Compressive Sensing Based Energy Efficient Wideband Cognitive Radio Network

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***Abstract*—In this paper, we present a cooperative spectrum sensing technique to identify the spectral location of primary users (PUs) in a wideband spectrum. Here we first divide the wideband into narrow bands of equal bandwidth and deploy arbitrarily located cognitive radio (CR) sensors to sense the narrowbands. The CR sensors operate on conventional energy detection principle and we assume that their spectrum sensing range overlap over each other so as to exploit diversity and mitigate deep fade problem. In order to have energy efficiency, we introduce probabilistic active and sleep state for individual CR sensors. CR sensors in active state compute energy in the specified sensing range and communicate it to a fusion center. Assuming sparse occupancy of PUs in the wideband and by Parseval's theorem, we represent energies of sub-bands in form of sparse vector. Next, we exploit concept of compressive sensing (CS) at fusion center to reconstruct the vector representing energies in the sub-bands. Since individual CR sensors randomly take active or sleep state in a time epoch, the sensing matrix for reconstruction is identified as random matrix. Extending the analysis, we also investigate the value of probability of active/sleep state for which sensing matrix satisfies Restricted Isometry Property (RIP). Finally, we compare the reconstructed energies of sub-band with a specified threshold to make decision on presence or absence of PU in the particular sub-bands. We validate our approach via simulations in which we show performance with (i) variation in probability of sleep state; (ii) variation in number of time epochs or measurements; (iii) varying degree of overlap in spectrum sensing range.**

***Index Terms*—Cognitive radio, wideband spectrum sensing, compressive sensing, energy efficiency.**

I. Introduction

The recent trends in wireless communication technology is witnessing an exponential increase in high data rate applications. However, scarcity in usable spectrum is putting limits on increasing demand of data rate and capacity. Cognitive Radio (CR) is an upcoming technology to overcome the drawbacks of static spectrum allocation policy by opportunistically exploiting the underutilized spectrum. The technology is built on concepts of an intelligent wireless system which on the basis of operating environment conditions provides reliable communications to both licensed (primary user - PU) and unlicensed (secondary user - SU) spectrum users [1]. Specifically, the SUs in CR networks seek transmission opportunities over a given band of spectrum that is temporarily unoccupied by PUs. For the success of CR technology, the foremost challenge is rapid and accurate identification of unused spectrum holes over a wide spectrum band. An efficient sensing of wide-band spectrum will not only increase throughput and capacity of SUs but also enables possibility of multi-secondary user communication [2].

Though wideband spectrum sensing is a promising technique for CR networks, it faces substantial challenge in Analog to Digital Converter (ADC) hardware design over GHz band. This is due to requirement of very high sampling rate for conventional spectral estimation methods which specifically operate at or above Nyquist criterion [2]. One simple strategy to sense entire wideband spectrum is to divide it into large number of small narrow bands and then perform sequential narrow band sensing. However, such a strategy is highly inefficient as it will have large sensing time resulting in small transmission time in dynamically changing spectrum environment [3]. A similar approach to detect the spectrum holes is to once again divide the complete wideband into non-overlapping narrow-bands and at the receiver side employ a bank of narrow band filter each of which is tuned to a particular sub-band [4]. This method is highly complex as it needs numerous RF components and introduces large latency in detection process. Many other approaches like multiband joint detection [5], wavelet detection [6], [7] have been proposed to improve the accuracy and reduce the hardware complexity; but all these methods still require sampling the wideband at or above Nyquist rate.

An alternative to large number of filter banks and high rate ADCs is to exploit specific feature of spectrum, such as sparsity. The PU occupancy in wideband spectrum is defined as sparse if the percentage of occupancy is very small, which is typical in open spectrum networks like Television white space (TVWS) [8]. In this context, several researchers have exploited concept of Compressive Sensing (CS) for CR networks considering sparsity in different domains. The basis concept of CS is to reconstruct sparse signals from far fewer measurements or samples than required by traditional Nyquist sampling criteria [9]. In order to apply CS, the key requirements are: (i) sparse representation of desired signal in some transform domain; and (ii) acquiring incoherent measurements of the signal to gather maximum information in fewer measurements [10].

Exploiting concept of CS for spectrum hole detection, [11] proposed a two-step compressed spectrum sensing scheme for efficient wideband sensing. Taking a slightly different approach, Bayesian compressive sensing framework is proposed in [12] to reduce sampling requirement and computational complexity by bypassing signal reconstruction. In [13], multiple CR sensors collaborate to exploit spatial diversity against wireless fading and establish consensus among local spectral estimates by running a decentralized consensus optimization algorithm. Similarly, compressive sampling is performed at local CRs in [6] to sense wideband spectrum in which measurements from local CR detectors are fused to improves the detection performance, specially in deep fading channels. Considering improvement in detection performance, authors in [14] have proposed optimization of threshold at fusion center on the basis of threshold exploited at local sensors. Similarly, authors in [15] have exploited multiple antennas to improve the detection performance in cooperative spectrum sensing network. It can be noted from above work that CS is a strong contender for success of wideband sensing in CR networks and we believe a more diverse and goal oriented approach like energy efficiency is still plausible. More specifically, minimizing the energy required by local sensors and mitigating the deep fade problem are still under explored concepts.

In this work, we exploit concept of CS to identify spectral location of primary users (PUs) in a given wideband spectrum. The wideband is first divided into narrow bands of equal bandwidth, and arbitrarily located CR sensors are deployed to sense the narrowbands. Here we assume that spectrum sensing range of individual CR overlap over each other so as to exploit diversity and mitigate deep fade problem. We also introduce probabilistic active and sleep state for individual CR sensors to have energy efficiency in spectrum sensing. The CR sensors in active state compute energy in the specified sensing range and communicate it to fusion center. Assuming sparse occupancy of PUs in the wideband and by Parseval's theorem, we represent energies of sub-bands in form of sparse vector. Next, we formulate a $l1$ minimization problem at fusion center to reconstruct the vector representing energies in sub-bands. Since individual CR randomly take active or sleep state in a time epoch, the sensing matrix for reconstruction is identified as random matrix. In order to improve energy efficiency, we investigate larger values of probability of sleep state for which sensing matrix satisfies Restricted Isometry Property (RIP).

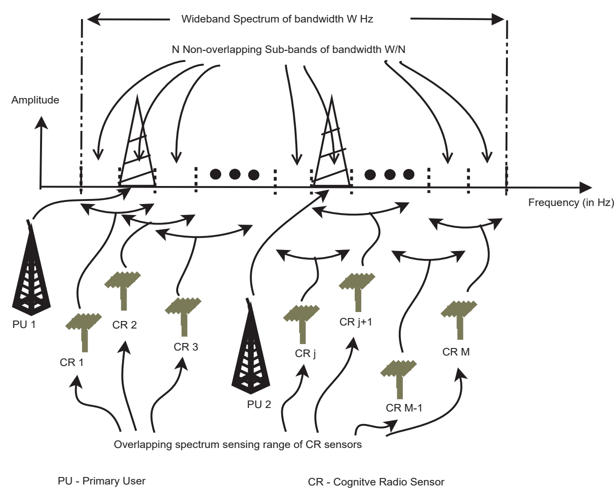

Fig. 1. Sparsely occupied wideband spectrum

II. System Model

Consider a wideband spectrum of bandwidth WHz which can divided into **N** non-overlapping narrow sub-bands of equal bandwidth, represented by $W_{sub} = \left(\frac{W}{N}\right)$Hz, as shown in Figure 1. For analytical simplicity, we assume primary users (PUs) occupy spectrum in sub-bands, i.e., PUs are located in specific sub-bands only. Further, we assume sparse occupancy of spectrum by PUs; thus, if l is the number of sub-bands occupied by primary users, then by sparsity $l << N$. Our objective in such a scenario is to identify PU occupied sub-bands which can be allocated to secondary users (SUs) on demand.

In order to sense the considered wide-band spectrum, we consider M cognitive radio (CR) sensors which are arbitrarily located in the space. Here we further assume that the sensing range of CR sensors overlap over each other. Overlap in spectrum sensing range helps in mitigating the deep fade problem. Thus, if a particular sensor is in deep fade, PU user occupancy can be detected by another CR sensor which has overlapped sensing range and is not in deep fade. In addition, overlap in spectrum sensing range reduces the quantity of CR sensors required to sense the wideband spectrum. The sub-bands sensed by the CR sensors can be represented by a $M \times N$ matrix A whose elements $a_{m,n}$ are given as,

$$a_{m,n} = \begin{cases} 1 & n^{th}\text{sub-band is sensed by } m^{th} \text{ CR sensor} \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

where, $m \in \{1, 2, \cdots M\}$ and $n \in \{1, 2, \cdots N\}$. The sensed signal at sensing time index i by m^{th} CR sensor can be expressed as,

$$x_m[i] = \begin{cases} n_m[i] & Hypothesis \quad H_0 \\ h_m[i]s_m[i] + n_m[i] & Hypothesis \quad H_1 \end{cases}, \tag{2}$$

in which, $s_m[i]$ denotes the sample of PU signal sensed by m^{th} CR sensor; $h_m[i]$ and $n_m[i]$ denotes fading coefficient sample and additive noise sample at m^{th} CR sensor, respectively. Here we assume flat fading with fading coefficient $h_m[i]$ to be Rayleigh distributed and additive noise $w_m[i]$ to be white Gaussian distributed with mean 0 and variance σ^2. The m^{th} CR sensor can sense the signal under binary hypothesis: H_0 - absence of PU and H_1 presence of PU signal.

Assuming conventional energy detector, if m^{th} CR sensor gathers L_e samples, then the computed energy can be expressed as,

$$g_m = \sum_{i=0}^{L_e - 1} |x_m[i]|^2. \tag{3}$$

Next, applying Parseval's theorem, the computed energy g_m is equal to the total energy present in sub-bands of of the m^{th} CR sensor sensing range. Thus, if $\bar{\mathbf{e}}_f$ is a $N \times 1$ vector, with n^{th} element represent energy in n^{th} sub-band then the energy sensed by m^{th} CR sensor is summation of energy in the corresponding spectrum sensing range. In vector form this can be represented,

$$g_m = a_m \cdot \bar{\mathbf{e}}_f \tag{4}$$

where a_m denotes m^{th} row of A matrix. The energies computed by all M sensors in form a vector $\bar{\mathbf{g}}$ can be expressed as,

$$\bar{\mathbf{g}} = A\bar{\mathbf{e}}_f \tag{5}$$

Here, we would like to remind our readers that due to assumption of sparse PU occupancy, vector $\bar{\mathbf{e}}_f$ is a sparse vector. However, depending upon the overlap in spectrum sensing range of individual CR sensors, $\bar{\mathbf{g}}$ may not be a sparse vector.

Next, we introduce two states for all M CR sensors: Active state in which CR sensor will sense the spectrum; and sleep state in which CR sensor will perform energy conservation and will not participate in sensing. We model probability of a particular sensor being in active or sleep state in an epoch by Bernoulli distribution. Here epoch is defined by decomposing the total sensing time in K equal parts. We further assume that state probability of sensors are identical and independent of each other, and denote probability of a CR sensor being in sleep state be p. Let L be the total number of samples that can be sensed in complete sensing time, then number of sample an active CR sensor collects in an epoch is L/K. Thus, if $L_e = L/K$ then g_m is the energy sensed by m^{th} CR sensor in each epoch.

Finally, we assume that the CR sensors in active state communicate the sensed energy at the end of each epoch to fusion center. Here we assume ideal communication between spatially located CR sensors and fusion center, i.e., all measurement transmission are synchronized and there are no loss of packets.

III. Compressive Sensing Equivalent Problem Formulation

In order to apply concept of compressive sensing (CS) on our system model we first need to identify the vector which is compressible, i.e., can be expressed by sparse representation in some known transform domain. It can be observed from equation (5) that $\bar{\mathbf{g}}$ is compressible vector which can be expressed by sparse representation $\bar{\mathbf{e}}_f$ in the transform domain spanned by columns of matrix $\mathbf{A}$. Next we define sparsity order as the number of non-zero elements in the sparse representation. Since, l out of total N sub-bands are occupied by PUs, $\bar{\mathbf{e}}_f$ is a sparse vector of sparsity order l and dimension N ($l << N$).

The task of compressive sensing is to recover sparse vector from reduced set of measurements. In our system model, weighted sum of energies sensed by active CR sensors in a given epoch denoted as measurement for that epoch. The weights for sensed energy are multiplied at CR sensors itself and summation of received weighted energies is performed at fusion center. Thus, fusion center receives a set of K measurements (since we have divided sensing time into K epochs) where weights for CR sensor can be expressed by a $K \times M$ matrix $\mathbf{C}$ whose elements $c_{k,m}$ are expressed as,

$$c_{k,m} = \begin{cases} 0 & \text{when } m^{th} \text{ CR sensor is in sleep state,} \\ & \text{with probability } p \\ \alpha & \text{when } m^{th} \text{ CR sensor is in active state,} \\ & \text{with probability } (1-p)/2 \\ -\alpha & \text{when } m^{th} \text{ CR sensor is in active state,} \\ & \text{with probability } (1-p)/2 \end{cases} \tag{6}$$

Matrix $\mathbf{C}$ forms measurement matrix and finally the $K \times 1$ measurement vector $\bar{\mathbf{y}}$ can be expressed as,

$$\bar{\mathbf{y}} = \mathbf{C}\bar{\mathbf{g}} = \mathbf{CA}\bar{\mathbf{e_f}} \tag{7}$$

The theory of compressive sensing indicates that respective energies in sub-band $\bar{\mathbf{e_f}}$ can be accurately recovered from measurements $\bar{\mathbf{y}}$, if characteristics of incoherent measurements and Restricted Isomerty Property (RIP) are satisfied. In next section we discuss the structure of $\mathbf{A}$ and $\mathbf{C}$ matrix and identify the value of probability of sleep state p required for satisfying RIP property.

IV. Representation and Measurement Matrix

The quality of sparse reconstruction in CS depends on the properties of sparse representation basis and sensing or measurement matrix.

Sparse representation basis: In case of CS, a signal may not appear sparse in a given domain, but it may have sparse representation in some known transform domain. For such representation, we need to identify application specific basis functions. It can be observed that for the considered CR network scenario, vector $\bar{e_f}$ represents energies in sub-bands and is sparse vector (based on assumption of sparse PU occupancy). However, energies computed by CR sensors represented by vector $\bar{\mathbf{g}}$ may not be sparse. Vectors $\bar{e_f}$ and $\bar{\mathbf{g}}$ are related by transforming matrix $\mathbf{A}$ which is essentially mapping the sparse sparse vector $\bar{e_f}$ in some sub space (assuming $M < N$).

Since matrix $\mathbf{A}$ denotes overlap in spectrum sensing range of local CR sensors, the structure of $\mathbf{A}$ is typically a non-square banded matrix. For example, if there is uniform spectrum sensing overlap in which local CR senses its sub-band and adjacent sub-bands, then matrix $\mathbf{A}$ can be expressed as,

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & \cdots & 0 & 0 & 0 \\ . & . & . & . & . & \cdots & . & . & . \\ 0 & 0 & 0 & 0 & 0 & \cdots & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 1 & 1 \end{bmatrix}.$$

Now if vector $\bar{e_f}$ is l_e sparse then the vector $\bar{\mathbf{g}}$ will also be sparse with sparsity order $l_g \leq 2l_e$. Thus, $\bar{\mathbf{g}}$ is also expected to be sparse with reduces sparsity order. In general, if r is the maximum number of CR sensors sensing the same sub-band, then sparsity order reduces to $l_g \leq rl_e$. In special case when M is quite small then $\bar{\mathbf{g}}$ may not be sparse. For example, when $M = 2$ the sparsity may be completely lost as both the sensors might be sensing the PU energies. Thus, there is tradeoff in performance in reducing number of CRs and the overlap in spectrum sensing range.

Random Sampling Matrix: Typically, Gaussian and Bernoulli matrices are used as sampling matrices as with high probability they satisfy the RIP property [16]. In [17], authors have proved Joshnson Lindenstrauss (JL) embedding for the matrix similar to measurement matrix $\mathbf{C}$ of equation (7) and can be stated here as:

Consider a arbitrary set of n points denoted by P in d-dimensional space; these point can be arranged in form of a $n \times d$ matrix $\mathbf{H}$. Let $\mathbf{R}$ be a $d \times k$ random matrix with elements $r_{i,j}$ ($i \in \{1, \cdots, d\}, j \in \{1, \cdots, k\}$) being independent and identical distributed as,

$$r_{i,j} = \sqrt{3} \begin{cases} +1 \text{ with probability } \frac{1}{6} \\ 0 \text{ with probability } \frac{2}{3} \\ -1 \text{ with probability } \frac{1}{6} \end{cases}. \quad (8)$$

Further let $\mathbf{E} = \frac{1}{\sqrt{k}} \mathbf{HR}$

Now, if $k \geq \frac{4+2\beta}{\epsilon^2/2 - \epsilon^3/3}$, for some $\epsilon, \beta > 0$; then with probability atleast $1 - n^{-\beta}$, for all $u, v \in P$,

$(1-\epsilon)||u-v||^2 \leq ||f(u) - f(v)||^2 \leq (1+\epsilon)||u-v||^2$

where, function f maps i^{th} row of $\mathbf{H}$ matrix to i^{th} row of $\mathbf{E}$ matrix.

It is later shown in [16] that by following concentration of inequalities, the random matrix presented in equation (8) also satisfies RIP property. It can be observed that matrix of equation (8) is equivalent to our sensing matrix $\mathbf{C}$ of equation (7) with $p = \frac{1}{3}$ and $\alpha = \sqrt(3)$. Thus, sparse energy vector $\bar{e_f}$ can be reconstructed with high probability even if $\frac{2}{3}$ times CR sensor can be in sleep sate, or in other words approximately, 66% energy can be conserved.

In order to investigate further improve energy efficiency, we follow the work presented in [18] where it is shown that the elements of matrix $\mathbf{R}$ can in general be represented as,

$$r_{i,j} = \sqrt{s} \begin{cases} +1 \text{ with probability } \frac{1}{2s} \\ 0 \text{ with probability } 1 - \frac{1}{s} \\ -1 \text{ with probability } \frac{1}{2s} \end{cases}. \quad (9)$$

It can be noted here that when $s = 1$, matrix $\mathbf{R}$ is a Bernoulli matrix, and when $s = 3$, $\mathbf{R}$ is same as equation (7). Thus, matrix of equation (10) is equivalent to our sensing matrix $\mathbf{C}$ of equation (7) with $p = \frac{1}{1-s}$ and $\alpha = \sqrt(s)$. It is further discussed in [18] that one does not restrict s to 3. In fact, it is shown that when considered vector is very sparse, $s = \frac{N}{log(N)}$, i.e., $log(N)$ fraction of samples is sufficient to accurately estimate the sparse vector where N is the length of sparse vector. This is due to exponential tail error bound, which is common in normal like distributions such as Binomial distribution. Once again following he concentration of inequalities one can prove RIP property for $\mathbf{R}$ of equation (10) as well with $s = \frac{N}{log(N)}$. However, it is recommended in the work to choose s less aggressively, for example $s = \sqrt{3}$. In our simulations we compare detection performance for various levels of energy efficiency by varying values of s.

V. Reconstruction Problem Formulation

After accumulating the measurements from local CR sensors and arranging them in form of measurement vector $\bar{\mathbf{y}}$ (7), fusion center reconstructs the sparse vector $\bar{\mathbf{e}}_f$ by solving following convex optimization problem,

$$\begin{array}{ll} min. & ||\bar{\mathbf{e}}_f||_1 \\ s.t. & ||\bar{\mathbf{y}} - \mathbf{CA}\bar{\mathbf{e_f}}||_2^2 \leq \sigma^2 \end{array} \quad (10)$$

where $||.||_1$ denotes l_1 norm; $||.||_2$ denotes l_2 norm; and σ^2 is the variance of AWGN noise in the wideband spectrum. Once sparse energy vector $\bar{\mathbf{e}}_f$ is recovered, the PU spectral location in wideband are identified by comparing the individual sub-band energies with a specified threshold. The threshold is computed based on system parameter like minimum desired probability of false alarm.

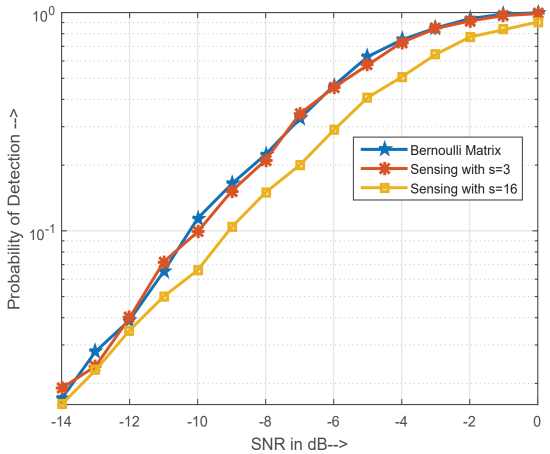

Fig. 2. Comparison of detection performance under different levels of energy conservation by varying value of 's'

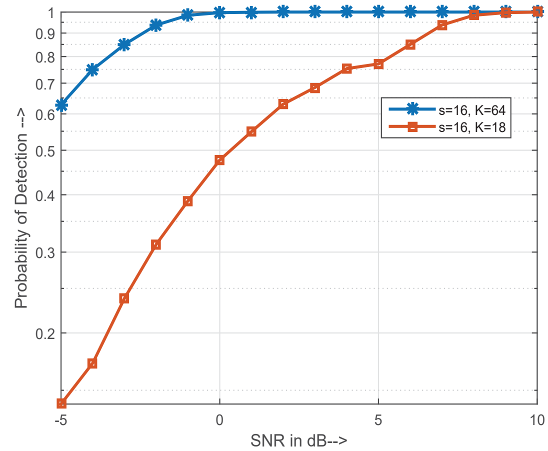

Fig. 3. Comparison of detection performance under different lengths of observation vector

VI. Simulation Results

In this section we present performance of the proposed wideband cooperative cognitive radio strategy under various sensing scenarios. Here we consider number of sub-bands $N = 256$ with sparsity order $l = 2$. The primary users (PU) are assumed to be located in space near to SU locations and signal power in an unknown occupied sub-band is assumed to be 0.5 watts; thus, total signal power is 1 watt in complete wideband spectrum. The CR sensors are also assumed to be spatially distributed near PU transmission. The threshold for energy detection is computed assuming probability of false alarm 10^{-3}.

Fig. 2 shows probability of detection performance vs signal to noise ratio (SNR) under no energy conservation i.e., Bernoulli distributed sensing matrix (s = 1) and energy conservation based sensing matrix with $s = 3$ and $s = \sqrt{N} = 16$. It can be observed that detection performance for s=1 and s=3 is almost equivalent, while inferior performance is achieved for s=16. This is due to increase in number of CR sensors going to sleep state in case of s=16, compared to no sensor going to sleep state for s=1 and $(2/3)^{rd}$ times CR sensor going for sleep state for s=3. Thus, we observe a tradeoff between accuracy in PU detection and the amount of energy conservation via CR sensors going to sleep state.

In previous scenario we considered sensing coefficient (M/N) to be 0.25 i.e. we had 64 observation vectors/measurements which is too high for a sparsity order of 2. In the following discussion we compare the detection performance with decreasing number of observation vectors. It can be observed from Fig. 3 that on reducing the number of observation vectors, there is degradation in probability of detection performance. Derivation of expression for minimum number of observation vector required for accurate reconstruction is scope of our future work.

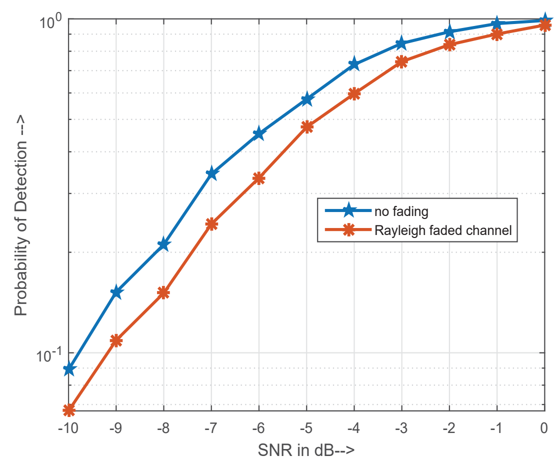

Fig. 4. Comparison of detection performance under zero fading and fading scenario

Next we incorporate fading in our analysis. As we have divide complete wideband into number of non-overlapping narrow bands, we assume flat fading over each sub-band. The channel fading coefficient is assumed to be zero mean with unity power. It can be observed from Fig. 4 that fading degrades detection performance. However, for high SNR, the signal strength becomes significant to mitigate deep fade and thus detection performance under fading converges towards no fading scenario.

Finally, to mitigate fading we analyze the scenario when CR sensors have overlap in spectrum sensing range. It can be observed from Fig. 5 that with increase in spectrum overlap, there is improvement in detection performance without any increase in probability of false alarm. However, care needs to be taken while increasing the overlap in spectrum sensing range as it reduces

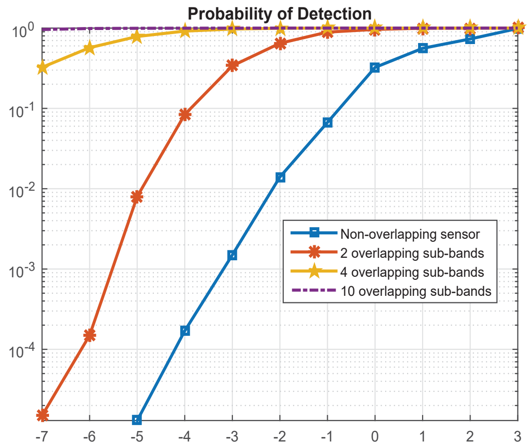

Fig. 5. Comparison of detection performance under different levels of overlap in spectrum sensing range

sparsity and may result in CS getting ineffective.

VII. Conclusion

In this paper, we have presented CS based cooperative spectrum sensing technique to identify spectral location of primary users (PUs) in a wideband spectrum. In order to mitigate fading and exploit diversity, we have assumed that spectrum sensing range of individual CR overlap over each other. Further, to improve energy efficiency in spectrum sensing, we also introduced probabilistic active and sleep state for individual CR sensors. Assuming sparse occupancy of PUs in the wideband and by Parseval's theorem, we formulated a l_1 minimization problem at fusion center to reconstruct the vector representing energies in sub-bands. Since individual CR random take active or sleep state randomly in a time epoch, the sensing matrix for reconstruction is identified as random matrix. Our analysis also identifies the larger values of probability of sleep state (so as to have more energy efficiency) for which sensing matrix satisfies restricted isometric property (RIP).

VIII. Acknowledgment

This research is sponsored by an Science and Engineering Research Board (SERB), Dept. of Science and Technology, Govt. of India under the funding for project: "Application of Cognitive Radio Technology for Smart Grid Communication" sanction order number ECR/2015/000008.

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