Effect of Heterogeneity on Amplitude Death Based Stability Solution of DC Microgrid

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***Abstract*—In real electrical systems, the parameters of the subsystems are inherently somewhat different. Synchronous behavior in DC micro-grids (MGs) can be achieved by means of the coupling of these subsystems under the condition of parameters mismatch. The method of detuning of one or more internal parameters in a statically coupled system can give rise to amplitude death (AD), a coupling induced stabilization of a dynamical system. This coupling technique has the advantage that this provides an open-loop stability solution for DC-DC converters in a DC MG in the presence of CPLs. The bifurcation analysis shows that the small detuning of parameters under strong coupling eliminates the standard oscillatory behavior from the large region of the parameter space, where the dominance of the AD can be observed. And this AD dominance produces a reliable dynamical control mechanism in the general case of coupled nonlinear oscillators. The goal of this work is to identify the dynamic states of coupled nonlinear oscillators and investigate how the heterogeneity of the system interacts with the coupling to produce coherent behavior.**

***Index Terms*—Constant power load (CPL), DC microgrid, amplitude death (AD), equilibrium point (EP).**

I. Introduction

When flexibility comes into the picture of AC as well as DC micro-grids (MGs), one can say that DC MGs are better choices than that of the AC grids. This is because they are more suitable for energy storage and renewable sources as almost all of the loads nowadays are inherently DC [1], [3], [4]. This increased reliability and flexibility of DC MGs have also helped engineers to select them in major applications such as telecommunication industries [5], low-power consumer electronics, vehicular technologies [6], industrial power systems [7], naval ships [8], residential homes [9], commercial buildings [10], and so on. Though DC MGs are more stable than AC MGs [11]; but there are some serious stability issues because of the interfacing power electronics for achieving different levels of voltages during the integration of sources, loads, and energy storage devices [2], [12]. In cascaded architecture, point-of-load (POL) converters with a resistive load ideally behaves as an instantaneous CPL [13]. The negative incremental resistance caused by CPL essentially brings the nonlinearity to the systems and results in a limit cycle oscillations. Moreover, this leads to the undesired oscillations [14] in the systems and thus the state variables can't converge to the desired equilibrium point (EP) or fixed point in terms of discrete time domain.

Therefore, it is well accepted that the destabilizing problem of DC power-grid networks needs to be solved for future practical use. Several strategies for enhancing the stability of an operating point of such grid have been demonstrated; mainly using passive damping [15], application of a bi-directional DC-DC converter, use of a virtual capacitor [16], feedback controller [13], and control for multiple power sources and loads [17]. These investigations have been tackled the destabilization problem from the power electronics viewpoint, and mostly follows the small signal linear stability analysis [18]. However, the stabilizing problem of such nonlinear systems cannot be analyzed by linear system dynamics. It is, therefore, necessary to apply the concept of nonlinear analysis. AD is a mathematical phenomenon of a nonlinear system by which the desired EP is stabilized due to the coupling [19]. There are mainly two reasons that can cause AD such as strong coupling and sufficiently different natural frequencies of interaction between the networks [20]. Recently, the model of the coupled systems are analyzed by Huddy and Skufca [21] using the concepts of nonlinear dynamics and synchronization of two interconnected converter topologies where the topology allows application of AD solution to this problem in a pair of DC bus systems.

Investigation reveals that the operations of power electronics converters can be characterized by cycle switching of the circuit topologies which give rise to a variety of chaotic phenomena. In such chaotic systems, one important feature is that even the fully identical oscillators can't generate synchronous waveforms. The reason behind this is their extreme sensitiveness to the change in initial conditions. In numerical simulations, this can be avoided by setting same initial conditions for each system; which results in the same chaotic waveforms at the output. Meanwhile, in the real electrical systems, this is not possible. However, in practice the parameters are inherently somewhat different; so the study of synchronous behavior is necessary by coupling the oscillators under parameter mismatch. In this paper, we will discuss in detail analysis about the effect of heterogeneity on AD based stability solving for constant power loaded converters system named DC MG. Using bifurcation analysis, we have

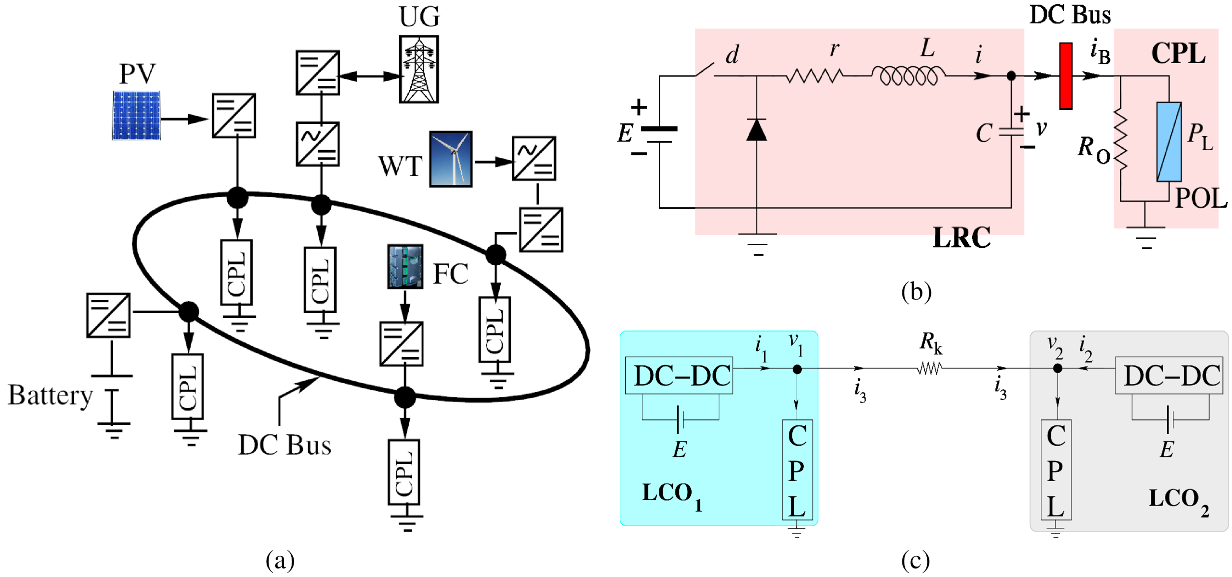


Fig. 1. (a) Conceptual diagram of a DC Bus system (PV: photovoltaic cell, UG: utility grid, WT: wind turbine, FC: fuel cells). (b) A simplified buck-based distributed power architecture. (c) Block diagram of coupled LRC systems for static coupling with a simple resistor R_{k} as a coupling link.

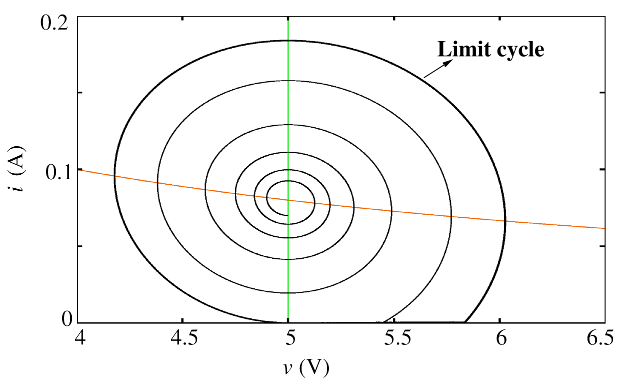


Fig. 2. Stable limit cycle behavior of the destabilised system due to CPL. Numerical results for the ideal buck LRC, with the parameters $P_{\mathrm{L}} = 0.4$ W, $E = 10$ V, $L = 4$ mH, $C = 50$ μF, $q = 0.5$, $f_{\mathrm{s}} = 20$ KHz.

investigated the behavior of coupled oscillators system under small parametric mismatch. It is shown that this mismatch can eliminate standard oscillatory solutions bringing the dominance of AD in a large region of parameter space.

II. DC MICROGRID AND CPL

As the output of photovoltaic and fuel cells are DC in nature, it is, therefore, easier and more efficient to connect them directly to a DC distribution system or, through a controlled DC/DC converter. For example, in traction power systems DC series motors are employed because of its high starting torque and better voltage regulation characteristics. Another example of DC distribution are the data centers, where the sensitive loads are connected for uninterrupted power supplies even if the main power sources are lost. Similarly, in the variety of power system applications based on advanced power-electronics technique such as international space station, spacecraft, electric and hybrid electric cars, telecommunications, terrestrial computer systems, and medical electronics etc. the supplies are mostly of the DC types. The multi-converter power electronics systems, also known as distributed power systems have various configurations like cascading, parallel, stacking, load/source splitting etc. based on different operational objectives and designs.

Therefore, considering a multi-converter DC bus system shown in Fig. 1(a), one could include many LRCs that regulate the main bus voltage, such as the one located between the main bus and the local microgrid source converters, which are loaded by other converters. The loads in such systems are the combination of tightly controlled POL converter and a fixed output resistor R_{o} [22]. Since efficiencies of POL converters are usually very high and this pair can be characterized as an instantaneous CPL with negative incremental resistance property. Due to this negative impedance instability, there is a decrease in the voltage stability margin which can cause significant oscillations. DC MGs consist of many sources and loads leads to the entire system to more complex, non-linear and coupled. However, DC microgrid stability analysis is constantly improving by various techniques like load shedding, direct connection of energy storage to the main bus, filtering, and control approaches [13]. Most of these studies are mainly relied on CPLs based small signal analysis [23], and their conclusions are that in constant power loaded dc systems, the EP of an LRC is unstable. Some other previous studies have large signal analysis [24] method to study the CPL effects on system stability. An instantaneous CPL can be represented by

$$i = \frac{P_{\mathrm{L}}}{v} \qquad \forall \quad v \geq \epsilon \tag{1}$$

where i is the input current, P_{L} is the CPL power, v is the input voltage of the main bus feeding the CPL, ϵ is an

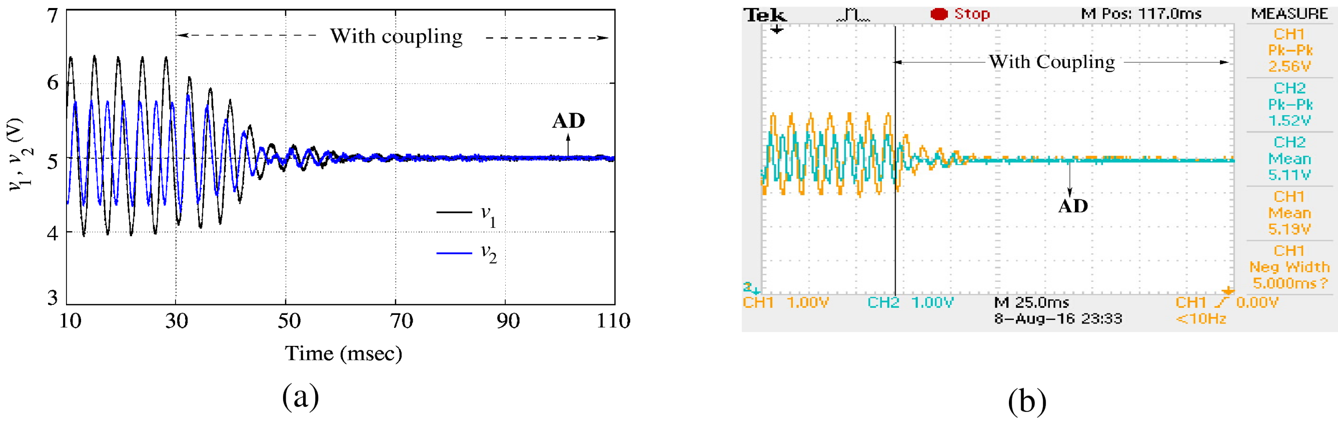


Fig. 3. Heterogeneous systems showing sustained oscillations before coupling and AD after coupling using parameters $P_{\rm L} = 0.4$ KW, $E = 10$ V, $L_1 = 8$ mH, $L_2 = \alpha L_1, \alpha = 0.5, C_1 = C_2 = 50\ \mu$F, $q = 0.5, f_{\rm s} = 20$ KHz, $R_{\rm k} = 35\ \Omega$. (a) Simulation results. (b) Oscilloscope traces. Y- channel: 1 division = 1 V.

arbitrarily small positive value. The switch model dynamics for the ideal buck LRC case $(r = 0, R_{\rm o} = \infty)$ with a CPL (see Fig.Fig. 1(b)) can be written as

$$\frac{di}{dt} = \frac{qE}{L} - \frac{v}{L}; \quad \frac{dv}{dt} = \frac{i}{C} - \frac{P_{\rm L}}{Cv}; \quad i \geq 0, \quad v \geq \epsilon \tag{2}$$

where i and v are the inductor current and capacitor voltage respectively. The switching function which controls the MOSFET is given by q and its fast average is given by the instantaneous duty cycle d.

During the transient, it's possible that the trajectories of the system can cross the boundary $(i = 0)$, but the converter topology only allows unidirectional current through the inductor $(i \geq 0)$. Therefore it is important to include the discontinuous conduction mode (DCM) operation of the converters [25] for low currents. Hence, the average state equations no longer describe the mathematical modeling of the converters and their behavior. A full order model of the converter is represented by

$$\frac{di}{dt} = \frac{qE}{L} - \frac{2ivf_{\rm s}}{q(E-v)}; \quad \frac{dv}{dt} = \frac{i}{C} - \frac{P_{\rm L}}{Cv}; \quad i < 0 \tag{3}$$

where $f_{\rm s}$ is the switching frequency of the converter. It is very difficult to calculate the eigenvalues of the system (Eqs. (2) and (3)), as these stability criteria involve different conduction modes of the converters for different conditions. Numerical computation method using XPPAUT [26] is thus used to determine the stability and/or eigenvalues of the system. It is observed that the EP [0] is unstable because of the eigenvalues have positive real parts [1] as shown in Fig. 2. The limit cycle marked in Fig. 2 shows the sustained oscillations in voltage as well as the current of the converter.

III. AD Caused By Heterogeneity

It is observed that stability of the EP remains unchanged when two identical systems are statically coupled [21], e.g. with a linear resistor as coupling link. However, modifying one or some internal parameters in each system help us to achieve AD for a certain range of coupling link resistance. For our model, we vary the inductance parameter because the EP of the system does not depend on inductance and changing the inductance value changes the frequency of oscillations. Then, for the sufficiently different frequencies and a strong enough coupling, oscillators will pull each other off its limit cycle and as an effect, both will settle to the EP. So the heterogeneity brings stability and all the oscillators come to the stationary. For the two coupled systems let $L_2 = \alpha L_1$, where L_1 and L_2 are inductance of the two limit cycle oscillators ($\mathrm{LCO_1}$ and $\mathrm{LCO_2}$) and α is the parameter which describes heterogeneity of the two systems. Considering the coupling of two systems as shown in Fig. 1(c), current through the coupling link $i_{\rm k}$ is given by

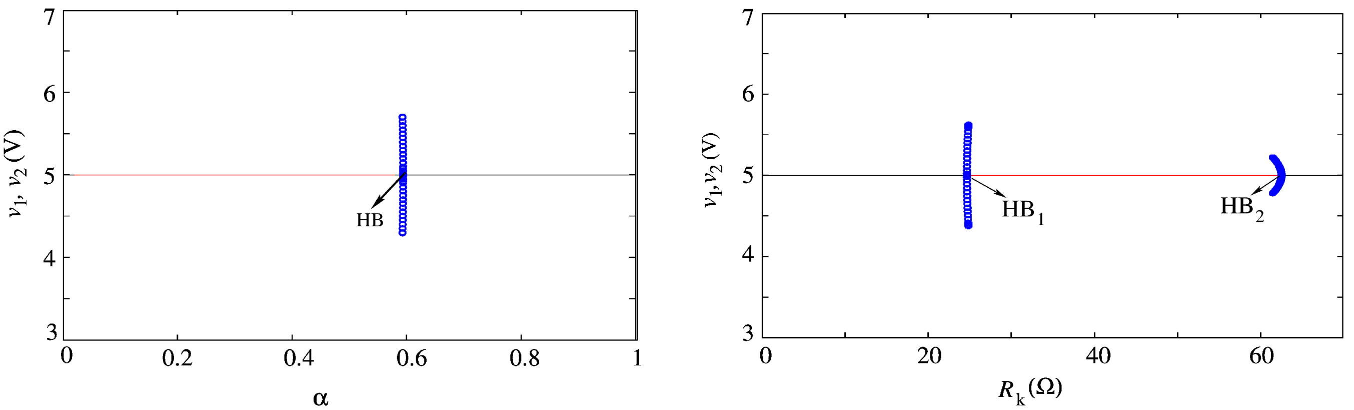
$$i_{\rm k} = \frac{v_1 - v_2}{R_{\rm k}} \tag{4}$$

where v_1 and v_2 are the capacitor voltages of two LRCs. The dynamics of the two diffusively coupled systems models are given by

$$\begin{aligned} \frac{dv_1}{dt} &= \frac{i_1}{C_1} - \frac{P_1}{C_1 v_1} - \frac{i_{\rm k}}{C_1} \\ \frac{di_1}{dt} &= \begin{cases} \dfrac{qE - v_1}{L_1} & ;\text{if } i_1 \geq 0 \\ \dfrac{qE}{L_1} - \dfrac{2i_1 v_1 f_{\rm s}}{q(E - v_1)} & ;\text{if } i_1 < 0 \end{cases} \end{aligned} \tag{5}$$

$$\begin{aligned} \frac{dv_2}{dt} &= \frac{i_2}{C_2} - \frac{P_2}{C_2 v_2} + \frac{i_{\rm k}}{C_2} \\ \frac{di_2}{dt} &= \begin{cases} \dfrac{qE - v_2}{L_2} & ;\text{if } i_2 \geq 0 \\ \dfrac{qE}{L_2} - \dfrac{2i_2 v_2 f_{\rm s}}{q(E - v_2)} & ;\text{if } i_2 < 0 \end{cases} \end{aligned} \tag{6}$$

[0]Given by the intersection of x-nullcline (orange) and y-nullcline (green)
[1]$\lambda_1 = 159.84 + j2230.35, \ \lambda_2 = 159.84 - j2230.35$



(a) (b)

Fig. 4. One parameter bifurcation diagram. (a) $\alpha \in (0, 1)$ is the bifurcation parameter and $R_\mathrm{k} = 40\ \Omega$. (b) $R_\mathrm{k} \in (0, 70\ \Omega)$ is the bifurcation parameter and $\alpha = 0.5$.

where $P_1 = P_2 = P_\mathrm{L}$ is the power of CPL used in both systems. i_1 and i_2 are inductor currents of two LRCs. C_1 and C_2 are the capacitances of two LRCs. The heterogeneous coupling brings AD as shown in Fig. 3(a). The same phenomenon is being validated experimentally as given in Fig. 3(b). It has been observed that heterogeneity stabilizes the statically coupled systems model which can be concluded from the eigen values computed by the numerical method ($\lambda_1 = -76.154 + j2102.8,\ \lambda_2 = -76.154 - j2102.8,\ \lambda_3 = -104.17 + j1677.01,\ \lambda_4 = -104.17 - j1677.01$).

IV. Bifurcation Analysis

As it is difficult to derive the range of parameters for the AD region, numerical analysis helps us to estimate this range. There are some useful tools for exploring how a dynamical system changes with respect to the variation of parameters. The most widely observed route to the AD is through Hopf bifurcation (HB), where coupling induces stability of the EP of the uncoupled systems. Continuation bifurcation is a straightforward technique to analyze the bifurcation in which a particular solution (such as EP or limit cycle) is followed as the parameter changes. The detection of EPs and limit cycles can be obtained automatically from the numerical tools provided by AUTO.

In Fig. 4(a), the plot gives the output voltages for the coupled systems as a function of the heterogeneous parameter α keeping the coupling resistance R_k constant. The stability analysis shows that AD occurs for $\alpha < 0.59$. The red line represents stable EPs and the black thin line represents the unstable EPs. A subcritical HB occurs at $\alpha = 0.59$ giving unstable periodic orbits implied by empty blue circles. This HB point separates the stable and unstable regions. Similarly in Fig. 4(b), the plot gives the output voltages as a function of R_k keeping α constant. The stability region is in the range $24.6\ \Omega < R_\mathrm{k} < 62.5\ \Omega$. Here two subcritical Hopf bifurcations exist denoted by HB_1 (high-frequency oscillations) and HB_2 (low-frequency oscillations) at $R_\mathrm{k} = 24.6\ \Omega$ and $R_\mathrm{k} = 62.5\ \Omega$ respectively. Stability region exists between these two points

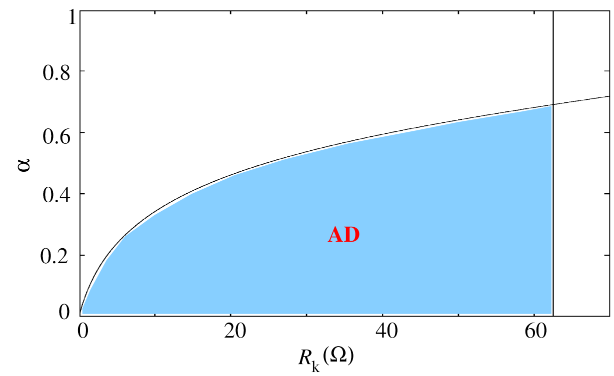


Fig. 5. Two parameters bifurcation diagram for $R_\mathrm{k} \in (0, 70\ \Omega)$ and $\alpha \in (0, 1)$ as the bifurcation parameters.

HB_1 and HB_2. The two parameters bifurcation diagram is given in Fig. 5 where AD island is given by the shaded area of α vs R_k plot.

V. CONCLUSION

The stability issues of DC MGs can be overcome by various feedback control methods. However, AD solution method has its uniqueness as it is free from any external controller circuits, hence, the implementation cost and complexity are reduced. Moreover, the coupling of inhomogeneous systems deals with the stability of DC MG when there is a mismatch of parameters in DC-DC converters systems with CPL. This heterogeneity scheme can also be a matter of interest for identical systems which have different initial conditions. The phenomena of the AD has been achieved in the coupled systems by numerically as well as experimentally. In this work, we study the change in bifurcation scenario due to mismatch of parameters. However, this work can be extended to the study of synchronization among multiple oscillators in DC MG networks.

References

[1] T. Dragicevic, J. C. Vasquez, J. M. Guerrero, and D. Skrlec, “Advanced lvdc electrical power architectures and microgrids: A step toward a new

generation of power distribution networks," *IEEE Electrific. Mag.*, vol. 2, no. 1, pp. 54-65, Mar. 2014.

[2] S. Singh, A. R. Gautam, and D. Fulwani, "Constant power loads and their effects in dc distributed power systems: A review," *Renewable and Sustainable Energy Reviews*, vol. 72, pp. 407-421, 2017.

[3] K. Chauhan and R. K. Chauhan, "Optimization of grid energy using demand and source side management for dc microgrid," *Journal of Renewable and Sustainable Energy*, vol. 9, pp. 035101-15, May 2017.

[4] R. K. Chauhan, C. Phurailatpam, B. S. Rajpurohit, F. M. Gonzalez-Longatt, and S. N. Singh, "Demand-side management system for autonomous dc microgrid for building," *Technology and Economics of Smart Grids and Sustainable Energy*, Springer, vol. 2, no. 4, pp. 1-11, Feb. 2017.

[5] A. Kwasinski and P. T. Krein, "Optimal configuration analysis of a microgrid-based telecom power system," in *Int. Telecommun. Energy Conf.*, Sep. 2006, pp. 1-8.

[6] C. Liu, K. T. Chau, C. Diao, J. Zhong, X. Zhang, S. Gao, and D. Wu, "A new dc micro-grid system using renewable energy and electric vehicles for smart energy delivery," in *Vehicle Power and Propulsion Conf. (VPPC)*, Sep. 2010, pp. 1-6.

[7] M. E. Baran and N. R. Mahajan, "Dc distribution for industrial systems: opportunities and challenges," *IEEE Trans. Ind. Appl.*, vol. 39, no. 6, pp. 1596-1601, Nov. 2003.

[8] A. Monti, D. Boroyevich, D. Cartes, R. Dougal, H. Ginn, G. Monnat, S. Pekarek, F. Ponci, E. Santi, S. Sudhoff, N. Schulz, W. Shutt, and F. Wang, "Ship power system control: a technology assessment," in *Electric Ship Technologies Symp.*, Jul. 2005, pp. 292-297.

[9] H. Kakigano, Y. Miura, T. Ise, T. Momose, and H. Hayakawa, "Fundamental characteristics of dc microgrid for residential houses with cogeneration system in each house," in *Power and Energy Soc. General Meet.-Convers. and Del. of Electr. Energy in the 21st Century*, Jul. 2008, pp. 1-8.

[10] D. Salomonsson and A. Sannino, "Low-voltage dc distribution system for commercial power systems with sensitive electronic loads," *IEEE Trans. Power Del.*, vol. 22, no. 3, pp. 1620-1627, Jul. 2007.

[11] R. S. Balog and P. T. Krein, "Bus selection in multibus dc microgrids," *IEEE Trans. Power Electron.*, vol. 26, no. 3, pp. 860-867, Mar. 2011.

[12] R. K. Chauhan, B. S. Rajpurohit, R. E. Hebner, S. N. Singh and F. M. G. Longatt, "Design and analysis of PID and fuzzy-PID controller for voltage control of DC microgrid," in *Proc. IEEE PES Innovative Smart Grid Technologies (ISGT) Asian Conf.*, Bangkok, pp.1-6, Nov. 04-06, 2015.

[13] A. Kwasinski and C. N. Onwuchekwa, "Dynamic behavior and stabilization of dc microgrids with instantaneous constant-power loads," *IEEE Trans. Power Electron.*, vol. 26, no. 3, pp. 822-834, Mar. 2011.

[14] A. Emadi, A. Khaligh, C. H. Rivetta, and G. A. Williamson, "Constant power loads and negative impedance instability in automotive systems: definition, modeling, stability, and control of power electronic converters and motor drives," *IEEE Trans. Veh. Technol.*, vol. 55, no. 4, pp. 1112-1125, Jul. 2006.

[15] M. Cespedes, L. Xing, and J. Sun, "Constant-power load system stabilization by passive damping," *IEEE Trans. Power Electron.*, vol. 26, no. 7, pp. 1832-1836, Jul. 2011.

[16] P. Magne, D. Marx, B. N. Mobarakeh, and S. Pierfederici, "Large-signal stabilization of a dc-link supplying a constant power load using a virtual capacitor: impact on the domain of attraction," *IEEE Trans. Ind. App.*, vol. 48, no. 3, pp. 878-887, May 2012.

[17] A. A. A. Radwan and Y. A. R. I. Mohamed, "Linear active stabilization of converter-dominated dc microgrids," *IEEE Trans. Smart Grid*, vol. 3, no. 1, pp. 203-216, Mar. 2012.

[18] C. H. Rivetta, A. Emadi, G. A. Williamson, R. Jayabalan, and B. Fahimi, "Analysis and control of a buck dc-dc converter operating with constant power load in sea and undersea vehicles," *IEEE Trans. Ind. Appl.*, vol. 42, no. 2, pp. 559-572, Mar. 2006.

[19] G. B. Ermentrout, "Oscillator death in populations of all to all coupled nonlinear oscillators," *Physica D: Nonlinear Phenomena*, vol. 41, no. 2, pp. 219-231, 1990.

[20] S. H. Strogatz, *Nonlinear dynamics and chaos: with applications to physics, biology, chemistry, and engineering*. Cambridge, MA: Westview press, 2000.

[21] S. R. Huddy and J. D. Skufca, "Amplitude death solutions for stabilization of dc microgrids with instantaneous constant-power loads," *IEEE Trans. Power Electron.*, vol. 28, no. 1, pp. 247-253, Jan. 2013.

[22] A. Kwasinski and P. T. Krein, "A microgrid-based telecom power system using modular multiple-input dc-dc converters," in *IEEE Int. Telecommun. Conf.*, Sept 2005, pp. 515-520.

[23] W. Du, J. Zhang, Y. Zhang, and Z. Qian, "Stability criterion for cascaded system with constant power load," *IEEE Trans. Power Electron.*, vol. 28, no. 4, pp. 1843-1851, Apr. 2013.

[24] D. Marx, P. Magne, B. Nahid-Mobarakeh, S. Pierfederici, and B. Davat, "Large signal stability analysis tools in dc power systems with constant power loads and variable power loads review," *IEEE Trans. Power Electron.*, vol. 27, no. 4, pp. 1773-1787, Apr. 2012.

[25] J. Sun, D. M. Mitchell, M. F. Greuel, P. T. Krein, and R. M. Bass, "Averaged modeling of pwm converters operating in discontinuous conduction mode," *IEEE Trans. Power Electron.*, vol. 16, no. 4, pp. 482-492, Jul. 2001.

[26] B. Ermentrout, *Simulating, Analyzing, and Animating Dynamical Systems. Society for Industrial and Applied Mathematics*, 2002. [Online]. Available: http://epubs.siam.org/doi/abs/10.1137/1.9780898718195.