Optimization of Majority Rule Threshold in Double Threshold Based Cooperative Cognitive Radio Network

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***Abstract*—In this paper, we investigate a double threshold based cooperative spectrum sensing scenario. Our objective is to determine the optimal threshold for majority rule which must be selected for minimum error in final decision. The CR sensors are assumed to make local hard decisions based on conventional energy detection technique and communicate one bit decision information to the fusion center. Here we assume that sensors whose test statistics fall in ambiguity region do not report to the fusion center. A majority rule is applied at the fusion center in which at least threshold n^* number of local sensor decision must favor for presence of primary user (PU) to make the final decision on presence of PU. Since choice of n^* decides error in final decision, we formulate an expression to compute optimal value n^*, i.e., n^*_{opt} which minimizes error in final decision. Further, due to uncertainty in number of sensors with test statistics in ambiguity region, the threshold n^*_{opt} also becomes a random variable. Hence we derive a statistical model to characterize the density function of number of sensors with test statistics in ambiguity region, and later exploit it to derive an expression for expected value of n^*_{opt}. Our simulation results validate our approach in which we show by selecting n^*_{opt} as threshold for majority rule, the error in final decision is at its minimum value.**

I. Introduction

In present scenario, cognitive radio (CR) technology has become one of the major research area in the field of wireless communication. This is due to its inherent capability in addressing the crucial challenge of spectrum scarcity by opportunistic utilization of under-utilized licensed spectrum [1][2]. In order to co-exist with the licensed spectrum allocation, CR technology allows unlicensed secondary users (SUs) to access the spectrum without causing interference to primary user (PU) communication. The non-interfering access of spectrum is through detection of spectrum holes (part of spectrum unused by PU) by CR sensors and is also termed as spectrum sensing [3].

Spectrum sensing can be realized by several techniques such as, energy detection, matched filter detection, cyclo-stationary detection [3]. Out of these techniques, energy detection is the simplest and most popular technique in which samples sensed from surrounding spectrum environment are exploited to compute the signal energy. The computed signal energy is then compared to a threshold to make decision on presence or absence of PU. The performance of energy detection technique is however degraded by several factors, like, shadowing, multi-path fading, noise uncertainty etc. In order to mitigate above factors, cooperative spectrum sensing has been proposed in [4] and [5].

In cooperative spectrum sensing, multiple CR sensors are deployed at different spatial locations. These sensors cooperate in either centralized or distributed manner to make final decision on presence or absence of PU signal [6]. For centralized cooperative spectrum sensing, a fusion center is employed at the apex to receive sensing information from all local CR sensors though reporting channel. In order to make final decision, fusion center combines the received sensing information through either hard or soft combining techniques [7]. In hard combining technique, CR sensors first make their local decisions which are later combined at fusion center and the final decision is made on the basis of AND, OR or majority rule [8]. A $n-outof-N$ majority rule is proposed in [9] where optimal value of threshold n is computed to minimize the total error in spectrum sensing. In this work, we have analyzed energy detection based cooperative hard combining technique as communication cost required for reporting the local decisions is less compared to soft decision techniques and hence is more bandwidth efficient.

Spectrum sensing based on principle of energy detection can be visualized as a binary hypothesis problem. It is observed that around the threshold for decision making, the probability density function of sensed energy under the two hypotheses have almost the same values. Thus, decisions made in the cases when sensed energy is close to threshold are highly unreliable. In order to improve reliability in decision making, a double threshold based energy detection is conceptualized in [10], [11]. Stressing on bandwidth efficiency, authors in [12] have investigated quantization techniques to reduce average number of bits required for reporting the local decision in view of bandwidth constraints. Based on our limited research survey, investigation of optimal threshold selection for majority rule in double threshold based cooperative spectrum sensing environment is still an unaddressed problem.

In this paper, we analyze a double threshold based cooperative spectrum sensing scenario with an objective to determine optimal majority rule threshold for minimum error in final decision. The CR sensors (assuming N sensor are deployed) exploit energy detection technique to make local hard decision and communicate one bit decision information to the central fusion center. Here we assume that sensors (assuming K such sensors) do not report any decision to the fusion center if their test statistics (sensed signal energy) fall in ambiguity region. The local decisions obtained from $N-K$ sensors are combined at fusion center according to $n^*\ outof\ (N-K)$ majority decision rule, where n^* is the integer threshold for majority rule. Since choice of n^* decides error in final decision, we formulate an expression to compute optimal value n^*, i.e., n^*_{opt} which minimizes the error in final decision. Further, number of sensors K not participating in final decision process being a random quantity, we derive a statistical model to characterize the density function of K as function of: (i) signal to noise ratio (SNR) observed at CR sensors; and (ii) width of ambiguity region used in local decision making. Finally, exploiting density function of K, we derive an expression for expected value of n^*_{opt} which on average will reduce error in final decision making.

II. System Model

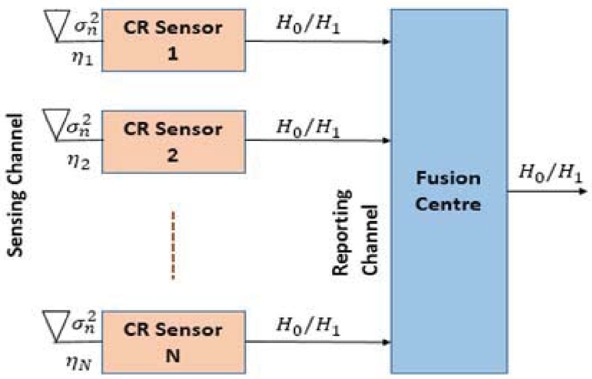

Fig. 1. Block diagram of cooperative spectrum sensing

Consider a spectrum sensing scenario shown in Fig. 1, in which N cognitive radio (CR) sensors cooperate to sense a given narrow band spectrum. The sensors compute local test statistics and send their decisions to a central fusion center which after accumulating the local decisions takes a final decision based on majority rule. In order to compute the test statistics, we further assume that every sensor n where $n \in \{1, 2, \cdots N\}$ gathers L number of samples in a given sensing time interval. Let $y_n(l)$ represent the signal sample sensed by n^{th} sensor at l^{th} time index where $l \in \{0, 1, ..., L-1\}$. The test statistic $T_n(y)$ of n^{th} sensor for energy detection based sensing mechanism can be expressed as,

$$T_n(y) = \sum_{l=0}^{L-1} |y_n(l)|^2. \tag{1}$$

The values taken by sensed sample $y_n(l)$ can be classified in either of two hypothesis,

$$\mathcal{H}_0 : y_n(l) = w_n(l)$$
$$\mathcal{H}_1 : y_n(l) = h_n(l) \cdot s(l) + w_n(l)$$

where, $\mathcal{H}_0$ and $\mathcal{H}_1$ denotes null and alternate hypothesis for absence and presence of PU signal $s(l)$; $h_n(l)$ and $w_n(l)$ denotes Rayleigh distributed flat channel fading coefficient and additive noise respectively, for n^{th} CR sensor. Here additive noise is assumed as white and is modeled by Gaussian distribution with zero mean and variance σ_w^2.

Next, we assume that the sensing time interval is small compared to coherence time of the channel. Thus, channel coefficients $h_n(l)$ can be identified as time invariant during sensing interval and hence without loss of generality, $h_n(l)$ can be denoted as h_n. The expected signal to noise ratio (SNR) for n^{th} sensor can be expressed as $\bar{\eta}_n = \frac{\mathrm{E}[|h_n|^2]P}{\sigma_w^2}$ in which, $\mathrm{E}[\cdot]$ is expectation operator; and P denotes power of PU signal. We further assume that the distance between PU and CR sensors is large compared to the distance between sensors. Thus, signal received at individual sensors observe identical path loss environment. Hence, under H_0 hypothesis in which PU is absent and channel is AWGN, we can assume $\bar{\eta}_n = \bar{\eta} := \eta, \quad \forall n \in \{1, \cdots N\}$. In case of H_1 hypothesis in which PU is present, it is reasonable to assume channel coefficients are independent and identical in distribution (i.i.d). Thus, for Rayleigh fading, instantaneous SNR $\eta_1, \cdots \eta_N$ are i.i.d. exponentially distributed with same mean $\bar{\eta} := \eta$.

Considering above assumptions, the distribution of test statistics defined in equation (1) follows chi square distribution [13] under binary hypothesis H_0 and H_1 and can be expressed as,

$$f_{T_n(y)}(y) = \begin{cases} \chi_L^2 = \dfrac{y^{\frac{L}{2}-1} e^{\frac{-y}{2\sigma_w^2}}}{\sigma_w^L 2^{\frac{L}{2}} \Gamma(\frac{L}{2})} & H_0 \\ \chi_L^2(\gamma) = \frac{1}{2\sigma_w^2} \left(\frac{y}{\gamma}\right)^{\frac{L}{4}-\frac{1}{2}} e^{\left(\frac{\gamma}{2}+\frac{y}{2\sigma_w^2}\right)} I_{\frac{L}{2}-1}\left(\sqrt{\frac{\gamma y}{\sigma_w^2}}\right) & H_1 \end{cases}$$

where, $\gamma = 2L\eta$ is non-centrality parameter with L degrees of freedom; $\Gamma(\cdot)$ denotes Gamma function; and $I_v(\cdot)$ denotes modified Bessel function of first kind and order v.

Conventional energy detectors exploit single threshold λ for decision making and via analytical simplicity λ can be computed as intersection point between distribution of test statistics $T_n(y)$ under hypothesis H_0 and H_1. In our analysis, we exploit double threshold based energy detection in which we define ambiguity region of width $\Delta_{th} = \lambda_2 - \lambda_1$ around threshold λ. The two thresholds can be computed as $\lambda_1 = \lambda - \frac{\Delta_{th}}{2}$ and $\lambda_2 = \lambda + \frac{\Delta_{th}}{2}$. Fig. 2 shows the distribution of $T_n(y)$ under two hypothesis with normalized ambiguity region of width $\Delta := \frac{\Delta_{th}}{\lambda}$ and thresholds $\lambda_1 = \lambda(1 - \frac{\Delta}{2})$ and $\lambda_2 = \lambda(1 + \frac{\Delta}{2})$.

Based on computed test statistics, local hard decision at n^{th} CR sensor is made in favor of either of the following choices:

- if $T_n(y) \geq \lambda_2 \implies$ PU signal is present
- if $T_n(y) \leq \lambda_1 \implies$ PU signal is absent

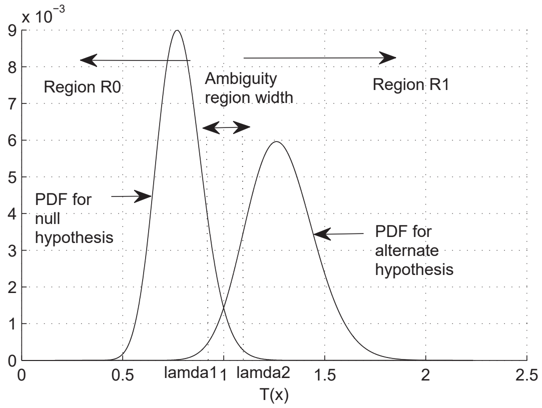

Fig. 2. Ambiguity region in null and alternate hypothesis

- $\lambda_1 < T_n(y) < \lambda_2 \implies$ No decision

After making local decisions, a one bit information B_n (B_n = 1/0 if PU is decided as present/absent) from n^{th} sensor is sent to fusion center through reporting channel. The local sensors whose test statistics fall in ambiguity region do not make any decision and hence do not communicate to fusion center. Thus, if K is the number of local sensors whose test statistics fall in ambiguity region, then only $N-K$ sensors participate in final decision making at fusion center.

Finally, at fusion center, the received one bit information from all $N-K$ sensors are fused together according to following combining rule:

$$D = \sum_{j=1}^{N-K} B_j \begin{cases} \geq n^*, & H_1 \\ < n^*, & H_0 \end{cases} \tag{2}$$

where, n^* is an integer threshold for $n^* - outof - (N-K)$ majority rule. It is easy to notice that $n^* = 1$ decision rule corresponds to OR rule; $n^* = N-K$ decision rule corresponds to AND rule; and $n^* = (N-K)/2$ decision rule corresponds to more than half majority rule.

III. Performance Characterization of CR Sensor

The probability of false alarm (P_{fa}) is defined as the probability of detecting the PU given that hypothesis H_0 is true, and can be computed as,

$$\begin{aligned} P_{fa} &= \int_{\lambda_2}^{\infty} P(T_n(y)|H_0)dy \\ &= \int_{\lambda_2}^{\infty} \frac{y^{\frac{L}{2}-1} e^{\frac{-y}{2\sigma_w^2}}}{\sigma_w^L 2^{\frac{L}{2}} \Gamma(\frac{L}{2})} dy \quad = \frac{\Gamma\left(\frac{L}{2}, \frac{\lambda_2}{2\sigma_w^2}\right)}{\Gamma\left(\frac{L}{2}\right)} \end{aligned} \tag{3}$$

where, $P(\cdot|\cdot)$ denotes conditional probability density function. Similarly, probability of correctly detecting the absence of PU conditioned over hypothesis H_0 being true can be computed as,

$$\begin{aligned} P_{cd,ab} &= \int_{\lambda_1}^{0} P(T_n(y)|H_0)dy \\ &= \int_{\lambda_1}^{0} \frac{y^{\frac{L}{2}-1} e^{\frac{-y}{2\sigma_w^2}}}{\sigma_w^L 2^{\frac{L}{2}} \Gamma(\frac{L}{2})} dy \quad = 1 - \frac{\Gamma\left(\frac{L}{2}, \frac{\lambda_1}{2\sigma_w^2}\right)}{\Gamma\left(\frac{L}{2}\right)}. \end{aligned} \tag{4}$$

Next, conditioned on Hypothesis H_1 to be correct, probability of correctly detecting the PU can be computed as,

$$\begin{aligned} P_{cd,pr} &= \int_{\lambda_2}^{\infty} P(T_n(y)|H_1)dy \\ &= \int_{\lambda_2}^{\infty} \frac{1}{2\sigma_w^2} \left(\frac{y}{\gamma}\right)^{\frac{L}{4}-\frac{1}{2}} e^{\left(\frac{\gamma}{2}+\frac{y}{2\sigma_w^2}\right)} I_{\frac{L}{2}-1}\left(\sqrt{\frac{\gamma y}{\sigma_w^2}}\right) dy \\ &= Q_{\frac{L}{2}}\left(\sqrt{(\gamma)}, \sqrt{(\frac{\lambda_2}{\sigma_w^2})}\right) \end{aligned} \tag{5}$$

where, $Q_m(a,b)$ is generalized Marcum Q-function. Similarly, probability of missed detection defined as probability of not detecting the PU given hypothesis H_1 is correct can be computed as,

$$\begin{aligned} P_{md} &= \int_{\lambda_1}^{0} P(T_n(y)|H_1)dy \\ &= \int_{\lambda_1}^{0} \frac{1}{2\sigma_w^2} \left(\frac{y}{\gamma}\right)^{\frac{L}{4}-\frac{1}{2}} e^{\left(\frac{\gamma}{2}+\frac{y}{2\sigma_w^2}\right)} I_{\frac{L}{2}-1}\left(\sqrt{\frac{\gamma y}{\sigma_w^2}}\right) dy \\ &= 1 - Q_{\frac{L}{2}}\left(\sqrt{(\gamma)}, \sqrt{(\frac{\lambda_1}{\sigma_w^2})}\right). \end{aligned} \tag{6}$$

It can be noticed that $P_{cd,pr}$ and P_{md} are function of instantaneous SNR η (via $\gamma = 2L\eta$) which is a random variable. For channel coefficients modeled by Rayleigh distribution, SNR η follows an exponential distribution [13] and can be expressed as,

$$f(\eta) = \frac{1}{\eta} e^{\frac{\eta}{\bar{\eta}}}, \qquad \eta \geq 0$$

where, $\bar{\eta}$ is mean of SNR. Substituting $y = \sqrt{2L\eta}$ in equation (5), expected $P_{cd,pr}$ can be expressed as,

$$\begin{aligned} \bar{P}_{cd,pr} &= \int_0^{\infty} P_{cd} f(\eta) d\eta \\ &= e^{-\frac{\lambda_2'}{2}} \left[(1+\rho^2)^{\frac{L}{2}-1} \left(e^{\frac{\lambda_2'}{2+2\rho^2}} - \sum_{k=0}^{\frac{L}{2}-2} \frac{1}{k!} \left(\frac{\lambda_2'}{2}\right)^k \right) \right] \end{aligned} \tag{7}$$

where, $\lambda_2' = \frac{\lambda_2}{\sigma_w^2}$ and $\rho^2 = \frac{1}{L\bar{\eta}}$. Similarly, substituting $y = \sqrt{2L\eta}$ in equation (6), expected $P_{md,pr}$ can be expressed as,

$$\begin{aligned} \bar{P}_{md} &= \int_0^{\infty} P_{md} f(\eta) d\eta \\ &= 1 - e^{-\frac{\lambda_1'}{2}} \left[(1+\rho^2)^{\frac{L}{2}-1} \left(e^{\frac{\lambda_1'}{2+2\rho^2}} - \sum_{k=0}^{\frac{L}{2}-2} \frac{1}{k!} \left(\frac{\lambda_1'}{2}\right)^k \right) \right] \end{aligned} \tag{8}$$

where, $\lambda_1' = \frac{\lambda_1}{\sigma_w^2}$.

Finally, probability that a test statistics lies inside the ambiguity region assuming hypothesis H_0 is correct can be computed as,

$$P_{L0} = \int_{\lambda_2}^{\lambda_1} P(T_n(y)|H_0)dy = 1 - P_{fa} - P_{cd,ab}. \tag{9}$$

Similarly, probability that the test statistics lies inside ambiguity region given hypothesis H_1 is correct can be computed as,

$$P_{L1} = \int_{\lambda_2}^{\lambda_1} P(T_n(y)|H_1)dy = 1 - \bar{P}_{cd,pr} - \bar{P}_{md}. \tag{10}$$

IV. Performance Characterization of Cooperative Spectrum Sensing

Assuming K our of total N sensors do not participate in decision making due to their test statistics falling in ambiguity region, the cooperative probability of false alarm $P_{fa}^{cop}(N-K)$ can expressed as,

$$P_{fa}^{cop}(N-K) = \binom{N}{K} P_{L0}^{K} \times \sum_{l=n^*}^{N-K} \binom{N-K}{l} P_{fa}^{l} P_{cd,ab}^{N-K-l} \tag{11}$$

Similarly, cooperative probability of miss detection $P_{cd}^{cop}(N-K)$ can be expressed as,

$$\begin{aligned} &P_{md}^{cop}(N-K) = 1 - P_{cd}^{cop}(N-K) = \\ &1 - \binom{N}{K} P_{L1}^{K} \times \sum_{l=n^*}^{N-K} \binom{N-K}{l} \bar{P}_{cd,pr}^{l} \bar{P}_{md}^{N-K-l} \end{aligned} \tag{12}$$

where, n^* is an integer threshold for $n^* - outof - (N-K)$ decision rule expressed in equation (2).

V. Optimization of Majority Rule Threshold

In this section, we analyze optimal voting rule for double threshold based energy detection in which we find optimal value of n^*, i.e., n_{opt}^* whose value gives the minimum error in final decision. The error in final decision can be defined as,

$$\begin{aligned} E_{fd} &= P_{fa}^{cop}(N-K) + P_{md}^{cop}(N-K) \\ &= P_{fa}^{cop}(N-K) + (1 - P_{cd}^{cop}(N-K)) \end{aligned} \tag{13}$$

Let $M(n^*) := P_{fa}^{cop}(N-K) - P_{cd}^{cop}(N-K)$, then $E_{fd} = 1 + M(n^*)$. On differentiating $M(n^*)$ with respect to n^* we get,

$$\begin{aligned} &\tfrac{\partial M(n)}{\partial n^*} \approx M(n^*+1) - M(n^*) \\ &\tbinom{N}{K}\tbinom{N-K}{n^*}\Big(-P_{L0}^{K} P_{fa}^{n^*} P_{cd,ab}^{N-K-n^*} \\ &\qquad + (P_{L1})^{K} (\bar{P}_{cd,pr})^{n^*} (\bar{P}_{md})^{N-K-n^*}\Big). \end{aligned} \tag{14}$$

To get optimal value of n^*, i.e., n_{opt}^* we equate $\frac{\partial M(n^*)}{\partial n^*} = 0$,

$$\frac{P_{fa}^{n^*} P_{cd,ab}^{N-K-n^*}}{\bar{P}_{cd,pr}^{n^*} \bar{P}_{md}^{N-K-n^*}} = \left(\frac{P_{L1}}{P_{L0}}\right)^{K}. \tag{15}$$

Next,taking logarithmic on both sides of equation (15), we get,

$$\begin{aligned} &n^*\left(lnP_{fa} - ln\bar{P}_{cd,pr}\right) + (N-K-n^*)\left(lnP_{cd,ab} - ln\bar{P}_{md}\right) \\ &= K\left(ln\bar{P}_{L1} - lnP_{L0}\right) \end{aligned} \tag{16}$$

It is generally observed that probability of test statistics falling in ambiguity region under hypothesis H_0 and H_1 is almost same, thus $\frac{P_{L1}}{P_{L0}} \approx 1$ resulting in right hand side of equation (16) to be zero. Thus, optimal value of n^* can be expressed as,

$$n_{opt}^* = (N-K)\left(\frac{ln\frac{\bar{P}_{md}}{P_{cd,ab}}}{ln\frac{P_{fa}\bar{P}_{md}}{\bar{P}_{cd,pr}P_{cd,ab}}}\right). \tag{17}$$

Since n_{opt}^* is a positive integer quantity,

$$n_{opt}^* = \left\lceil \frac{N-K}{\psi+1} \right\rceil, \quad \text{where} \quad \psi = \frac{ln\frac{P_{fa}}{\bar{P}_{cd,pr}}}{ln\frac{\bar{P}_{md}}{P_{cd,ab}}}. \tag{18}$$

VI. Statistical Characterization of Optimal Threshold

It can be noticed from equation (18) that optimal threshold n_{opt}^* for final decision is function of number of sensors not participating in decision making $K \in \{1, 2, \cdots N\}$ for a given number of deployed sensors N. The number of sensors which do not participate in decision making have their test statistics in ambiguity region and is a random quantity. Thus, n_{opt}^* is also a random variable and for its statistical characterization we first derive a statistical model for K.

For the system model considered in section I, number of sensors K whose test statistics fall in ambiguity region depends on the two parameters: (i) normalized width of ambiguity region Δ; and (ii) SNR observed at individual CR sensors η. For a specified η, K takes large values if Δ is large and vice-versa. Similarly, for a specified Δ, K takes smaller values if η is large and vice-versa. Thus, to derive probability mass function (PMF) of K we take exploit Monte Carlo simulations to plot PMF for different values of Δ and η. Fig. 3 and 4 shows plot of simulated PMF of K (considering N = 10) for different normalized ambiguity region widths and different SNR values. It is easy to notice that the PMF plot is very close to a Gaussian distribution. Thus, we next fit Gaussian distribution function on the sample PMF values with expressions of mean and variance as function of SNR η and ambiguity region width Δ.

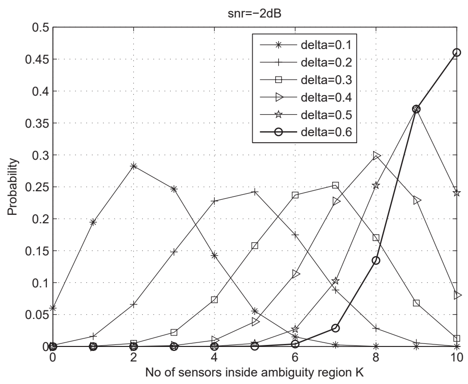

Fig. 3. PMF of K for $N = 10$, $\eta = -0.2dB$ and $\Delta = 0.1, 0.2, \cdots 0.6$

Here we use statistical tool box of Matlab to first fit Gaussian distribution on probability sample points of Fig. 3 and 4. The fitting parameters, mean and standard deviation obtained from the toolbox are tabulated in Table I and II.

Next we fit polynomial equations with η and Δ as argument on sample values of mean μ_b in Table II and variance σ_a^2, $\frac{p}{\sqrt{2\pi\sigma_a^2}}$ from Table I. Let μ and η be related by quadratic equation,

$$\mu = a\eta^2 + b\eta + c. \tag{19}$$

Using polynomial fit on six sample mean values of μ_b for $\Delta = (0.1, 0.2, \cdots, 0.6)$, six set of coefficient a, b and c values can be identified. Next we fit another polynomial with Δ as

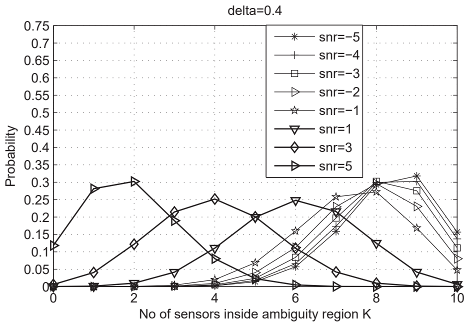

Fig. 4. Probability mass function of K for and $\eta = -5, -4, \cdots 3, 4$ and $\Delta = 0.4$

Δ	Mean μ_a	$\frac{p}{\sqrt{2\pi\sigma_a^2}}$	Std. dev. σ_a
0.1	2.3146	0.2869	1.4112
0.2	4.6389	0.2488	1.6108
0.3	6.5241	0.2594	1.5482
0.4	7.9315	0.2976	1.3660
0.5	8.9534	0.3620	1.1963
0.6	9.7637	0.4677	1.1312

TABLE I
MEAN AND STANDARD DEVIATION PARAMETERS OF GAUSSIAN FITTING AT SNR $\eta = -2dB$

η dB	Mean μ_b	$\frac{p}{\sqrt{2\pi\sigma_b^2}}$	std. dev. σ_b
-5	8.5251	0.3267	1.2761
-4	8.4032	0.3213	1.2859
-3	8.2277	0.3110	1.3187
-2	7.9354	0.2976	1.3659
-1	7.5079	0.2827	1.4261

TABLE II
MEAN AND STANDARD DEVIATION PARAMETERS OF GAUSSIAN FITTING AT $\Delta = 0.4$

arguments on six set of coefficient a, b and c values. The best polynomial fit on coefficients is identified as, $a = 0.97\Delta^3 - 0.91\Delta^2 + 0.17\Delta - 0.054$, $b = 2.9\Delta^2 - 2.5\Delta - 0.16$, $c = -12\Delta^2 + 22\Delta - 0.33$.

Similarly, let σ and Δ be related by cubic equation,

$$\sigma^2 = d\Delta^3 + e\Delta^2 + f\Delta + g \qquad (20)$$

Once again, using polynomial fit on five sample mean values of σ_a for $\eta = (-5, \cdots, -1)dB$, five set of coefficient d, e, f and g values can be identified. A best polynomial fitting with η as arguments is identified as, $d = -0.083\eta^3 - 1.1\eta^2 - 6.8\eta + 54$, $e = 0.21\eta^2 + 2.3\eta - 72$; $f = 0.21\eta^2 + 1.8\eta + 26$; $g = -0.023\eta^2 - 0.28\eta - 0.19$.

Finally, let $\frac{p}{\sqrt{2\pi\sigma^2}}$ and Δ be related by cubic equation,

$$\frac{p}{\sqrt{2\pi\sigma^2}} = h\Delta^3 + i\Delta^2 + j\Delta + l \qquad (21)$$

Using polynomial fit on five sample mean values of $\frac{p}{\sqrt{2\pi\sigma_a^2}}$ for $\eta = (5, \cdots, 1)dB$, five set of coefficient h, i, j and l values can be identified. The best polynomial fitting with η as arguments is identified as, $h = 0.26\eta^2 - 0.27\eta - 1.9$; $i = -0.22\eta^2 + 0.11\eta + 3.2$; $j = 0.045\eta^2 - 0.11\eta - 1.4$; $l = 0.032\eta + 0.42$.

The continuous probability density function (PDF) of K can be finally expressed as:

$$f_K(k) = \frac{p}{\sqrt{2\pi\sigma^2}} e^{-(k-\mu)^2/(2\sigma^2)} \qquad (22)$$

where, expression of μ, σ, and $\frac{p}{\sqrt{2\pi\sigma^2}}$ in terms of η and Δ can be obtained from equation (19),(20), and (21), respectively. In Fig. 5 we verify accuracy of our statistical modeling in which we can easily observe tight closeness between simulated PMF and PDF obtained form equation (6) for $\Delta = 0.1, 0.3, 0.6$ and $\eta = -2$dB.

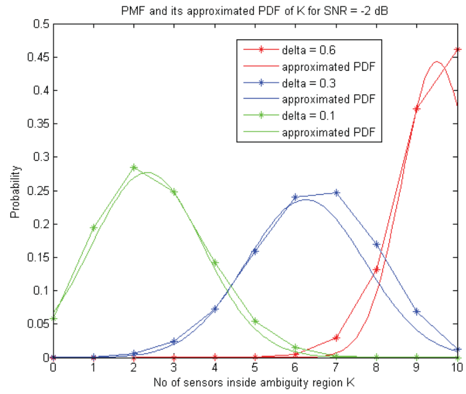

Fig. 5. Simulate PMF and approximated PDF of K for various Δ and η values

In order to compute expected value of n^*_{opt}, the PDF of $N-K$ can be expressed as,

$$\begin{aligned} f_{N-K}(k) &= \frac{\partial F_{N-K}(k)}{\partial k} = \frac{\partial P(N-K<x)}{\partial k} \\ &= \frac{\partial(1-P(K<N-k))}{\partial k} = \frac{\partial(1-F_K(N-k))}{\partial k} \\ &= f_K(N-k) = \frac{p}{\sqrt{2\pi\sigma^2}} e^{-\frac{(N-k-\mu)^2}{2\sigma^2}} \end{aligned} \qquad (23)$$

where, $F_K(k)$ is the cumulative distribution function of K. Finally, exploiting equation (18) expected value of optimal threshold n^*_{opt} can be expressed as,

$$E[n^*_{opt}] = E\left[\left\lceil \frac{N-K}{\psi+1} \right\rceil\right] = \left\lceil \frac{N-\mu}{\psi+1} \right\rceil. \qquad (24)$$

VII. SIMULATION AND RESULT

In order to illustrate efficacy of our analysis, we simulate a cooperative CR network with $N = 10$ number of CR sensors. The sensing channels between PU and CR sensors are assumed to be i.i.d. with Rayleigh distributed flat channel coefficients. Further, reporting channels are also assumed to be ideal i.e., loss less communication. The PU signal is assumed to be BPSK modulated. The conventional single threshold λ is obtained from the intersection point of PDFs of test statistics under hypothesis H_0 and H_1. At the fusion center, final decision on presence or absence of PU is done by combining local hard decisions of CR sensors using Majority rule as specified in equation (2).

We validate the analysis of section VI where we have derived the expression of optimal threshold n^*_{opt} for majority

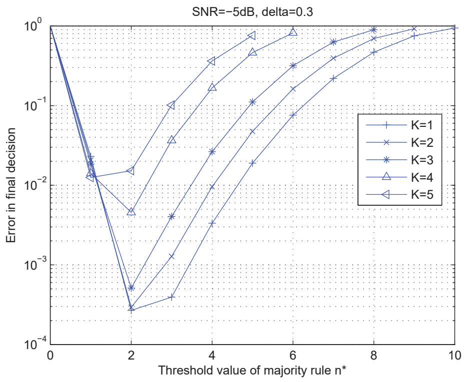

Fig. 6. Error in final decision versus n^*_{opt} for different values of K

decision. Fig. 6 shows the simulated plot of error in final decision E_{fd} versus optimal value of threshold n^*_{opt} under various values of K, number of CR sensors whose test statistics fall inside the ambiguity region. It is easy to observe that for $K = 2$, the error in final decision E_{fd} reaches its minimum value when $n^*_{opt} = 2$. The theoretical value of n^*_{opt} from equation (18) with computed values of P_{fa}, $P_{cd,ab}$, $P_{cd,pr}$ and P_{md} from section III for $N = 10$, $K = 5$, $\eta = -5dB$ and $\Delta = 0.3$,

$$n_{opt}(theoretical) = \left[\frac{10-2}{\frac{ln\frac{0.0017}{0.4869}}{ln\frac{0.7606}{0.7311}}+1} \right] = 2$$

Thus, theoretical value of n^*_{opt} in equation (18) matches with optimal value of n^*_{opt} obtained from simulations.

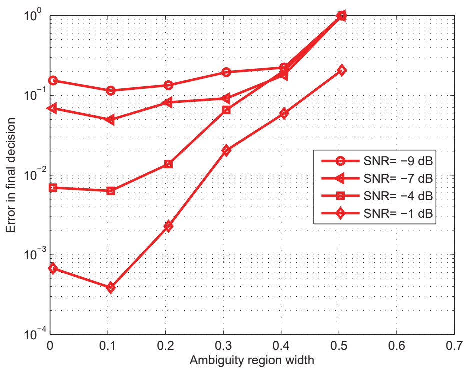

Fig. 7. Error in final decision versus Δ for different values of SNR η

Next, we simulate error in final decision for different widths of ambiguity region. It can be observed from Fig. 7 that for a specified SNR η, there exists a ambiguity region width Δ for which error in final decision is at minimum value. Further, we can observe that for $\Delta = 0.1$, the error in final decision is minimum for different values of SNR. Thus, we can compute expected optimal value of threshold $E[n^*_{opt}]$ for different values of SNR with $\Delta = 0.1$ using equation (24). For example, when $\eta = -4dB$, computed value $E[N - K]$ is 7 and $E[n^*_{opt}] = 2$.

VIII. Conclusion

In this paper, a double threshold based cooperative spectrum sensing scenario is investigated. The CR sensors make local hard decisions based on conventional energy detection technique and communicate one bit decision information to the fusion center. The sensors whose test statistics fall in ambiguity region do not report to the fusion center. At fusion center, a majority rule is applied in which integer threshold n^* is used to make the final decision on presence of PU. Since choice of n^* decides error in final decision, we formulate an expression to compute optimal value n^*, i.e., n^*_{opt} which minimizes the error in final decision. In order to address uncertainty due to sensors with test statistics in ambiguity region, a statistical model is proposed which is later exploit it to derive an expression for expected value of n^*_{opt}.

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