Analysis of Wind Characteristics using ARMA & Weibull Distribution

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***Abstract*—A wind energy conversion system (WECS) converts the available wind potential at a particular wind site into electrical energy to meet the ever-increasing power demand. Assessing the adequacy of a WECS requires the consideration of some factors one of them is random nature of wind resources at the site location. So a model has to be developed in order to portray the intermittent characteristics of the wind. Here efforts have been made to present two popular methods to portray the behaviour of wind speed, one-time series autoregressive moving average (ARMA) model and other Weibull distribution. The fitting to the wind speed data using those methods follow their own different procedures, and the accuracies are checked indifferent approaches. In case of ARMA model, the accuracy is checked using F-criterion following Box-Jenkins guidelines. Weibull distribution fitting is checked through its parameter estimation using some statistical analysis.**

Keywords—ARMA; renewable source; Weibull distribution; wind energy conversion system; wind speed

I. Introduction

Alarming increase of population demands the cumulative power growth. As of now, the major part of the demand is met by conventional sources; those have panic effect on the environment. So the system planners are thinking for less harmful alternatives. Among all the available alternatives, wind energy is getting more potential [1]. Power production from wind is site-specific and speed dependent. As the wind speed is random, so some predictive models must be developed to describe the stochastic nature [2]. Here two popular models are fitted to the observed speed, time series ARMA model and Weibull distribution.

First of all, wind speed data are collected from a desired location. The wind data are then simulated using different ARMA models [3, 4 & 5]. The best fit ARMA model is determined using F criterion followed by Box-Jenkins guidelines [5]. The model simulates the wind speed as well as forecasts the speed using regression.

Other method to predict the wind speed behaviour is Weibull distribution. Generally, two-parameter Weibull distributions is preferred over one parameter Rayleigh distribution as it is giving more accurate fit for the observed wind speed [1, 6]. The parameters of the Weibull distribution are determined using some of the popular approaches like maximum likely hood method (MLM), modified maximum likely hood method (MMLM), method of moments (MOM), power density method (PDM), and graphical method [1, 6 & 7]. MMLM and MOM are special cases of MLM. Graphical method is less accurate [8]. So here MLM and PDM are presented.

II. Simulating Wind Speed using ARMA

A. ARMA Model

The speed values of wind at a particular location are collected. The collected data are used to develop time series ARMA model to simulate the wind speed [3, 4 & 5]. The model is given by

$$\begin{aligned} y_t &= \phi_1 y_{t-1} + \phi_2 y_{t-2} + \ldots + \phi_n y_{t-n} + \alpha_t \\ &-\theta_1 \alpha_{t-1} - \theta_2 \alpha_{t-2} - \ldots - \theta_m \alpha_{t-m} \end{aligned} \tag{1}$$

where ϕ_i (i=1,2,…,n) and Θ_j (j=1,2,3…,m) are the autoregressive and moving average parameters of the model, $\{\alpha_t\}$ is a normal white noise process with zero mean and variance σ_a^2, i.e. $\alpha_t \epsilon\ NID(0, \sigma_a^2)$, where *NID* denotes Normally Independent Distributed.

Several ARMA models with different orders are fitted with the observed wind speed. Following the Box-Jenkins guidelines and F-criterion, the order of the best ARMA model is determined [5].

After establishing the model, the wind speed can be simulated [3 & 4] as

$$SW_t = \mu_t + \sigma_t y_t \tag{2}$$

where σ_t is the standard deviation of the collected wind speed at hour t & μ_t is the collectedd mean wind speed at hour t.

B. Checking the Accuracy of the Fit

The accuracy of ARMA model fitted to the observed data can be checked using F-criterion, following Box-Jenkins guidelines. In the beginning, an index F is calculated with the knowledge of residual sum of squares of the ARMA models of orders of n^{th}, $(n-1)^{th}$ and $(n+1)^{th}$. The calculated index is then compared with F-distribution with degrees of freedom 2 and $N-s$ at a probability level p. Here N is sum total of observation, $s=2n+2$ with 95% probability. If the calculated index is less than the F-distribution value, then the model is adequate, otherwise higher order models are generated, and the condition is to be checked again.

III. SIMULATING WIND SPEED USING WEIBULL DISTRIBUTION

A. Wind Speed Distribution

Generally, histogram is preferred as the common portrayal of wind speed data. The histogram is commonly achieved by segregating the total data into bins of equal sizes, called classes [9]. The mid value of each bin represents a class. Therefore, the corresponding frequency associated with each bin b_j with $\Delta\upsilon$ width is

$$fr_j = {n_j}/{n} \tag{3}$$

where n_j is the number of data points that come under the class represented by the wind speed; υ_j and fr_j is the corresponding frequency linked with bin "*j*".

Thus,

$$\sum_{j=1}^{N} n_j = n \text{ and } \sum_{j=1}^{N} fr_j = 1$$

where N is the number of classes.

B. Weibull Distribution

For statistical analysis, frequency distribution format is quite convenient as compared to time series format. So the hourly wind speed data were converted to frequency format [1, 9]. In Weibull distribution, the variable characteristic of wind speed is presented by two functions: the probability density and the cumulative distribution. The Probability Density Function (PDF) is given as

$$f(\upsilon) = \frac{\rho}{\lambda}\left(\frac{\upsilon}{\lambda}\right)^{\rho-1} e^{-\left(\frac{\upsilon}{\lambda}\right)^{\rho}} \tag{4}$$

where υ is speed of the wind measured in *m/s*; λ is scale parameter, measured using the same unit of the wind speed and ρ is shape parameter which is dimensionless. The scale parameter (λ) is generally described the average wind speed using the shape parameter (ρ) that decides the range of variation of the wind speed near to a desired site [1].

The Cumulative Distribution Function (CDF) is the additive corresponding recurrence of individual class of the wind speed. Thus, the CDF is given by

$$F(\upsilon) = 1 - e^{-\left(\frac{\upsilon}{\lambda}\right)^{\rho}} \tag{5}$$

C. Determination of Weibull parameters

Several approaches are available to find the value of the parameters of the Weibull distribution. The most popular approaches among them are MLM, PDM and graphical method. As per some literature, graphical method is not accurate as compared to MLM. Some special methods of MLM are also available in the literature. Here MLM and PDM are presented. A comparative analysis of both the methods is also presented.

1) Maximum likelihood method

The universal method used to estimate the parameters of the Weibull distribution is Maximum Likelihood method [1 & 7]. The parameters are estimated through numerical analysis, using Newton–Raphson method,

$$\rho = \left[\frac{\sum_{i=1}^{n} \upsilon_i^{\rho} \ln(\upsilon_i)}{\sum_{i=1}^{n} \upsilon_i^{\rho}} - \frac{\sum_{i=1}^{n} \ln(\upsilon_i)}{n}\right]^{-1} \tag{6}$$

$$\lambda = \left(\frac{1}{n}\sum_{i=1}^{n} \upsilon_i^{\rho}\right)^{1/\rho} \tag{7}$$

2) Power Density Method

In this approach, an index called energy pattern factor E_{pf} is initially calculated using average wind speed [1, 7], that is applicable for the aerodynamic layout of the wind turbine. The factor is expressed as

$$E_{pf} = \frac{\frac{1}{n}\sum_{i=1}^{n} \upsilon_i^3}{\left(\frac{1}{n}\sum_{i=1}^{n} \upsilon_i\right)^3} \tag{8}$$

After finding the value of the energy factor, the shape parameter ρ can be determined as

$$\rho = 1 + \frac{3.69}{E_{pf}^2} \tag{9}$$

and the scale parameter λ can be computed as

$$\lambda = \frac{\bar{\upsilon}}{\Gamma\left(1 + \frac{1}{\rho}\right)} \tag{10}$$

D. Checking goodness of fit

Here approaches have been made to fit theoretical probability density function to the observed data. To check how effectively it is fitted, some statistical tests are performed by determining the errors like the mean percentage error (MPE), mean absolute percentage error (MAPE), root mean square error (RMSE), and the chi-square test are performed [1, 6, 7 & 9]. MPE shows the mean deviation in percentage of the Weibull distribution values from that of the collected sample while MAPE shows mean absolute divergence in percentage. The accuracy is high when these deviations are approximated to null. The Chi-square statistical test evaluates the adequacy of the considered ideal distribution to the collected sample.

$$MPE = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{x_{i,w} - y_{i,m}}{y_{i,m}}\right) \times 100 \tag{11}$$

$$MAPE = \frac{1}{N}\sum_{i=1}^{N}\left|\frac{x_{i,w} - y_{i,m}}{y_{i,m}}\right| \times 100 \tag{12}$$

$$RMSE = \left[\frac{1}{N} \sum_{i=1}^{N} (y_{i,m} - x_{i,w})^2 \right]^{1/2} \tag{13}$$

$$\chi^2 = \sum_{i=1}^{N} \left(\frac{(x_{i,w} - y_{i,m})^2}{x_{i,w}} \right) \tag{14}$$

where N is number of observations, $y_{i,m}$ is the frequency of observation or i^{th} calculated value from the observed data, $x_{i,w}$ is the frequency of Weibull or i^{th} calculated value from the Weibull distribution.

IV. Case Study

Wind speed data on hourly basis are collected for a time horizon of one year near the wind farm in Kayathar, Tamilnadu, India [10]. The hourly wind speed data for a typical 31 days month is depicted in the Fig. 1. The wind speed values for one year are divided into different bins [9] with a range value of 1m/s as shown in table I. ARMA and Weibull distribution are implemented using MATLAB R2015a.

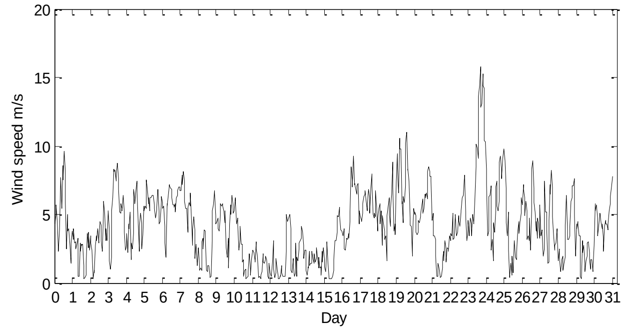

Fig. 1. Hourly wind speed data for a typical one month

TABLE I. Wind Speed Frequency Distribution

Bin	Frequency	Bin	Frequency
0	5	11	223
1	472	12	133
2	722	13	93
3	1028	14	43
4	1219	15	30
5	1209	16	14
6	1168	17	4
7	953	18	2
8	669	19	1
9	467	20	0
10	304	21	1

V. Result & Discussion

A. ARMA model

Different orders of ARMA models starting from ARMA(2, 1) as per the Box-Jenkins guidelines are fitted to the observed wind speed. The best fit model is determined using the F criterion. ARMA(4,3) is chosen the best fit as the F value resulting from the residual sum of ARMA(4,3), and ARMA(5,4) is coming less than the F-distribution $F_{0.95}(2,\infty)=3$ [11].

The comparative plots as shown in the Fig. 2, is validating the efficacy of the ARMA(4,3) model in simulating the wind speed.

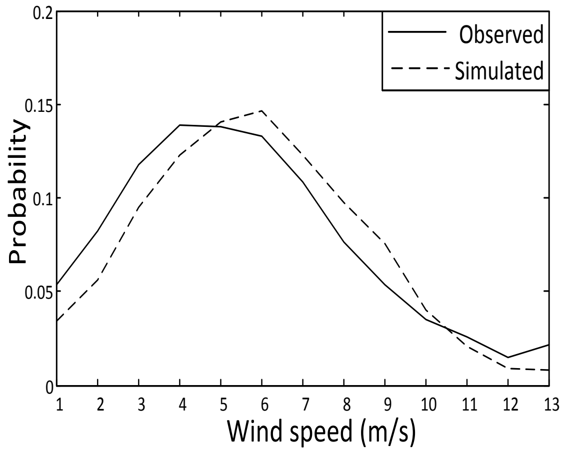

Fig. 2. Comparative plot of fitted and overserved wind speed using ARMA model

B. Weibull Distribution

The comparative plot of simulated wind speed using MLM and observed wind speed is presented in Fig. 3. The observed data is presented using the bar plot, and the estimated one is shown in the curve plot.

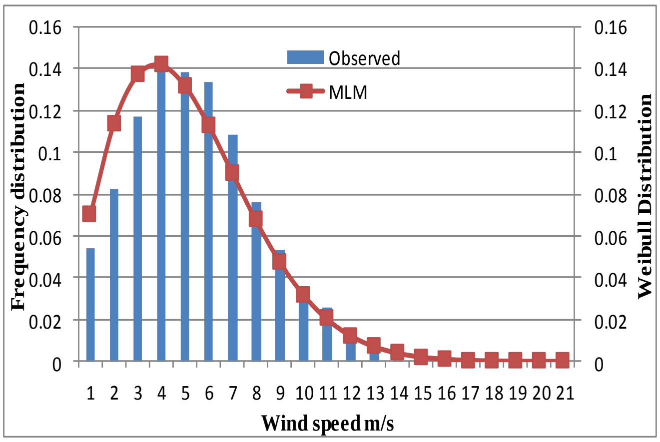

Fig. 3. Comparison of the observed wind speed and simulated wind speed using MLM

The comparative plot of estimated wind speed with PDM and observed wind speed is shown in Fig. 4. Observed wind speed is represented using bar plot and the estimated one is shown in curve plot. From Fig. 3 and 4, it can be observed that, the Weibull distributions resulting from both the cases almost fit to the observed wind speed data.

The parameters ρ and λ for both the considered methods for analysis are presented in table II.

TABLE II. Shape and Scale Parameter

Weibull Parameter	MLM	PDM
ρ	1.85	1.86
λ	5.75	5.75

Statistical tests can check the goodness of fit of the distribution to the observed wind speed data by finding the indices as discussed in the section III D. Among the four statistical errors checking indices, RMSE and chi-square errors (χ^2) are generally chosen for error detection. From the table III, it is seen that both RMSE and χ^2 are almost same for both the methods. Therefore the probability distribution functions using both the methods are coinciding as shown in Fig. 5. The wind characteristic is reciprocal with the value of Weibull shape parameter ρ. A low value of ρ indicates the large variation of wind speeds, while greater value ok ρ points to precise wind speeds [12]. In this consideration, ρ values found nearer to 1.85. Parameter λ determines the value of the annual average wind speed. Here it is settled nearer to 6 m/s. Thus, these indices determine the operational range as well as the mean wind speed for a considered wind regime.

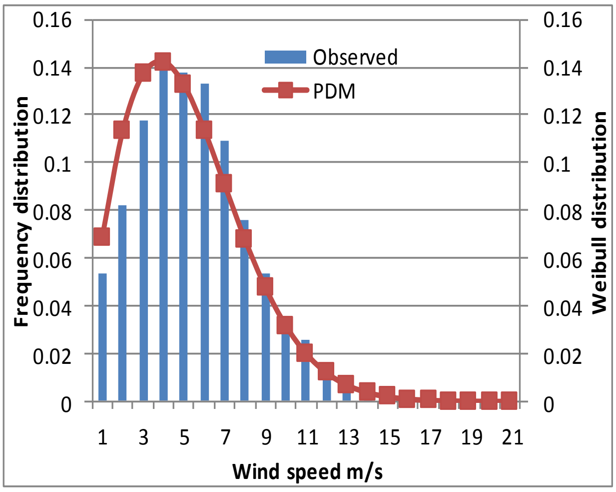

Fig. 4. Comparison of observed wind speed and estimated wind speed with PDM

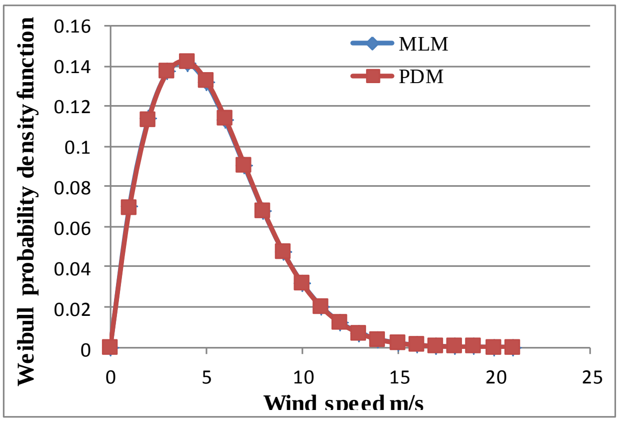

Fig. 5. Comparison of Weibull distribution function generated by MLM and PDM

TABLE III. GOODNESS OF FIT

Tests	MLM	PDM
MPE	-0.0819	-0.0004
MAPE	0.1218	0.0006
RMSE	0.478	0.465
χ^2	0.0324	0.0307

VI. CONCLUSION

In this paper, wind speed is analyzed using two most popular method, time series ARMA model and Weibull distribution. The comparison of the probability distribution of observed wind speed and simulated wind speed resulting from ARMA model is giving a fair explanation of the effectiveness of representing the wind speed using ARMA. The goodness of fit of different ARMA models are checked through F-criterion following Box-Jenkins guidelines.

The Weibull parameters ρ and λ are determined using two methods. The best fit values are determined statistically by finding the error values. The calculated error values are pointing towards the efficient method of estimating the parameters ρ and λ. Both MLM and PDM are giving almost same ρ and λ values, as well as their probability distributions, are coinciding. Again, these parameters in return are giving a fair account of the wind behaviour like the range of operation as well as the average value.

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