studies the important phenomenon of roll and pitch motions of ships. It is necessary to know and analyze the maximum amplitudes of ships under stable and unstable beam seas considering the effect of the non-linearity. Due to large displacement and hydrodynamic effects creates complicated nonlinear behaviors of the free floating ship. For certain classes of vessels, static stability standards based on statistical and other analyses of intact static condition are sufficient for design purpose and can give a qualitative understanding of the stability behavior for the naval architect. These include hydrodynamics and viscous roll damping, as affected by the wave exciting force. The motion of the roll pitch coupled nonlinear systems creates complicated dynamic behavior including high amplitude vibration, resonances, quasiperiodic and chaotic motion. In this paper, we discussed the coupled motion of roll pitch of the ship and introduced the time delay feedback control which has been control the response of the two degree of freedom of coupling roll pitch motion of ship. Significant efforts have been made in designing control method for vibration.

MATHEMATICAL MODELLING FOR THE GENERAL ROLL-PITCH COUPLED MOTION

This paper considers a nonlinear coupled roll and pitch motion of the ship under sinusoidal harmonic excitation.

$$\begin{aligned}\ddot{Y}_1 + 2\varepsilon\mu_1\dot{Y}_1 + \omega_1^2 Y_1 + \varepsilon\alpha_1 Y_1 Y_2 &= G_0\cos(\Omega t)\\ \ddot{Y}_2 + 2\varepsilon\mu_2\dot{Y}_2 + \omega_2^2 Y_2 + \varepsilon\alpha_2 Y_1^2 &= F_0\cos(\Omega t)\end{aligned} \tag{1}$$

Where, Y_1 and Y_2 are the roll and pitch mode amplitudes; μ_1 and μ_2 are the mode damping coefficients; ω_1 and ω_2 are the natural angular frequencies of the roll and pitch modes; Ω the excitation or wave frequency; G_0 and F_0 are the excitation force amplitudes of the roll and pitch modes; α_1 and α_2 are nonlinear coefficients; ε is a small perturbation parameter.

Let, $Y_1 = y(1)$ and $\dot{Y}_1 = y(2)$

$Y_2 = y(3)$ and $\dot{Y}_2 = y(4)$

Substituting the values $Y_1, Y_2, \dot{Y}_1$ and $\dot{Y}_2$ in the above equation, we get the following

$$\dot{y}(1) = y(2)$$
$$\dot{y}(2) = G_0\cos(\Omega t) - 2\varepsilon\mu_1 y(2) - \omega_1^2 y(1) - \varepsilon\alpha_1 y(1)y(3)$$
$$\dot{y}(3) = y(4)$$
$$\dot{y}(4) = F_0\cos(\Omega t) - 2\varepsilon\mu_2 y(4) - \omega_2^2 y(3) - \varepsilon\alpha_2 y(1)^2$$

After applying the feedback control the equation becomes

$$\begin{aligned}\ddot{Y}_1 + 2\varepsilon\mu_1\dot{Y}_1 + \omega_1^2 Y_1 + \varepsilon\alpha_1 Y_1 Y_2 &= G_0\cos(\Omega t) + g_{d1}Y_1(t-d) + g_{v1}\dot{Y}_1(t-d)\\ \ddot{Y}_2 + 2\varepsilon\mu_2\dot{Y}_2 + \omega_2^2 Y_2 + \varepsilon\alpha_2 Y_1^2 &= F_0\cos(\Omega t) + g_{d2}Y_2(t-d) + g_{v2}\dot{Y}_2(t-d)\end{aligned} \tag{2}$$

The following different sets of parameters used for analyzing the response behavior of coupled roll-pitch ship motion expressed by Eq. (2)

Table 1: Different sets of parameters

Set	μ_1	μ_2	α_1	α_2	ω_1	ω_2	Ω	G_0	F_0	$\varepsilon = 0.1$
1	0.1	0.2	1.0	2.0	0.94	1.68	1.97	0.01	0.03	0.1
2	0.1	0.2	1.0	2.0	0.94	$2\omega_1$	1.97	0.01	0.03	0.1
3	0.1	0.2	1.0	2.0	0.94	1.68	1.97	1.0	0.03	0.1
4	0.1	0.2	1.0	2.0	0.94	1.68	1.97	0.01	1.2	0.1

RESULTS AND DISCUSSION

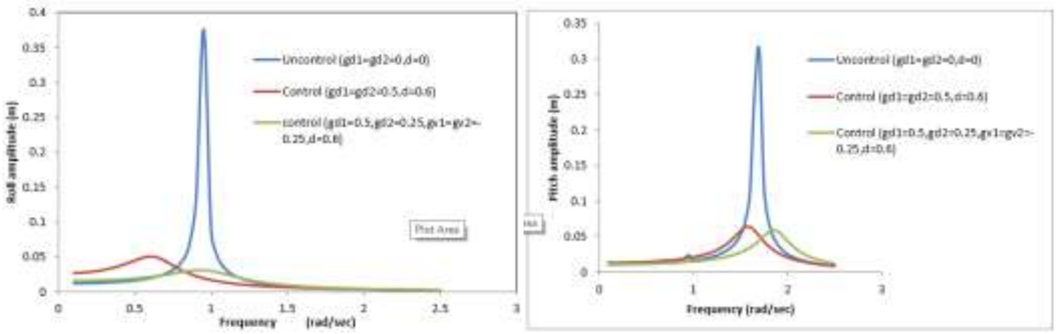
The non-linear coupled roll-pitch motion of 2-DOF systems expressed by Eq. (2) has been investigated for vibration control using time-delayed state (displacement and velocity) feedback. Numerical integration (Runge-Kutta 4th order) method has been applied using MATLAB/SIMULINK software for this purpose. To get a good understanding of controlled responses, the uncontrolled system is also studied. To study the uncontrolled roll-pitch motion we have set all control parameters (i.e., g_{d1}, g_{d2}, g_{v1}, g_{v2}, d) to zero. Studies of vibration control of our said model has been performed by frequency domain analysis. In the frequency domain analysis four different sets of parameters shown in Table (1) are used from Kamel [7]. The first set is for non-resonance condition, second one for resonance condition, third one for studying the effect increasing the force amplitude (G_0) of the roll motion and finally set-4 for studying the effect of increasing the force amplitude (F_0) of the pitch motion. We should mention that the control strategy is based on only displacement feedback (i.e. keeping velocity gain zero), and/or both the displacement and velocity feedback.

FREQUENCY DOMAIN ANALYSIS

Set-1: Non-resonance condition

The set-1 parameters of table 1 are repeated here for convenience, $\mu_1 = 0.1,\ \mu_2 = 0.2,\ \alpha_1 = 1.0,\ \alpha_2 = 2.0,\ \omega_1 = 0.94,\ \omega_2 = 1.68,\ G_0 = 0.01,\ F_0 = 0.03,\ \varepsilon = 0.1,\ h = .025$.

Fig 1(a) shows that the frequency response plots of roll response for a frequency range of 0 to 2.5 rad/sec. It is observed that the roll peak response at 0.375 rad at frequency excitation of Ω=0.9rad/sec. On application of the delayed displacement feedback control with (gd1=gd2=0.5, d=0.6), the peak resonance has been reduced to 0.05 rad at a frequency of 0.65 rad/sec to the left of resonating frequency of 0.9 rad/sec. When both velocity feedback and displacement feedback control (gd1=0.5, gd2=0.25, gv1=gv2=-0.25, d=0.6). The peak response has been reduced to 0.025 rad with almost 93% control peak displacement.



(a) **(b)**

Fig 1: Frequency response curve for the values of $\mu_1 = 0.1,\ \mu_2 = 0.2,\ \alpha_1 = 1.0,\ \alpha_2 = 2.0,$
$\omega 1 = 0.94,\ \omega 2 = 1.68,\ G_0 = 0.01,\ F_0 = 0.03,\ \varepsilon = 0.1,\ h = .025$.

Fig 1 (b) shows that the frequency response plots of pitch response for a frequency range of 0 to 2.5 rad/sec. However the pitch response is observed at 0.325 rad at frequency excitation o

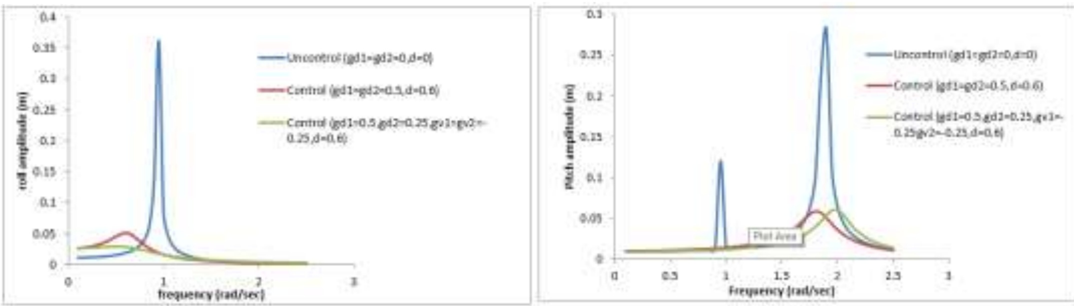
Ω=1.75 rad/sec. After applying the delayed displacement feedback control (gd1=gd2=0.5, d=0.6), the peak response has been decreased from 0.325 rad to 0.06 rad which occurs at Ω =1.6 rad/sec and also reduced to about 81.53% control peak displacement. Similarly when displacement feedback control along with the velocity feedback is applied (gd1=0.5, gd2=0.25, gv1=gv2=0.25, d=0.6), it has been suppressed to about the value of 0.055 rad which occurs at the frequency of 1.85 rad/sec to the right of resonating frequency of 1.6 rad/sec which has a maximum response of uncontrolled motion.

Set-2: Resonance condition

The following system parameters are used for from Table 1. Note that here the values of ω_2 is change as compared to set-1. $\mu_1 = 0.1,\ \mu_2 = 0.2,\ \alpha_1 = 1.0,\ \alpha_2 = 2.0,\ \omega_1 = 0.94,\ \omega_2 = 2\omega_1$ $G_0 = 0.01,\ F_0 = 0.03,\ \varepsilon = 0.1,\ h = .025,\ d = 0.6.$

Now, here we concentrate on the internal resonance case of pitch-roll motion of the ship. Fig 2(a) represents the frequency response plot of roll motion for an excitation frequency range of 0 to 2.5 rad/s. It is observed that the roll response is reached the peak value of 0.36 rad at frequency Ω=0.9 rad/s when all control parameters are set to zero (i.e. uncontrolled system). After introducing the delayed displacement feedback with parameters $g_{d1} = g_{d2} = 0.5$, $g_{v1} = g_{v2} = 0$, and $d = 0.6$, it is observed that the peak response has been reduced up to 0.05 rad which occurs at a frequency of 0.65 rad/s. The resonance curve is shifted towards left due to the application of feedback control .In this case the suppression is observed to be 86%. Further applying both displacement and velocity feedback control of values $g_{d1} = 0.5,\ g_{d2} = 0.25$, $g_{v1} = g_{v2} = -0.25$ and $d = 0.6$, the peak response has been reduced to 0.025 rad at excitation frequency of 0.5 rad/s with a percentage reduction of 93.05.

Fig 2(b) represents the frequency-amplitude response plots of pitch motion for internal resonance case. It shows that in the uncontrolled case we are getting two peaks at frequencies 0.95 and 1.9rad/s with amplitudes respectively 0.12 and 0.28rad. The reason is that with this selection of system parameters, a strong coupling situation takes place. It is interesting to observed that the feedback scheme in the controlled response converts the two peaks to one. This implies the fact that introduction of delayed feedback control is capable to reduce the coupling effect. With control parameters $g_{d1} = g_{d2} = 0.5$, $g_{v1} = g_{v2} = 0$, and $d = 0.6$, the peak response is skewed towards left reducing amplitude to 0.06 rad at frequency of 1.8 rad/s. Finally application of state feedback reduces the peak amplitude as shown in the figure 2.



(a) **(b)**

Fig 2: Frequency response curve for the values of $\mu_1 = 0.1,\ \mu_2 = 0.2,\ \alpha_1 = 1.0,\ \alpha_2 = 2.0,$ $\omega 1 = 0.94,\ \omega 2 = 1.68,\ G_0 = 0.01,\ F_0 = 0.03,\ \varepsilon = 0.1,\ h = .025.$

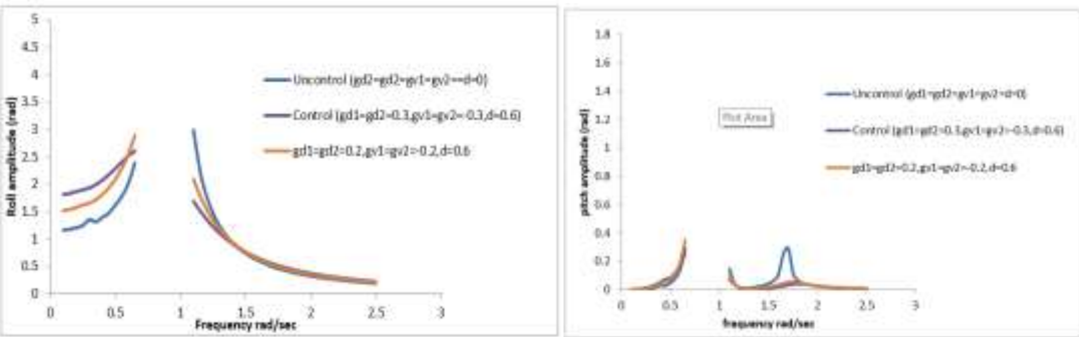
The periodic solutions of the system at various parameters near the internal and sub harmonic resonance cases i.e., $\Omega \cong 2\omega_1, \omega_2 \cong 2\omega_1$ and also for simultaneous resonance cases i.e. $\Omega \cong \omega_1, \omega_1 \cong \omega_2$ have been investigated.

Set-3: Non-Resonance condition

$\mu_1 = 0.1,\ \mu_2 = 0.2,\ \alpha_1 = 1.0,\ \alpha_2 = 2.0,\ \omega 1 = 0.94,\ \omega 2 = 1.68,\ G_0 = 1.0,\ F_0 = 0.03,\ \varepsilon = 0.1,$
$h = .025$.

Next, from figure 3(a) the frequency amplitudes plot for G_0=1.0 and F_0=0.03 has been obtained in the backward sweep of frequency up to the values of $\Omega = 1.15$ rad/sec with the amplitudes response of 2.24 rad. On further sweeping of frequencies, amplitudes of roll response in resonances band of frequencies could not obtained because of numerical instability and convergence problem of numerical integration techniques of the applied method for the said range of frequency. However below $\Omega = 0.65$ rad/sec, the branch of periodic solution has been obtained up to the frequencies of 0.1 rad/sec. After applying the displacement and velocity feedback control $(gd1 = gd2 = 0.3, gv1 = gv2 = -0.3, d = 0.6)$, the same frequency up to the values of $\Omega = 1.15$ rad/sec with the amplitudes is reduced up to 1.512 rad by backward sweeping and some part of the frequency response could not be obtained due to instability of the system. Integrates the system by forward sweeping, $\Omega = 0.15$ rad/sec with amplitudes 1.8311 rad to $\Omega = 0.65$ rad/sec with amplitudes 2.8955rad is obtained. Again applying the system with the different gain and delay values $(gd1 = gd2 = 0.2, gv1 = gv2 = -0.2, d = 0.6)$, the amplitudes is reduced to 1.5126 rad at same frequency of $\Omega = 1.15$ rad/sec.

The frequency response of the pitch motion for G_0=1.0 and F_0=0.03 is shown in Fig 3 (b). Band of frequency is obtained by backward sweeping starting from $\Omega = 2.5$ rad/sec to 1.15 rad/sec and the maximum peak amplitude is obtained at $\Omega = 1.7$ rad/sec of amplitude 0.292 rad. In between $\Omega = 0.65$ to 1.15 rad/sec could not give response due to instability of the system. And from $\Omega = 0.15$ to 0.65 rad/sec gives the band of unstable solution. After application of the delayed feedback control gd1=gd2=0.3, gv1=gv2=-0.3, d=0.6, a set of frequency response occurred with the maximum peak amplitude of 0.0475 rad at $\omega = 1.85$ rad/sec and also get the unstable solution from $\Omega = 0.15$ to 0.65 rad/sec. Applying another set of delayed feedback control gd1=gd2=0.2, gv1=gv2=--0.2, d=0.6, the maximum peak response is obtained at 1.7 rad/sec with amplitude 0.5 rad. In this case also a set of instability solution could not occur from $\Omega = 0.65$ to 1.15 rad/sec.



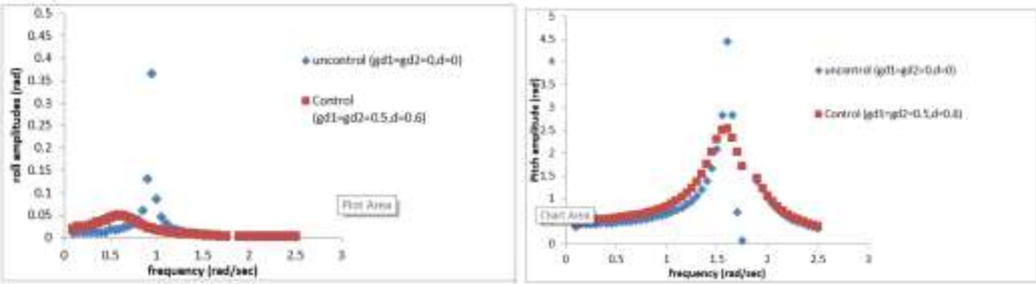
(a) **(b)**

Fig 3: Frequency response curve values of $\mu_1 = 0.1,\ \mu_2 = 0.2,\ \alpha_1 = 1.0,\ \alpha_2 = 2.0,\ \omega 1 = 0.94,$ $\omega 2 = 1.68,\ G_0 = 1.0,\ F_0 = 0.03,\ \varepsilon = 0.1,\ h = .025.$

Set-4: Non-Resonance condition

$\mu_1 = 0.1,\ \mu_2 = 0.2,\ \alpha_1 = 1.0,\ \alpha_2 = 2.0,\ \omega 1 = 0.94,\ \omega 2 = 1.68,\ G_0 = 0.01,\ F_0 = 1.2,\ \varepsilon = 0.1,$ $h = .025.$

Figure 4 (a) shows the frequency response when F_0=1.2 and G_0=0.01. The roll peak amplitude is occurred when $\Omega = 0.95$ rad/sec with amplitude 0.364 rad. In between $\Omega = 1.75$ to 1.9 rad/sec could not obtained the roll amplitude response because of the instability and converge problems. After application of feedback control, the roll peak amplitude is reduced up to $\Omega = 0.55$ rad/sec with amplitude 0.047 rad.in case of the pitch motion, the peak amplitude is obtained at $\Omega = 1.6$ rad/sec with amplitude response 4.44rad. After application of the delayed feedback control the peak amplitude is reduced up to $\Omega = 1.55$ rad/sec with amplitude 2.508 rad. In case of the instability part of the roll motion take $\Omega = 1.75$ rad, the time trace gives a quasi-periodic solution and obtained a chaotic motion as shown in figure 5 and 6.



(a) **(b)**

Fig 4: Frequency response curve values of $\mu_1 = 0.1,\ \mu_2 = 0.2,\ \alpha_1 = 1.0,\ \alpha_2 = 2.0,\ \omega 1 = 0.94,$ $\omega 2 = 1.68,\ G_0 = 0.01,\ F_0 = 1.2,\ \varepsilon = 0.1,\ h = .025.$

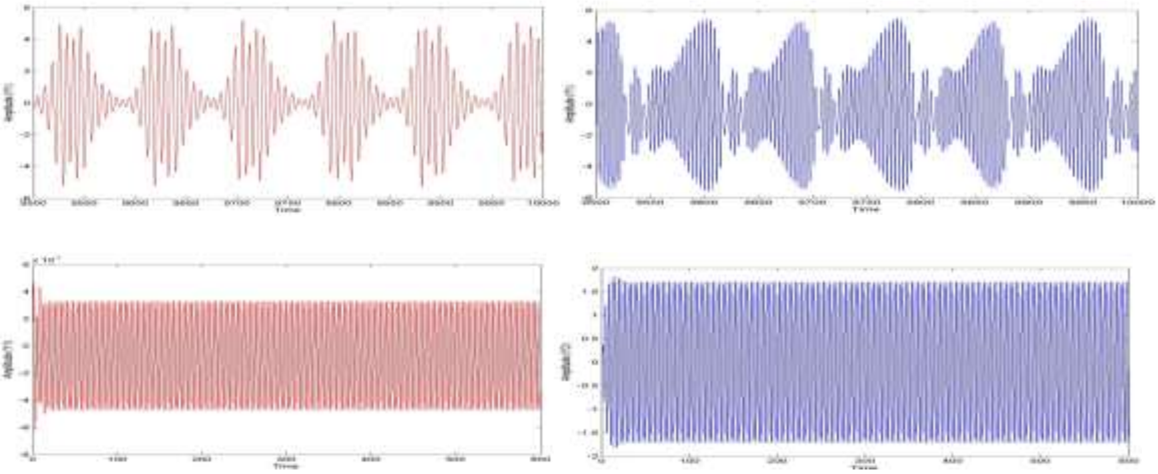


Fig 5: Time histories of one periodic solution of roll and pitch motion at Ω=1.75 rad/sec (controlled)

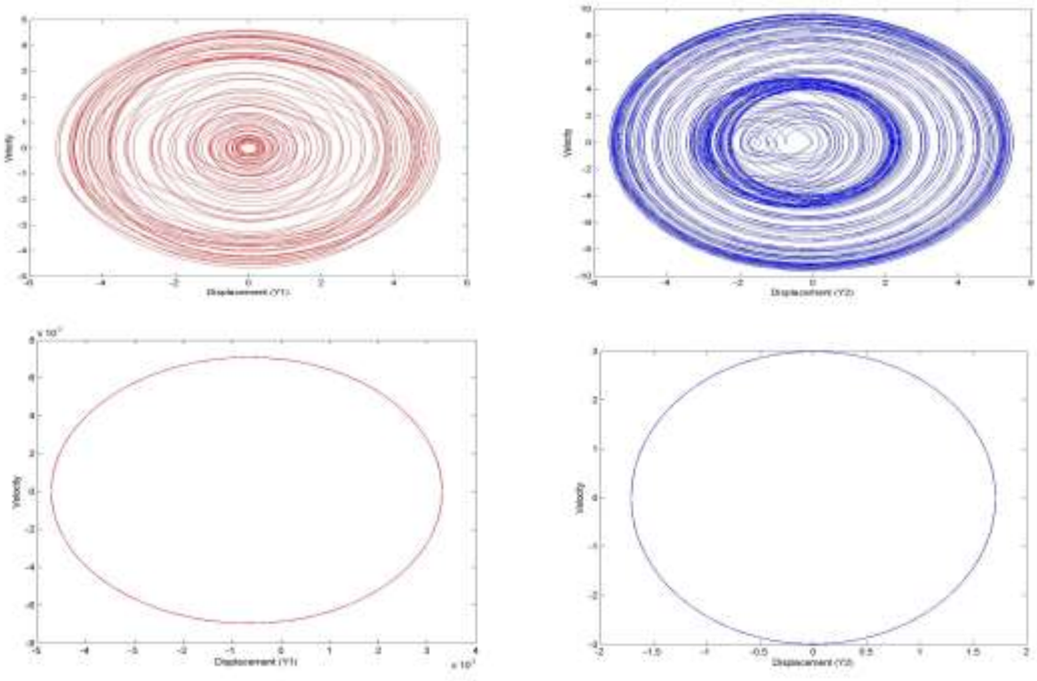


Fig 6: Phase plot diagram for roll and pitch motion at Ω=1.75 (uncontrolled and controlled)

CONCLUSIONS

Here the ship of two modes of the system is considered and the response under sinusoidal harmonic excitation of the system is calculated. The method of numerical techniques is applied to obtain the behavior of the roll pitch coupled motion using MATLAB Simulink. Both the frequency response and phase plot techniques are studied for the stability of the system. The effect of different parameters of the system is studied numerically. From the above study, the following important conclusions may be drawn.

1. For small values of excitation amplitudes G_0 and F_0 with $\omega_1 = 0.94$ and $\omega_2 = 1.68$, it is observed that the peak response of roll motion is reached up to 0.375 rad with excitation frequency Ω=0.9 rad/sec. After application of displacement feedback control, the peak response is suppressed up to 82 % and 93 % is suppressed when both the velocity and displacement feedback control is applied. The peak amplitude of pitch motion is also reduced up to 81.53 % after applying the displacement feedback control and is reduced up to 83.02% after applying both the velocity and displacement feedback control.
2. For the internal resonating case $\omega_2 = 2\omega_1$ the peak amplitude is reach up to 0.36 rad with excitation frequency 0.9 rad/sec. After application of the delayed feedback control the roll peak amplitude is reduced up to 86.11 % and further application of both displacement and velocity feedback control is reduced up to 93.05 % as compared with uncontrolled peak amplitude.
3. Only when the roll excitation amplitude G_0 increased, some of the roll response could not been obtained because of numerical instability and convergence problem of numerical integration techniques of the applied method the peak roll amplitude is reached up to 2.24 rad with Ω=1.15 rad/sec and the peak pitch amplitude is reached up

to 0.292 rad/sec with 1.7 rad/sec. After application of the delayed feedback control, some of the responses could not be obtained.

4. When the pitch excitation amplitudes F_0 is increased, some of the response could not been obtained because of numerical instability and convergence problem of numerical integration techniques of the applied method the peak roll amplitude is reached up to 0.364 rad with Ω=0.95 rad/sec. In case of the instability part of the roll motion take $\Omega = 1.75$ rad, the time trace gives a quasi-periodic solution and obtained a chaotic motion. After application of the delayed feedback control the system gives a limiting cycle solution of period 1 has been obtained and as a result, chaotic responses and consequent instability of the system is obviated.

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