

Performance Evaluation and Optimization of Multiantenna Two-Way Relaying System with CCI.

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Abstract—In this paper, we investigate the performance of multiple antenna two-way relay network with transmit beamforming and maximum ratio combining, in the presence of multiple interferers and noise over Nakagami-m fading channels. We have evaluated the signal-to-interference-plus-noise ratio (SINR) in closed-form, along with the tight lower bound on the outage probability (OP). The average symbol error probability (SEP) for binary phase shift keying is evaluated by using cumulative distribution function of the SINR. Furthermore, to render insights into the performance degradation due to the effect of interference, asymptotic expression of OP and SEP are obtained, which easily enables us to evaluate diversity order and coding gain. We further investigate the joint optimization problem of relay location and power allocation to minimize the OP. Finally, the tightness of our analysis is attested through Monte Carlo simulation.

Index Terms—Two-way relay systems, co-channel interference, multi-antenna, beamforming, AF relaying, outage probability, symbol error probability .

I. INTRODUCTION

Reliability, reduced power consumption and wider coverage is the primary concern of future wireless systems. Relay network is the promising candidate to fulfill these requirements [1]. Among the relaying schemes, one-way relaying networks (OWRN) is quite popular because of its simplicity and thus low implementation complexity. On the other hand, two-way relaying networks (TWRN) have recently gained significant popularity due to their higher spectral efficiency as compared to OWRN [2]. To further enhance the communication reliability multiple-input-multiple-output (MIMO) technologies can be used effectively. Specifically, beamforming (BF) is used as a transmission strategy in multi-antenna systems. Maximal ratio combining (MRC) and maximal ratio transmission (MRT) are included in BF technique [3].

However, in the real-world scenario, increasing frequency reuse causes severe co-channel interference (CCI). The performance of the relay assisted system is also degraded in the presence of CCI and satisfactory diversity gains can be achieved only if the limited interferer power is observed [4]. Thus, the analysis becomes complicated when CCI is assumed in the multi-antenna relay system. Therefore, in this paper, we analyze the relay based multiantenna BF scheme in the presence of co-channel interference (CCI).

Prior related research: In [5], approximate analysis of interference limited TWRN is evaluated in Rayleigh fading environment. Authors in [6] studied the approximate analysis of TWR amplify-and-forward (AF) relaying system in Nakagami-m fading channel, where CCI is assumed only at the relay. It is further investigated in [4], where authors studied the bounds on overall system performance in Nakagami-m fading. Recently, the effect of CCI is evaluated in decode-and-forward (DF)

based spectrum sharing TWRN system in Nakagami-m fading [7]. Authors in [8] investigate the joint impact of hardware impairment and CCI in the system performance of DF-OWRN, where N^{th} best relay selection employed and CCI is assumed only at the relay.

Only a few studies, [9]–[12], address the issue of CCI with the multi-antenna relay assisted system. In [9] authors addressed performance comparison of different beamforming techniques in the presence of CCI in OWRN, where interference is assumed only at the multi-antenna relay node. Recently, authors in [10] proposed optimal beamforming in the multi-antenna system in the presence of CCI but the analysis is limited to OWRN. Authors in [11] evaluate the performance of multi-antenna BF-TWRN in the presence of antenna correlation and CCI is assumed only at the relay node. The issue of CCI in multi-antenna TWRN is studied in [12], where beamforming is used among the terminals and the end outage probability (OP) is evaluated in Rayleigh fading environment.

Motivation and our contribution: In [3], authors considered multiantenna system but didn't consider the CCIs. Authors in [12] take into account both CCIs and multi-antenna; while interference is assumed only at the relay terminal. However, only a few works [4] and [5] includes the CCI at both the source nodes and the relay node in TWRN, which represents the generalized approach. In [4] and [5], authors didn't consider multi-antenna system. Therefore, to the best of our knowledge, analysis of multi-antenna TWRNs with CCI at all the terminals have not yet been studied completely. Thus, this paper fills the gap by deriving the performance analysis of multi-antenna BF scheme in the presence of CCI at all the terminals.

II. SYSTEM AND CHANNEL MODEL

We consider a two-way relaying system where source S_a and S_b exchange information via intermediate relay R as shown in Fig. 1. We have assumed that there is no direct link between sources S_a and S_b which may be due to the large distance between the transceiver and/or heavy shadowing [2]. We have also considered that S_a and S_b are equipped with L and M antennas respectively, while R is equipped with a single antenna [3]. All nodes are assumed to operate in half-duplex mode. The communication between nodes takes place in two phases namely multiple access (MAC) and broadcast (BC) phase [2]. In MAC phase, S_a and S_b communicate with R, afterwards, R amplifies the received signal and forwards it to S_a and S_b in BC phase. Furthermore, we assume that all the terminals are inflicted by CCI from other users in the network.

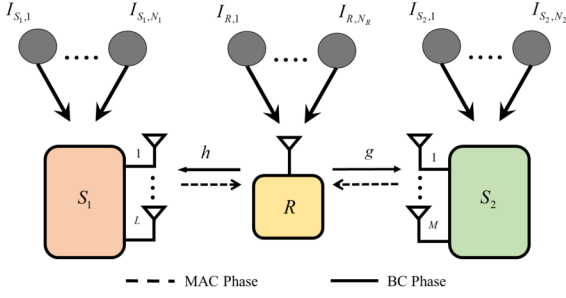


Fig. 1. Multi-antenna two-way AF relaying

We consider channels are reciprocal and channel amplitudes undergo flat Nakagami-m fading.

Here sources and relay i.e., S_a , S_b and R are affected by N_1 , N_2 and N_R interferers respectively. In MAC phase (which is represented by dashed lines in Fig. 1), signal received at R is given by

$$y_R = \sqrt{P_a} \mathbf{h}^T \mathbf{w}_a x_a + \sqrt{P_b} \mathbf{g}^T \mathbf{w}_b x_b + \sum_{i=1}^{N_R} \sqrt{P_{R,i}} h_{R,i} x_{R,i} + n_R \quad (1)$$

where, x_τ and P_τ denote the transmit signal (with unit energy) and power from the source S_τ , for $\tau \in \{a, b\}$, and n_R represents the additive white Gaussian noise (AWGN) at R . $\mathbf{h} = [h_1, h_2, \dots, h_L]$ and $\mathbf{g} = [g_1, g_2, \dots, g_M]$ defines¹ the channel fading coefficient between $S_a \rightarrow R$ and $S_b \rightarrow R$ respectively. $\mathbf{w}_a = (\mathbf{h}^\dagger / \|\mathbf{h}\|)^T$ and $\mathbf{w}_b = (\mathbf{g}^\dagger / \|\mathbf{g}\|)^T$ defines the transmit weight vector at S_a and S_b respectively. The transmit power from S_a and S_b are given as $\mathbb{E}[\|\sqrt{P_a} \mathbf{w}_a\|^2] = P_a$ and $\mathbb{E}[\|\sqrt{P_b} \mathbf{w}_b\|^2] = P_b$, respectively. Besides, $\{h_{R,i}\}_{i=1}^{N_R}$ represent channel coefficient between R and i -th interferer. $P_{R,i}$ and $x_{R,i}$ are the received power and transmitted signal of i -th interferer at R . In BC phase (which is represented by solid lines in Fig. 1), the received signal at S_a and S_b are given by

$$y_{S_a} = \mathbf{w}_a^\dagger \left(\mathcal{G} \mathbf{h} y_R + \sum_{j=1}^{N_1} \sqrt{P_{S_a,j}} \mathbf{h}_{S_a,j} x_{S_a,j} + n_a \right) \quad (2)$$

$$y_{S_b} = \mathbf{w}_b^\dagger \left(\mathcal{G} \mathbf{g} y_R + \sum_{k=1}^{N_2} \sqrt{P_{S_b,k}} \mathbf{h}_{S_b,k} x_{S_b,k} + n_b \right) \quad (3)$$

respectively, where $\mathbf{h}_{S_a,j} = [h_{S_a,j,1}, h_{S_a,j,2}, \dots, h_{S_a,j,L}]$ and $\mathbf{h}_{S_b,k} = [h_{S_b,k,1}, h_{S_b,k,2}, \dots, h_{S_b,k,M}]$ represent the channel fading coefficient between j -th interferer $\rightarrow S_a$ and k -th interferer $\rightarrow S_b$ respectively. $P_{S_a,j}$, $P_{S_b,k}$ and $x_{S_a,j}$, $x_{S_b,k}$ are the signal powers and symbols of j -th, k -th interferer received at nodes S_a and S_b respectively. n_a and n_b represents AWGN at S_a and S_b respectively. \mathcal{G} defines the variable gain of relay which is given as² $\mathcal{G}^2 = P_R / (P_a \|\mathbf{h}\|^2 + P_b \|\mathbf{g}\|^2 + \sum_{i=1}^{N_R} P_{R,i} |h_{R,i}|^2 + N_0)$. Now

¹Notations: In this paper scalar and vectors are represented as italic symbol and lower case boldface symbols respectively. For any scalar a , absolute value is denoted by $|a|$. For a given complex vector \mathbf{a} , $(\mathbf{a})^T$ represents the transpose, $(\mathbf{a})^\dagger$ represent the conjugate transpose and $\|\mathbf{a}\|$ denotes the Euclidean norm. $\mathcal{CN}(\mu, \sigma^2)$ denotes a complex circular Gaussian random variable with mean μ and variance σ^2 . $\Gamma(\cdot)$, $\Upsilon(\cdot, \cdot)$ and $\Gamma(\cdot, \cdot)$ represent, respectively, the gamma function [13, Eq. (8.310.1)], the lower incomplete gamma function [13, Eq. (8.350.1)] and the upper incomplete gamma function [13, Eq. (8.350.2)]. $\mathbb{E}[\cdot]$ shows the expectation.

²Without loss of generality we have assumed that the noise at all the terminal (n_R, n_a, n_b) follows $\mathcal{CN}(0, N_0)$.

canceling the self interference terms, the received signals at S_a leads to

$$\tilde{y}_{S_a} = \mathcal{G} \sqrt{P_b} \mathbf{w}_a^\dagger \mathbf{h} \mathbf{g}^\dagger \mathbf{w}_b x_b + \mathcal{G} \mathbf{w}_a^\dagger \mathbf{h} \sum_{i=1}^{N_R} \sqrt{P_{R,i}} h_{R,i} x_{R,i} + \mathcal{G} \mathbf{w}_a^\dagger \mathbf{h} n_R + \mathbf{w}_a^\dagger n_a + \mathbf{w}_a^\dagger \sum_{j=1}^{N_1} \sqrt{P_{S_a,j}} \mathbf{h}_{S_a,j} x_{S_a,j}. \quad (4)$$

The elements of $\mathbf{h}_{S_a,j}$ are independent and identically distributed (i.i.d) complex Gaussian random variables (RVs), the channel coefficients of each interferer link is still a complex Gaussian RV even after the MRC [14]. Thus for the ease of further analysis, we can represent \tilde{y}_{S_a} as

$$\tilde{y}_{S_a} = \mathcal{G} \sqrt{P_b} \mathbf{w}_a^\dagger \mathbf{h} \mathbf{g}^\dagger \mathbf{w}_b x_b + \mathcal{G} \mathbf{w}_a^\dagger \mathbf{h} \sum_{i=1}^{N_R} \sqrt{P_{R,i}} h_{R,i} x_{R,i} + \mathcal{G} \mathbf{w}_a^\dagger \mathbf{h} n_R + \mathbf{w}_a^\dagger n_a + \mathbf{w}_a^\dagger \sum_{j=1}^{N_1} \sqrt{P_{S_a,j}} h_{S_a,j} x_{S_a,j}. \quad (5)$$

Similarly, we will get \tilde{y}_{S_b} as:

$$\tilde{y}_{S_b} = \mathcal{G} \sqrt{P_a} \mathbf{w}_b^\dagger \mathbf{g} \mathbf{h}^\dagger \mathbf{w}_a x_a + \mathcal{G} \mathbf{w}_b^\dagger \mathbf{g} \sum_{i=1}^{N_R} \sqrt{P_{R,i}} h_{R,i} x_{R,i} + \mathcal{G} \mathbf{w}_b^\dagger \mathbf{g} n_R + \mathbf{w}_b^\dagger n_b + \mathbf{w}_b^\dagger \sum_{k=1}^{N_2} \sqrt{P_{S_b,k}} h_{S_b,k} x_{S_b,k} \quad (6)$$

Thus the end-to-end signal-to-interference-noise ratio (SINR) from S_ρ to S_ω in generalized form is given by

$$\gamma_{S_\rho \rightarrow S_\omega} = \frac{\xi \gamma_\rho \gamma_\omega}{\xi \gamma_\omega \gamma_R + (\gamma_\rho + \gamma_\omega) \gamma_\lambda + \gamma_R \gamma_\lambda} \quad (7)$$

where we have assumed $P_{S_a} = P_{S_b} = P_S$ then $\xi = \frac{P_R}{P_S}$. Here $(\rho, \omega, \lambda) \in \{(a, b, \eta), (b, a, \delta)\}$, $\gamma_\eta = U + 1$, $\gamma_R = V + 1$ and $\gamma_\delta = W + 1$, where $U = \sum_{k=1}^{N_2} \frac{P_{S_b,k} |h_{S_b,k}|^2}{N_0}$, $V = \sum_{i=1}^{N_R} \frac{P_{R,i} |h_{R,i}|^2}{N_0}$ and $W = \sum_{j=1}^{N_1} \frac{P_{S_a,j} |h_{S_a,j}|^2}{N_0}$. Here, $\gamma_1 = \bar{\gamma} \|\mathbf{h}\|^2$ and $\gamma_2 = \bar{\gamma} \|\mathbf{g}\|^2$, where $\bar{\gamma} = P_S / N_0$.

III. PERFORMANCE ANALYSIS

In this section, we analyze the performance of the proposed system model. Lower bound expressions for OP and SEP are derived as follows:

A. Outage Probability (OP)

OP of a system can be defined as the probability with which the instantaneous SINR, $\gamma_{S_\rho \rightarrow S_\omega}$ drops below a predefined threshold γ_{th} . Here, $\gamma_{th} = 2^{R_s} - 1$ is the SNR threshold [2] for a given transmission rate R_s . The SINR at terminal S_a can be obtained by $(\rho, \omega, \lambda) \in (b, a, \delta)$ in (7). After doing some mathematical manipulations, (7) can be written as

$$\gamma_{S_b \rightarrow S_a}^{BF} \leq (\gamma_{S_b \rightarrow S_a}^{BF})^{up} = \min \left(\frac{\xi \gamma_a}{\gamma_\delta}, \frac{\xi \gamma_b}{(\xi \gamma_R + \gamma_\delta)} \right) \quad (8)$$

$\|\mathbf{h}\|^2$ and $\|\mathbf{g}\|^2$ will follow Gamma distribution with fading severity parameters m_h, m_g and channel powers Ω_h, Ω_g respectively. The cumulative distribution function (CDF) of \mathcal{X} , for $\mathcal{X} \in \{\|\mathbf{h}\|^2, \|\mathbf{g}\|^2\}$ is given by $F_{\mathcal{X}}(x) = \Upsilon(m, \alpha x) / \Gamma(m)$, where $\Upsilon(m, \alpha x) = \Gamma(m) - \Gamma(m, \alpha x)$, $\alpha \in \{\alpha_h = m_h / \Omega_h, \alpha_g = m_g / \Omega_g\}$ and $m \in \{L m_h, M m_g\}$.

Proposition 1: The OP expression of instantaneous SINR at S_a is given as

$$\begin{aligned} \mathcal{P}_{out}(\gamma_{th}) &= 1 - e^{-\gamma_{th}\mathcal{A}} \sum_{i=0}^{m_h L - 1} \sum_{j=0}^{m_g M - 1} \sum_{l=0}^{i+j-k} \sum_{k=0}^j \sum_{n=0}^k \\ &\times \binom{i+j-k}{l} \binom{j}{k} \binom{k}{n} \frac{\xi^{k-i-j} m_h^i m_g^j \gamma_{th}^{i+j}}{i! j! \Omega_h^i \Omega_g^j (\bar{\gamma})^{i+j}} \left(\frac{m_1}{P_1^I \Omega_1} \right)^{m_1 N_1} \\ &\times \frac{\Gamma(l + m_1 N_1)}{\Gamma(m_1 N_1)} \left(\frac{m_R}{P_R^I \Omega_R} \right)^{m_R N_R} \frac{\Gamma(n + m_R N_R)}{\Gamma(m_R N_R)} \\ &\times \left(\frac{m_1}{P_1^I \Omega_1} + \gamma_{th} \mathcal{B} \right)^{-l - m_1 N_1} \left(\frac{m_R}{P_R^I \Omega_R} + \frac{\gamma_{th} m_g}{\bar{\gamma} \Omega_g} \right)^{-n - m_R N_R} \end{aligned} \quad (9)$$

where $\mathcal{A} = (m_h \Omega_g + m_g \Omega_h (\xi + 1)) / (\xi \bar{\gamma} \Omega_h \Omega_g)$, $\Omega_h = \mathbb{E}\{\|\mathbf{h}\|^2\}$, $\Omega_g = \mathbb{E}\{\|\mathbf{g}\|^2\}$, $\mathcal{B} = (m_h \Omega_g + m_g \Omega_h) / (\xi \bar{\gamma} \Omega_h \Omega_g)$.

Proof: See Appendix.

B. Symbol Error Probability (SEP)

To evaluate the system performance, SEP is one of the important performance metric. Generally, it is defined [2], in terms of CDF of end-to-end SNR. In our case the SEP can be lower bounded as

$$\mathcal{P}_e = \frac{\alpha}{2} \sqrt{\frac{\beta}{\pi}} \int_0^\infty \frac{e^{-\alpha\gamma}}{\sqrt{\gamma}} F_{\gamma_{S_b \rightarrow S_a}}^{up}(\gamma) d\gamma, \quad (10)$$

where (α, β) are predefined constants depending on the type of modulation used [2]. $F_{\gamma_{S_b \rightarrow S_a}}^{up}(\gamma)$ is the CDF of $\gamma_{S_b \rightarrow S_a}^{up}$ which can be easily obtained from OP expression given in (9) by interchanging γ_{th} to γ . Furthermore, while deriving the expression for SEP given in (17), we will use the following equality:

$$\begin{aligned} T &= \frac{1}{\left(x + \frac{m_1}{P_1^I \Omega_1 D}\right)^{l+m_1 N_1} \left(x + \frac{m_R \Omega_g \bar{\gamma}}{P_R^I \Omega_R m_g}\right)^{n+m_R N_R}} \\ &= \sum_{u=1}^{l+m_1 N_1} \frac{\mathcal{L}_u}{\left(x + \frac{m_1}{P_1^I \Omega_1 D}\right)^u} + \sum_{v=1}^{n+m_R N_R} \frac{\mathcal{L}_v}{\left(x + \frac{m_R \Omega_g \bar{\gamma}}{P_R^I \Omega_R m_g}\right)^v}, \end{aligned} \quad (11)$$

where \mathcal{L}_u and \mathcal{L}_v are given as follows:

$$\begin{aligned} \mathcal{L}_u &= \lim_{x \rightarrow -\frac{m_1}{P_1^I \Omega_1 D}} \frac{d^{l+m_1 N_1 - u} \left(x + \frac{m_R \Omega_g \bar{\gamma}}{m_g P_R \Omega_R}\right)^{-n - m_R N_R}}{dx^{l+m_1 N_1 - u} (l + m_1 N_1 - u)!} \\ \mathcal{L}_v &= \lim_{x \rightarrow -\frac{m_R \Omega_g \bar{\gamma}}{m_g P_R \Omega_R}} \frac{d^{n+m_R N_R - v} \left(x + \frac{m_1}{P_1^I \Omega_1 D}\right)^{-l - m_1 N_1}}{dx^{n+m_R N_R - v} (n + m_R N_R - v)!}. \end{aligned} \quad (12)$$

We will also use the identity $\int_{y=0}^\infty \frac{y^\rho e^{-ry}}{(y+z)^\epsilon} dy = \frac{\Gamma(\rho+1)}{z^{\epsilon-\rho-1}} \psi(\rho+1, \rho-\epsilon+2; rz)$, where $\psi(\cdot; \cdot; \cdot)$ is Tricomi confluent hypergeometric function defined in [13, Eq. (9.210.2)]. Furthermore, by using the fact ${}_2F_0(v, \kappa; -; -1/\sigma) = \sigma^v \psi(v, v - \kappa + 1; \sigma)$ and after some mathematical manipulation we will obtain the lower bound closed form expression of SEP given in (17), where ${}_2F_0(\cdot, \cdot; \cdot; \cdot)$ is generalised hypergeometric function and $C = \Gamma(i+j+0.5)(\mathcal{A} + \beta)^{-i-j-0.5}$.

C. Asymptotic Analysis

In order to gain better insight of the relationship between system parameters and performance (OP and SEP), we present asymptotic expression of (9) and (17). Using [13, Eq. (8.354.1)] for series representation of $\Upsilon(m, \alpha x) \stackrel{x \rightarrow 0}{=} (\alpha x)^m / m$, thus the asymptotic CDF of \mathcal{X} is obtained as $F_{\mathcal{X}}(x) = (\alpha x)^m / \Gamma(m+1)$. Using the above CDF and neglecting higher order terms in (30), an asymptotic expression for (9) can be derived as

$$\mathcal{P}_{out}^\infty \underset{\bar{\gamma} \rightarrow \infty}{\approx} \mathcal{G}_c \left(\frac{\gamma_{th}}{\bar{\gamma}} \right)^{\mathcal{G}_d}, \quad (13)$$

where diversity order $\mathcal{G}_d = \min(m_h L, m_g M)$ and coding gain \mathcal{G}_c given below

$$\mathcal{G}_c = \begin{cases} \mathcal{G}_{c_1}, & m_h L > m_g M \\ \mathcal{G}_{c_2}, & m_g M > m_h L \\ \mathcal{G}_{c_1} + \mathcal{G}_{c_2}, & m_h L = m_g M, \end{cases} \quad (14)$$

while \mathcal{G}_{c_1} and \mathcal{G}_{c_2} are defined as

$$\begin{aligned} \mathcal{G}_{c_1} &= \frac{(m_h / \Omega_h \xi \bar{\gamma})^{m_h L}}{\Gamma(m_h L + 1)} \sum_{i=0}^{m_h L} \binom{m_h L}{i} \frac{\Gamma(i + m_1 N_1)}{\Gamma(m_1 N_1)} \left(\frac{P_1^I \Omega_1}{m_1} \right)^i \\ \mathcal{G}_{c_2} &= \frac{(m_g / \Omega_g \xi \bar{\gamma})^{m_g M}}{\Gamma(m_g M + 1)} \sum_{i=0}^{m_g M} \sum_{j=0}^i \sum_{k=0}^{m_g M - i} \binom{m_g M}{i} \binom{i}{j} \\ &\quad \binom{m_g M - i}{k} \frac{\Gamma(k + m_1 N_1) \Gamma(j + m_R N_R)}{\Gamma(m_1 N_1) \Gamma(m_R N_R)} \xi^{-i} \left(\frac{P_1^I \Omega_1}{m_1} \right)^k \left(\frac{P_R^I \Omega_R}{m_R} \right)^j \end{aligned} \quad (15)$$

In the asymptotic analysis, by using (13) in (10), the SEP at node S_a can be derived as:

$$\begin{aligned} \mathcal{P}_e^\infty \underset{\bar{\gamma} \rightarrow \infty}{\approx} \frac{\alpha}{2\sqrt{\pi}} \left(\left(\frac{m_h}{\Omega_h \bar{\gamma} \xi} \right)^{m_h L} \sum_{i=0}^{m_h L} \binom{m_h L}{i} \frac{\Gamma(i + m_1 N_1)}{\Gamma(m_1 N_1) \beta^{m_h L}} \right. \\ \times \frac{\Gamma(0.5 + m_h L)}{\Gamma(1 + m_h L)} \left(\frac{P_1^I \Omega_1}{m_1} \right)^i + \left. \left(\frac{m_g}{\Omega_g \bar{\gamma} \xi} \right)^{m_g M} \frac{\Gamma(0.5 + m_g M)}{\Gamma(1 + m_g M)} \right. \\ \times \sum_{i=0}^{m_g M} \sum_{j=0}^i \sum_{k=0}^{m_g M - i} \binom{m_g M}{i} \binom{i}{j} \binom{m_g M - i}{k} \left(\frac{P_1^I \Omega_1}{m_1} \right)^k \\ \times \left. \frac{\Gamma(k + m_1 N_1) \Gamma(j + m_R N_R)}{\Gamma(m_1 N_1) \Gamma(m_R N_R)} \beta^{m_g M} \left(\frac{P_R^I \Omega_R}{m_R} \right)^j \right). \end{aligned} \quad (16)$$

Remark (Assume $P_1^I = P_2^I = P_R^I = P_I$): From (13) and (16), we can deduce that the achievable diversity order depends on the interference power level. If interference power remains fixed on condition $P_I \ll P$, then the achievable diversity order of the proposed system is $\min(m_h L, m_g M)$. However, when P_I increases to the same level of P , so that $\frac{P}{P_I}$ remains constant, then the diversity order is reduced to zero, which is in consent with [4].

IV. OPTIMIZATION OF POWER ALLOCATION AND RELAY LOCATION

In this section, we address the optimization problem of relay location and power allocation to minimize the OP of our considered system in the presence of CCI. The problem of power allocation optimization can be formulated for power

$$\begin{aligned}
\mathcal{P}_e &= \frac{\alpha}{2} - \frac{\alpha\sqrt{\beta}}{2\sqrt{\pi}} \sum_{i=0}^{m_h L-1} \sum_{j=0}^{m_g M-1} \sum_{l=0}^{i+j-k} \sum_{k=0}^j \sum_{n=0}^k \binom{i+j-k}{l} \binom{j}{k} \binom{k}{n} \frac{\mathcal{B}^{-l-m_1 N_1} \xi^{k-i-j} m_h^i m_g^j \gamma^{i+j}}{i! j! \Omega_h^i \Omega_g^j (\bar{\gamma})^{i+j}} \left(\frac{m_g}{\Omega_g \bar{\gamma}} \right)^{-n-m_R N_R} \left(\frac{m_1}{P_1^I \Omega_1} \right)^{m_1 N_1} \\
&\times \frac{\Gamma(l+m_1 N_1)}{\Gamma(m_1 N_1)} \left(\frac{m_R}{P_R^I \Omega_R} \right)^{m_R N_R} \frac{\Gamma(n+m_R N_R)}{\Gamma(m_R N_R)} \left[\sum_{u=1}^{l+m_1 N_1} c \left(\frac{m_1}{P_1^I \Omega_1 \mathcal{B}} \right)^{-u} {}_2F_0 \left(i+j+0.5, u; ; -\frac{P_1^I \Omega_1 \mathcal{B}}{(\mathcal{A}+\beta) m_1} \right) \mathcal{L}_u \right. \\
&\quad \left. + \sum_{v=1}^{n+m_R N_R} c \left(\frac{m_R \Omega_g \gamma}{m_g P_R^I \Omega_R} \right)^{-v} {}_2F_0 \left(i+j+0.5, v; ; -\frac{m_g P_R^I \Omega_R}{(\mathcal{A}+\beta) m_R \Omega_g \bar{\gamma}} \right) \mathcal{L}_v \right] \quad (17)
\end{aligned}$$

constraint $P_T = P_{S_a} + P_{S_b} + P_R$, where P_T is the total transmit power, while d_1 and d_2 denote the distance between $S_a \leftrightarrow R$ and $S_b \leftrightarrow R$ respectively. For our analysis, we have assumed $P_{S_a} = P_{S_b} = P_S$. Thus, the objective function will be given as

$$\begin{aligned}
P_S^*, P_R^* &= \arg \min_{P_S, P_R, d_1, d_2} \mathcal{P}_{out}^\infty(\gamma_{th}) \\
&\text{subject to } 2P_S + P_R \leq P_T, P_S, P_R, d_1, d_2 > 0. \quad (18)
\end{aligned}$$

For the ease of our analysis, we have assumed $m_h = m_g = 1$ and $L = M = 1$ in (9)³. Thus the simplified expression of OP at S_a is given as

$$\tilde{\mathcal{P}}_{out}(\gamma_{th}) = 1 - \frac{e^{-\frac{\gamma_{th}}{\xi\bar{\gamma}}[e+f(\xi+1)]}}{\left(\frac{P_1^I \Omega_1 \gamma_{th} (e+f)}{m_1 \xi \bar{\gamma}} + 1 \right)^{m_1 N_1} \left(\frac{P_R^I \Omega_R \gamma_{th} f}{m_R \bar{\gamma}} + 1 \right)^{m_R N_R}} \quad (19)$$

where $e = \frac{m_h}{P_S \Omega_h}$ and $f = \frac{m_g}{P_S \Omega_g}$. In the high SNR regime, the above expressed outage expression is reduced to

$$\begin{aligned}
\tilde{\mathcal{P}}_{out}^\infty(\gamma_{th}) &= \frac{\gamma_{th} (e+f)}{\xi \bar{\gamma}} \left(1 + N_1 P_1^I \Omega_1 \right) + \frac{\gamma_{th} f}{\bar{\gamma}} \left(1 + N_R P_R^I \Omega_R \right) \\
&\quad + O(\gamma_{th}) \quad (20)
\end{aligned}$$

We assume that the average fading power Ω_h and Ω_g follow exponential distribution with distance such that $\Omega_h = d_1^{-z}$ and $\Omega_g = d_2^{-z}$. Note that z is the path-loss exponent, which lies in the range of 2 to 6. Now, substituting the path loss definition of e and f and neglecting higher order terms, (20) can be expressed as

$$\tilde{\mathcal{P}}_{out}^\infty(\gamma_{th}) \triangleq \mathcal{K}(\gamma_{th}) = \mathcal{E}_1 \frac{d_1^z}{P_R P_S} + \mathcal{E}_1 \frac{d_2^z}{P_R P_S} + \mathcal{F}_1 \frac{d_2^z}{P_S^2}, \quad (21)$$

where, $\mathcal{K}(\gamma_{th})$ is the objective function of γ_{th} , $\mathcal{E}_1 = (1 + N_1 P_1^I \Omega_1) \gamma_{th}$ and $\mathcal{F}_1 = (1 + N_R P_R^I \Omega_R) \gamma_{th}$.

A. Optimization of Power Allocation (OPA) under Fixed Relay Location

The OPA can be formulated for power constraint P_T of given system as

$$\begin{aligned}
P_S^*, P_R^* &= \arg \min_{P_S, P_R} \mathcal{K}(\gamma_{th}) \\
&\text{subject to } 2P_S + P_R \leq P_T, P_S, P_R. \quad (22)
\end{aligned}$$

The second derivative of (21) with respect to P_S is equal to

$$\frac{\partial^2 \mathcal{K}(\gamma_{th})}{\partial P_S^2} = (\mathcal{E}_1 d_1^z + \mathcal{E}_1 d_2^z) \frac{(2P_R^2 + 8P_S^2 - 4P_R^2 P_S^2)}{P_S^4 P_R^4} + \frac{6\mathcal{F}_1 d_2^z}{P_S^4} > 0 \quad (23)$$

³Root finding algorithms such as Newton-Raphson method or bisection method can be used in solving optimal power or distance for higher values of m_h, m_g, L and M .

By observing above equation we can say it is strictly convex with respect to $P_S \in (0, P_T/2)$. Thereby, the optimal power can be obtained by equating the first derivative to 0 as

$$\frac{\partial \mathcal{K}(\gamma_{th})}{\partial P_S} = (\mathcal{E}_1 d_1^z + \mathcal{E}_1 d_2^z) \frac{(2P_S - P_R)}{P_R^2} - \frac{2\mathcal{F}_1 d_2^z}{P_S} = 0, \quad (24)$$

$$(24) \text{ can be solved to obtain } P_S^* = \frac{P_T \left(1 + \sqrt{1 + \frac{16\mathcal{F}_1 d_2^z}{\mathcal{E}_1 d_1^z + \mathcal{E}_1 d_2^z}} \right)}{\left(6 + 2\sqrt{1 + \frac{16\mathcal{F}_1 d_2^z}{\mathcal{E}_1 d_1^z + \mathcal{E}_1 d_2^z}} \right)}.$$

B. Optimization of Relay Location (ORL) under Fixed power Allocation

The total distance between S_a and S_b is normalized to unity so that $d_1 + d_2 = 1$. The optimal relay position can be formulated as

$$\begin{aligned}
d^* &= \arg \min_{d_1} \mathcal{K}(\gamma_{th}) \\
&\text{subject to } 0 < d_1 < 1. \quad (25)
\end{aligned}$$

By substituting $d_1 = d$ and $d_2 = 1 - d$, the objective function in (21) is reduced to

$$\mathcal{K}(\gamma_{th}) = \mathcal{E}_1 \frac{d^z + (1-d)^z}{P_R P_S} + \mathcal{F}_1 \frac{(1-d)^z}{P_S^2}. \quad (26)$$

By following the same procedure as optimal power allocation, we can show that $\mathcal{K}(\gamma_{th})$ in (21) is strictly convex function of d_1 for $d_1 \in (0, 1)$. The optimal relay position can be obtained from

$$\frac{\partial \mathcal{K}(\gamma_{th})}{\partial d} = \frac{\mathcal{E}_1 z}{P_R P_S} (d^{z-1} - (1-d)^{z-1}) - \frac{\mathcal{F}_1 z}{P_S^2} (1-d)^{z-1} = 0, \quad (27)$$

$$(27) \text{ can be solved to obtain } d^* = \frac{1}{1 + \left(\frac{\mathcal{E}_1 P_S}{\mathcal{E}_1 P_S + \mathcal{F}_1 P_R} \right)^{\frac{1}{z-1}}}.$$

C. Joint Optimization of Relay Location and Power Allocation

The problem of joint optimization can be formulated as

$$\begin{aligned}
P_S^*, P_R^*, d^* &= \arg \min_{P_S, P_R, d_1, d_2} \mathcal{K}(\gamma_{th}) \\
&\text{subject to } 2P_S + P_R \leq P_T, P_S, P_R, d_1, d_2 > 0. \quad (28)
\end{aligned}$$

By differentiating (21) with respect to P_S , (26) with respect to d and equating both to 0, we will get the following equations

$$\frac{d^{z-1}}{(1-d)^{z-1}} = 1 + \frac{\mathcal{F}_1 P_R}{\mathcal{E}_1 P_S}, \quad \frac{2\mathcal{F}_1 \left(\frac{P_R}{P_S} \right)^2}{\mathcal{E}_1 \left(2 - \frac{P_R}{P_S} \right)} - 1 = \frac{d^z}{(1-d)^z}$$

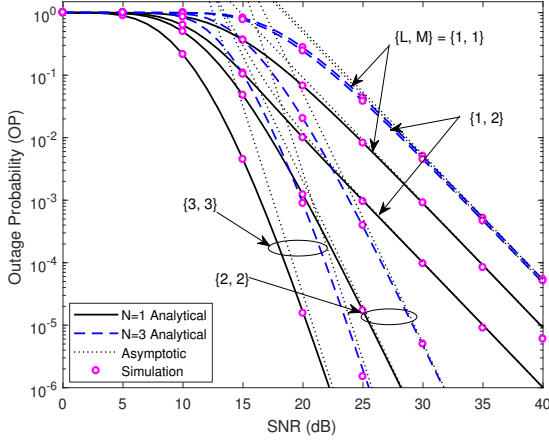


Fig. 2. OP vs SNR of BF Multi-antenna TWR system under constant interference power

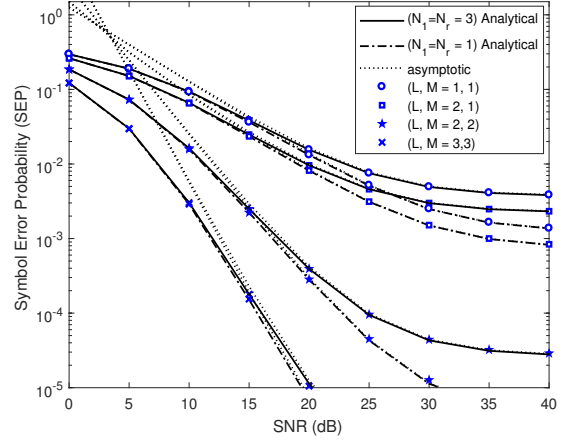


Fig. 4. SEP vs SNR of BF Multi-antenna TWR system under different system parameter

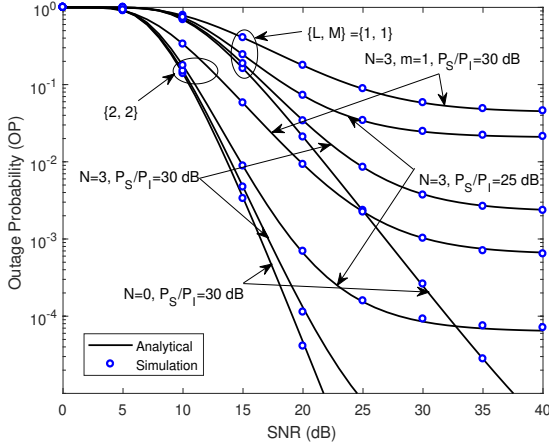


Fig. 3. OP vs SNR of BF Multi-antenna TWR system under fixed P_S/P_I ratio

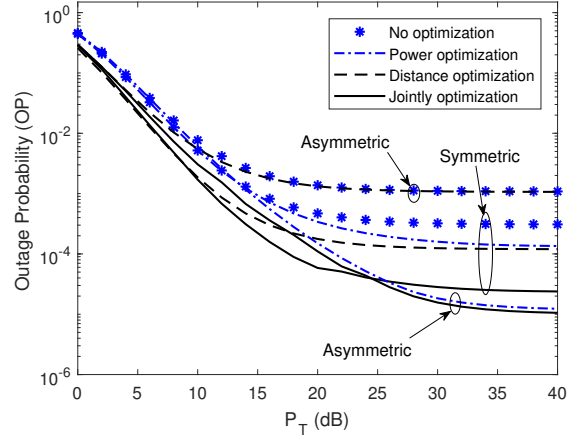


Fig. 5. OP vs P_T of BF Multi-antenna TWR system under different system parameter

We are able to get optimal power allocation and relay location by solving above equations $P_S^* = \frac{P_T \left(1 + \sqrt{1 + \frac{16\mathcal{F}_1}{\mathcal{E}_1 \mathcal{D}_1}}\right)}{(6 + 2\sqrt{1 + \frac{16\mathcal{F}_1}{\mathcal{E}_1 \mathcal{D}_1}})}$, where,

$$\mathcal{D}_1 = 1 + \frac{2\mathcal{E}_1 \left(\left(\frac{d}{1-d} \right)^{z-1} - 1 \right)^2 + \mathcal{E}_1 \left(\frac{d}{1-d} \right)^{z-1} - \mathcal{E}_1 - 2\mathcal{F}_1}{2\mathcal{F}_1 + \mathcal{E}_1 - \mathcal{E}_1 \left(\frac{d}{1-d} \right)^{z-1}} \quad (29)$$

Utilizing P_S^* defined above with $P_R = P_T/2$ and $P_{S_b} = P_T/2 - P_{S_a}$ in (27) and using root finding techniques, we can find the d^* and accordingly P_S^* can be obtained from (24).

V. NUMERICAL AND SIMULATION RESULTS

Without loss of generality, we have assumed transmission rate $R_s = 3$ bps/Hz, $m_h = m_g = 2$, $m_1 = m_R = 2$, noise variances and all the average channel gains to be unity. Simulations are averaged over 1 million iterations.

Fig. 2 illustrates the analytical lower bound OP (given in (10)) versus (vs) transmit signal-to-noise ratio (SNR), $\bar{\gamma} = P_S/N_0$, where interference power is kept constant. Here we have assumed $P_1^I = P_R^I = 20$ dB. Here, we have shown two sets of simulation results, one with a different number of

interferers and another with varying the number of antennas on each source terminal. We observed that system OP at S_a decreases with increasing L and increases with increasing interferers, while increasing transmitting antenna (M) doesn't reduce the effect of CCI which is in the consent of [15]. The analytical curve shows excellent agreement with Monte Carlo simulation across the whole SNR range, thus validates our analysis. Similarly, the asymptotic outage follows exactly at high SNR and very effectively reflects the exact OP.

Fig. 3 demonstrate the lower bound outage expression (9), with P_S/P_I as constant and $\xi = 0.5$. We also consider the Rayleigh fading scenario for the sake of completeness, where all shape parameters (m) are equal to 1. We observe that the OP increases by increasing the number of interferers. The OP reaches an error floor as soon as the SNR increases, it is due to the pronounced effect of CCI thus reduces the diversity order to 0. On the contrary, the diversity order is maintained at $\min(m_h L, m_g M)$ for the interference-free case. It is also evident that increasing number of antenna sufficiently alleviate the effect of CCI.

Fig. 4 presents the lower bounded SEP given in (17) for

BPSK modulation ($\alpha = \beta = 1$) plotted as a function of SNR for two sets of simulation. One with a different number of interferers and another with varying the number of antennas on each source terminal. The simulation result having a good match with the analytical curve shows the validity of our analysis. A similar observation can be made by varying the shape parameters.

The optimization scenarios outlined in Section IV is presented in Fig. 5, where P_S/P_I is kept constant and OP curves are plotted against P_T . We have assumed two cases namely symmetric and asymmetric interference profile, former represent $P_S - P_I^1 = 15\text{dB}$, $P_R - P_R^I = 15\text{dB}$, while later represents $P_S - P_I^1 = 0\text{dB}$, $P_R - P_R^I = 30\text{dB}$. Our results show that the joint optimization in asymmetric power case outperforms the symmetric in high SNR and vice versa. In the asymmetric case, the ORL and non-optimized case ($d = 0.5$ and $P_R = 2P_S = P_T/2$) are having same results. On the other hand, OPA leads to sufficient performance gain same as the joint optimization. In the symmetric case, joint optimization gives an impressive gain in performance by ten orders of magnitude as compared to the non-optimized case. As a concluding point, for low SNR regime the best choice is $d = 0.5$ and $P_R = 2P_S = P_T/2$.

VI. CONCLUSIONS

In this paper, we have examined the BF scheme for a two-way multiantenna relaying network, while all the terminals were experiencing the multiple CCIs. We have derived closed-form expressions of a tight lower bound on the OP and SEP. We analyzed the influence of CCIs, antenna configurations, power allocation and relay location on the outage performance of the considered system. Moreover, asymptotic result providing further insights into diversity order and array gain were also obtained. With the aim to minimize the OP, we analyzed the joint optimization of relay location and power allocation. Our finding suggests that deployment of multiple antenna improves the performance when the relay assisted system is affected by CCI of low level, thus proved to be an attractive solution. However, if the CCI effect is more pronounced, the performance significantly deteriorates and the diversity order is reduced to zero. Additionally, interference cancellation in multi-antenna schemes could be considered and implemented for future work.

APPENDIX

From (7), we know that OP can be written as function of $(\gamma_{S_b \rightarrow S_a})^{up}$ as

$$\begin{aligned} \mathcal{P}_{out}(\gamma_{th}) &= F_{\gamma_{S_b \rightarrow S_a}}^{up}(\gamma_{th}) = \Pr((\gamma_{S_b \rightarrow S_a})^{up} \leq \gamma_{th}) \\ &= 1 - \Pr\left(\frac{\xi\gamma_1}{\gamma_\delta} > \gamma_{th}\right) \Pr\left(\frac{\xi\gamma_2}{\xi\gamma_R + \gamma_\delta} > \gamma_{th}\right) \\ &= 1 - \left(1 - \Pr\left(\|\mathbf{h}\|^2 \leq \frac{\gamma_\delta \gamma_{th}}{\xi\bar{\gamma}}\right)\right) \\ &\times \left(1 - \Pr\left(\|\mathbf{g}\|^2 \leq \frac{(\xi\gamma_R + \gamma_\delta)\gamma_{th}}{\xi\bar{\gamma}}\right)\right) \\ &= 1 - \frac{1}{\Gamma(m_h L)\Gamma(m_g M)} \int_{w=0}^{\infty} \int_{v=0}^{\infty} \Gamma\left(m_h L, \frac{m_h(w+1)\gamma_{th}}{\Omega_h \xi \bar{\gamma}}\right) \end{aligned}$$

$$\times \Gamma\left(m_g M, \frac{m_g \gamma_{th} (\xi(v+1) + (w+1))}{\Omega_g \xi \bar{\gamma}}\right) f_W(w) f_V(v) dw dv \quad (30)$$

In order to simplify the derivation we have considered that $P_{S_{1,1}} = P_{S_{1,2}} = \dots = P_{S_{1,N_1}} = P_I^I$ [5] and obtain the probability density function of W under Nakagami- m fading with m_1 and $\frac{\Omega_1}{m_1}$ are shape and scale parameter of interference link as $f_W(w) = \frac{w^{m_1 N_1 - 1}}{\Gamma(m_1 N_1)} \left(\frac{m_1}{P_I^I \Omega_1}\right)^{m_1 N_1} e^{-\frac{m_1 w}{P_I^I \Omega_1}}$. Similarly, we have considered that $P_{R,1} = P_{R,2} = \dots = P_{R,N_1} = P_R^I$ and obtain the probability density function of V under Nakagami- m fading with m_R and $\frac{\Omega_R}{m_R}$ are shape and scale parameter of interference link as $f_V(v) = \frac{v^{m_R N_R - 1}}{\Gamma(m_R N_R)} \left(\frac{m_R}{P_R^I \Omega_R}\right)^{m_R N_R} e^{-\frac{m_R v}{P_R^I \Omega_R}}$. Utilizing $f_W(w)$ and $f_V(v)$ in (30), evaluating the integrals using [13, Eq. (3.351.3)] and after some mathematical manipulation we will get (9).

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