DYNAMIC STABILITY OF CURVED PANELS

WITH CUTOUTS

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Total number of pages including figures and tables =30

Total number of tables=3

Total number of figures=8

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Summary

The parametric instability behaviour of curved panels with cutouts subjected to in-plane static and periodic compressive edge loadings are studied using finite element analysis. The first order shear deformation theory is used to model the curved panels, considering the effects of transverse shear deformation and rotary inertia. The theory used is the extension of dynamic, shear deformable theory according to the Sander's first approximation for doubly curved shells, which can be reduced to Love's and Donnell's theories by means of tracers. The effects of static load factor, aspect ratio, radius to thickness ratio, shallowness ratio, boundary conditions and the load parameters on the principal instability regions of curved panels with cutouts are studied in detail using Bolotin's method. Quantitative results are presented to show the effects of shell geometry and load parameters on the stability boundaries. Results for plates are also presented as special cases and are compared with those available in the literature.

1. INTRODUCTION

Structural elements subjected to in-plane periodic forces may lead to dynamic stability, due to certain combinations of the values of load parameters. The instability may occur below the critical load of the structure under compressive loads over a range or ranges of excitation frequencies. Several means of combating resonance such as damping and vibration isolation may be inadequate and sometimes dangerous with reverse results [1]. Cutouts are inevitable in aerospace, civil, mechanical and marine structures mainly for practical considerations. In aerospace structures, cutouts are commonly found as access ports for mechanical and electrical systems, or simply to reduce weight. Cutouts in wing spars and cover panels of commercial transport wings and military fighter wings are

needed to provide access for hydraulic lines and for damage inspection. Cutouts are also made to lighten the loads, provide ventilation and for altering the resonant frequency of the structures. In addition, the designers, often need to incorporate cutouts or openings in a structure to serve as doors and windows. Thus the dynamic stability of structures with cutouts are of great technical importance for understanding the dynamic systems under periodic loads.

Despite the practical importance of these structures, the number of technical papers and reports dealing with the subjects are very limited due to complexity involved. An extensive bibliography of earlier works on these problems are given in review papers [1-3] through 1987. Most of the investigators [4-6] studied the dynamic stability of closed cylindrical shells with a simply supported boundary condition, using analytical approach. The dynamic instability of conical shells were studied by Ng *et al.* [7] using Generalized Differential Quadrature method. The study of the parametric instability behaviour of curved panels is new. The dynamic stability of uniaxially loaded cylindrical panels with transverse shear effects is studied by Ng, Lam and Reddy [8]. Previous investigations involving cutouts were mainly confined to free vibration of plates. A finite element analysis of a clamped plate with different cutout sizes, along with experiments using holographic interferometry, was carried out by Monahan *et al.* [9]. Paramsivam [10] used a finite difference approach in analyzing the effects of openings on the fundamental frequencies of plates with simply supported and clamped boundary conditions. The dynamic characteristics of rectangular plates with one or two cutouts using a finite difference formulation, based on variational principles were obtained by Aksu and Ali [11], along with experimental verifications. Ali and Atwal [12] studied the natural frequencies of simply supported rectangular plates and rectangular cutouts using the Raleigh Ritz method. The linear and

large amplitude flexural vibration of isotropic and composite plates with cutout was studied by Reddy [13] using finite element method. Mundkur *et al.* [14] studied the vibration of square plates with square cutouts by using boundary characteristics orthogonal polynomials satisfying the boundary conditions. Chang and Chiang [15] studied the vibration of the rectangular plate with an interior cutout by using finite element method. Lam and Hung [16] investigated on the flexural vibrations of plates with discontinuities in the form of cracks and cutouts using orthogonal polynomial functions, generated using the Gram Schmidt process. Lee, Lim and Chow [17] predicted the natural frequencies of rectangular plates with an arbitrarily located rectangular cutout. Huang and Sakiyama [18] analyzed the free vibration of rectangular plates with variously shaped holes. Rossi [19] has dealt with the frequency of transverse vibrations of the structural systems, especially with the fixed boundaries. Young, Yuan and Dickinson [20] presented the free vibration of thick rectangular plates with depression, grooves or cutouts using 3D elasticity and Ritz method. Ritchie and Rhodes [21] have investigated theoretically and experimentally the behaviour of simply supported uniformly compressed rectangular plates with central holes, using a combination of Raleigh-Ritz and finite element methods. The buckling of rectangular plates with central cutouts are investigated by Willium.L. Ko [22] using structural performance and resizing (SPAR) finite element computer program. Brogan *et al.* [23] presented analytical and experimental investigations of the dynamic behaviour of closed circular cylindrical shell with a rectangular cutout. Toda and Komatsu [24] studied analytically and experimentally the effect of circular cutouts on the resonant frequencies of thin cylindrical shells using simplified Raleigh-Ritz type approximations. The effects of cutouts on the natural frequencies of curved panels have been treated sparsely in the literature. Liew and Lim [25] examined the natural frequencies and vibratory characteristics

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of shallow shells having an outer super-elliptical periphery and an inner super-elliptical cutout using Ritz procedure. The free vibration characteristics of unstiffened [26] and longitudinally stiffened [27] square panels with symmetrical square cutouts are investigated by Sivasubramonium *et al.*[] using finite element method. To the best of authors' knowledge, there is no study involving dynamic stability of curved panel with a cutout. The application of cut out on the structural component will alter the global quantities such as stresses, free vibration frequency, buckling load and dynamic instability region(DIR).

In the present study, the dynamic stability of curved panels with cutouts are investigated. The influences of various parameters like effects of static & dynamic load factors, size of cutout, geometry and various boundary conditions on the instability behaviour of curved panels have been examined. The present formulation of the problem is made general to accommodate a doubly curved panel with finite curvatures in both the directions having arbitrary load and boundary conditions.

2. THEORY AND FORMULATIONS

The basic configuration of the problem considered here is a doubly curved panel with cutout as shown in Figure 1, subjected to harmonic in-plane edge loading.

2.1 GOVERNING EQUATIONS

The equation of equilibrium for free vibration of a shear deformable doubly curved panel with cutout subjected to in-plane external loading can be written as:

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} - \frac{1}{2}C_2\left(\frac{1}{R_y} - \frac{1}{R_x}\right)\frac{\partial M_{xy}}{\partial y} + C_1\frac{Q_x}{R_x} + C_1\frac{Q_y}{R_{xy}} = \rho h\frac{\partial^2 u}{\partial t^2} \qquad (1)$$

$$\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} + \frac{1}{2}C_2\left(\frac{1}{R_y} - \frac{1}{R_x}\right)\frac{\partial M_{xy}}{\partial x} + C_1\frac{Q_y}{R_y} + C_1\frac{Q_x}{R_{xy}} = \rho h\frac{\partial^2 v}{\partial t^2}$$

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} - \frac{N_x}{R_x} - \frac{N_y}{R_y} - 2\frac{N_{xy}}{R_{xy}} + N_x^0\frac{\partial^2 w}{\partial x^2} + N_y^0\frac{\partial^2 w}{\partial y^2} = \rho h\frac{\partial^2 w}{\partial t^2}$$

$$\frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x = \frac{\rho h^3}{12}\frac{\partial^2 \theta_x}{\partial t^2}$$

$$\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial y} - Q_y = \frac{\rho h^3}{12}\frac{\partial^2 \theta_y}{\partial t^2}$$

where N_x^0 and N_y^0 are the external loading in X and Y directions respectively. C_1 and C_2 are tracers by which the analysis can be reduced to that of Sander's, Love's and Donnell's theories. The equation of motion can be written in matrix form as:

$$[M]\{\ddot{q}\} + [[K_e] - \lambda[K_g]]\{q\} = 0 \tag{2}$$

The in-plane load N(t) is periodic and can be expressed in the form

$$N(t) = N_s + N_t cos\Omega t \tag{3}$$

where N_s is the static portion of N(t). N_t is the amplitude of the dynamic portion of N(t) and Ω is the frequency of excitation. The static buckling load of the panel N_{cr} is the measure of the magnitude of N_s and N_t,

$$N_s = \alpha N_{cr}, \qquad\qquad N_t = \beta N_{cr} \tag{4}$$

where α and β are termed as static and dynamic load factors respectively. Using Equation (3), the equation of motion is obtained as:

$$[M]\{\ddot{q}\} + [[K_e] - \alpha N_{cr}[K_g] - \beta N_{cr}[K_g]cos\Omega t]\{q\} = 0 \tag{5}$$

Equation (5) represents a system of second order differential equations with periodic coefficients of the Mathieu-Hill type. The development of regions of instability arises from Floquet's theory which establishes the existence of periodic solutions. The boundaries of the dynamic instability regions are formed by the periodic solutions of period T and 2T,

where $T = 2\pi/\Omega$. The boundaries of the primary instability regions with period 2T are of practical importance [28] and the solution can be achieved in the form of the trigonometric series

$$q(t) = \sum_{k=1,3,5}^{\infty} \left[\{a_k\}sin\frac{k\Omega t}{2} + \{b_k\}cos\frac{k\Omega t}{2} \right] \qquad (6)$$

Putting this in Equation (4) and if only first term of the series is considered, equating coefficients of $sin\frac{\Omega t}{2}$ and $cos\frac{\Omega t}{2}$, the equation (4) reduces to

$$\left[[\mathbf{K_e}] - \alpha \mathbf{N_{cr}}[\mathbf{K_g}] \pm \frac{1}{2}\beta \mathbf{N_{cr}}[\mathbf{K_g}] - \frac{\Omega^2}{4}[\mathbf{M}] \right] \{\mathbf{q}\} = \mathbf{0} \qquad (7)$$

Equation (7) represents an eigenvalue problem for known values of α, β and N_{cr}. The two conditions under a plus and minus sign correspond to two boundaries of the dynamic instability region. The eigenvalues are Ω, which give the boundary frequencies of the instability regions for given values of α and β. In this analysis, the computed static buckling load of the panel is considered as the reference load in line with Moorthy *et el.* [29] and Ganapathi *et al.* [30].

An eight-nodded curved isoparametric quadratic element is employed in the present analysis with five degrees of freedom u, v, w, θ_x and θ_y per node. First order shear deformation theory (FSDT) is used and the shear correction coefficient has been employed to account for the nonlinear distribution of the shear strains through the thickness. The displacement field assumes that mid-plane normal remains straight but not necessarily normal after deformation, so that

$$\bar{u}(x, y, z) = u(x, y) + z\theta_x(x, y) \qquad (8)$$

$$\bar{v}(x, y, z) = v(x, y) + z\theta_y(x, y)$$

$$\bar{w}(x, y, z) = w(x, y)$$

where,

θ_x, θ_y are the rotations of the mid surface

Also $\bar{u}, \bar{v}, \bar{w}$ and u, v, w are the displacement components in the x, y, z directions at any section and at mid- surface respectively. The constitutive relationships for the shell are given by

$$F = [D]\{\varepsilon\} \tag{9}$$

where

$$F = [N_x, N_y, N_{xy}, M_x, M_y, M_{xy}, Q_x, Q_y]^T \tag{10}$$

$$[D] = \begin{bmatrix}
\frac{Eh}{1-\nu^2} & \frac{\nu Eh}{1-\nu^2} & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{\nu Eh}{1-\nu^2} & \frac{Eh}{1-\nu^2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{Eh}{2(1+\nu)} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{Eh^3}{12(1-\nu^2)} & \frac{\nu Eh^3}{12(1-\nu^2)} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{Eh^3\nu}{12(1-\nu^2)} & \frac{Eh^3}{12(1-\nu^2)} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{Eh^3}{24(1+\nu)} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{Eh}{2.4(1+\nu)} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{Eh}{2.4(1+\nu)}
\end{bmatrix} \tag{11}$$

A Reissner's shear correction factor of 5/6 is included for all numerical computations. Extension of shear deformable Sander's kinematic relations for doubly curved shells [31,32] are used in the analysis. The linear strain displacement relations are

$$\varepsilon_{xl} = \frac{\partial u}{\partial x} + \frac{w}{R_x} + z\kappa_x \tag{12}$$

$$\varepsilon_{yl} = \frac{\partial v}{\partial y} + \frac{w}{R_y} + z\kappa_y$$

$$\gamma_{xyl} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + z\kappa_{xy}$$

$$\gamma_{yz} = \frac{\partial w}{\partial y} + \theta_y - C_1 \frac{v}{R_y}$$

$$\gamma_{xz} = \frac{\partial w}{\partial x} + \theta_x - C_1 \frac{u}{R_x}$$

where

$$\kappa_x = \frac{\partial \theta_x}{\partial x}, \kappa_y = \frac{\partial \theta_y}{\partial y} \tag{13}$$

$$\kappa_{xy} = \frac{\partial \theta_x}{\partial y} + \frac{\partial \theta_y}{\partial x} + \frac{1}{2}C_2 \left(\frac{1}{R_y} - \frac{1}{R_x} \right) \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)$$

The element geometric stiffness matrix for the doubly curved panel is derived using the

nonlinear strain components as:

$$\varepsilon_{xnl} = \frac{1}{2}\left(\frac{\partial u}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial v}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial w}{\partial x} - \frac{u}{R_x}\right)^2 + \frac{1}{2}z^2\left[\left(\frac{\partial \theta_x}{\partial x}\right)^2 + \left(\frac{\partial \theta_y}{\partial x}\right)^2\right] \tag{14}$$

$$\varepsilon_{ynl} = \frac{1}{2}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{1}{2}\left(\frac{\partial v}{\partial y}\right)^2 + \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{v}{R_y}\right)^2 + \frac{1}{2}z^2\left[\left(\frac{\partial \theta_x}{\partial y}\right)^2 + \left(\frac{\partial \theta_y}{\partial y}\right)^2\right]$$

$$\gamma_{xynl} = \left[\left(\frac{\partial u}{\partial x}\right)\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\left(\frac{\partial v}{\partial y}\right) + \left(\frac{\partial w}{\partial x} - \frac{u}{R_x}\right)\left(\frac{\partial w}{\partial y} - \frac{v}{R_y}\right)\right]$$

$$+z^2\left[\left(\frac{\partial \theta_x}{\partial x}\right)\left(\frac{\partial \theta_x}{\partial y}\right) + \left(\frac{\partial \theta_y}{\partial x}\right)\left(\frac{\partial \theta_y}{\partial y}\right)\right]$$

The element matrices are derived as:

Elastic stiffness matrix

$$[k_e]_e = \int [B]^T [D][B] dx dy \tag{15}$$

Geometric stiffness matrix

$$[k_g]_e = \int [G]^T [S][G] dv \tag{16}$$

Consistent mass matrix

$$[m]_e = \int [N]^T [I][N] dx dy \tag{17}$$

The overall matrices $[K_e], [K_g]$ and $[M]$ are obtained by assembling the corresponding element matrices.

2.2 COMPUTER PROGRAM

A computer program has been developed to perform all the necessary computations. Element elastic stiffness matrices and mass matrices are obtained using a standard procedure. The geometric stiffness matrix is essentially a function of the in-plane stress distribution in the element due to applied edge loadings. Since the stress field is non-uniform, due to cutout, plane stress analysis is carried out using the Finite element method to determine the stresses and these are used to formulate the geometric stiffness matrix. Reduced integration technique is adopted in order to avoid possible shear locking. Element matrices are assembled into global matrices, using skyline technique. Subspace iteration method is adopted throughout to solve the eigenvalue problems.

3. RESULTS AND DISCUSSIONS

The convergence studies have been carried out for fundamental frequencies of vibration of the SSSS square plates with square hole of size ratio c/a=0.5, for different mesh divisions. As shown in Table 1, the frequencies of vibration are computed and the results are compared with the values by Lee, Lim and Chow [17]. It shows the sufficient accuracy of the numerical solutions by the present method. From the above convergence study, 20×20 mesh has been employed to idealise the panel in the subsequent analysis. The accuracy and efficiency of the present formulation is also established through comparison of frequency parameters of plates with different cutout sizes with the finite element results using a standard package PACFAC 75 [16], the numerical scheme using orthogonal polynomial functions by Lam and Hung [16], the numerical method based on Raleigh quotient by Lee, Lam and Chow [17], results using boundary characteristics orthogonal

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polynomials functions in the Ralegh-Ritz method by Mundakar, Bhat and Neriya [14], the results of Koushal and Bhat [33] and are presented in Table 2. Good agreement exists between the present FEM results with the literature. The present formulation is then validated for buckling of plate with cutout with the graphical results using the structural performance and resizing (SPAR) finite element computer program by Ko [22]. As seen from Fig 2, good agreement exists between both the finite element results. To validate the formulation further, the free vibration frequency parameters with computational results for clamped (CCCC) curved panel having cutout are compared with the work carried out by Sivasubramonian *et al.* [27]. The results are shown in Table 3. The above studies indicate good agreement between the present study and those from the literature. Once the free vibration and buckling results are validated, the dynamic instability studies are made.

3.1 DYNAMIC STABILITY STUDIES

The dynamic instability regions (DIR) are plotted for a flat and cylindrical panel with/without static component to consider the effects of static load factor, size of cutout, different panel geometry and boundary conditions. A simply supported plate of dimensions a=b=500mm, h=5mm, E=70 Gpa, ν=0.3, $\rho = 2800 kg/m^3$ is described as a standard case and the computed buckling load of this panel is taken as the reference load in line with Moorthy et el. [29]. The non-dimensional excitation frequency $\Omega = \bar{\Omega}a^2\sqrt{\rho h/D}$ is used throughout the dynamic instability studies (unless otherwise mentioned), where $\bar{\Omega}$ is the excitation frequency in radian/second, $D = \frac{Eh^3}{12(1-\nu^2)}$. For a given panel, the effect of size of cutout on the instability region are studied from 0.0 (no cutout) to 0.8 at an interval of 0.1. However, for clarity, the plots are shown for size of cutout 0.0 (no cutout) to 0.8 at an interval of 0.2 and c/a=0.5 and are presented in Fig 3. It can be observed that, the

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onset of instability occurs with lower excitation frequencies for small cutouts in simply supported plates up to c/a=0.2. With increase of cutout size, the onset of excitation frequency increases along with wider dynamic instability regions . The onset of instability occurs with higher excitation frequency for plates with cutout size of c/a=0.4 onwards than that of plate without cutout (c/a=0.0). The onset of instability occurs at a higher excitation frequencies up to plate with cutout of c/a=0.8 with wider instability regions. This may be attributed to the predominance of the boundary restraints over the entire plate. The effect of static component of load on the instability regions of the plate with a cutout of size c/a=0.5 is studied for α=0.0, 0.2 and 0.4, as shown in Figure 4. Due to increase of static component, the instability regions tend to shift to lower frequencies and become wider. All further studies are made with a static load factor of 0.2 (unless otherwise mentioned). Studies have also been made for comparison of instability regions for different shell geometries. The dynamic instability regions are plotted for plate and different curved panels such as cylindrical (b/R_y=0.25), spherical ($a/R_x = b/R_y$=0.25) and hyperbolic paraboloids ($a/R_x = -0.25, b/R_y = 0.25$) with cutouts of c/a=0.5 and are compared in Fig 5. It is observed, that the excitation frequency increases with introduction of curvatures from plate to curved panels with cutout. However, the hyperbolic paraboloid with cutout shows similar instability behaviour as that of a flat panel with no stiffness being added due to the curvature of the panel with cutout. Similar observations were also obtained by Leissa and Kadi [34] on a study of free vibration of shells, especially on a supported planform. The effect of size of cutout on instability regions of a simply supported cylindrical panel is investigated for c/a=0.0, 0.2, 0.4, 0.5, 0.6 and 0.8. As shown in Fig 6, the onset of instability occurs earlier with increase of size of cutout up to c/a=0.5. With further increase of cutout, the excitation frequency gradually

increases having wider dynamic instability regions. The onset of instability occurs with higher excitation frequency for the cylindrical panel with cutout size c/a=0.8 than that of without cutout with very wide DIR. The onset of instability will even occur earlier for a cylindrical panel with cutout c/a=0.8 for a higher value of dynamic load beyond β=0.6. Figure 7 shows the influence of different boundaries (SSSS, SCSC, CSCS, CCCC) on the principal instability regions. As expected, the instability occurs at a higher excitation frequency from simply supported to clamped edges due to the restraint at the edges. The width of the instability regions are also decreased with the increase of restraint at the edges. The study is then extended to dynamic stability of cylindrical panel with cutout subjected to biaxial loading. The instability regions are plotted for the cylindrical panel with cutout c/a=0.5 and compared with that of the panel without cutout. As shown in Fig 8, the instability appears at lower excitation frequency with increasing dynamic instability region for biaxial loading of cylindrical panel with cutout.

4. CONCLUSION

The results of the stability studies of the plates and shells with cutout can be summarised as follows:

1. The onset of instability occurs at lower excitation frequencies with increase of cutout size in plates with wider instability regions. With further increase of size of cutout the excitation frequency increases and sometimes higher than that of plate with cutout.

2. Due to static component, the instability regions tend to shift to lower frequencies with wide instability regions showing destabilizing effect on the dynamic stability behaviour of the curved panel with cutout.

3. The effect of curvature is reduced with increase of size of cutout.

4. The curved panels with cutout show more stiffness with addition of curvatures. But

the hyperbolic paraboloid panels, especially on a supported planform, behave like a plate with no stiffness being added due to curvature of the shell.

5. The instability regions have been influenced due to restraint provided at the edges of the curved panel with cutout. The predominance of the boundary restraints may be responsible for increase of excitation frequencies especially for plate and curved panels with higher size of cutout. The onset of instability occurs for curved panels with restraints at the straight edges than that of curved boundary.

6. It was observed that instability appears at lower excitation frequency with increasing dynamic instability region for cylindrical panel with cutout, subjected to biaxial loading.

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Table 1 Convergence of non-dimensional fundamental frequencies of a SSSS square plate with a hole of size ratio c/a=0.5.

$$a/b = 1, b/h = 100, \nu = 0.3$$

Non dimensional frequency, $\omega = \bar{\omega} a^2 \sqrt{(\frac{\rho h}{D})}$

Mesh	Non-dimensional frequencies			
Division	1	2	3	4
8×8	23.570	40.5227	40.5227	72.2728
12×12	23.4703	40.1072	40.1072	71.4209
16×16	23.4364	39.9793	39.9793	71.1964
20×20	23.4218	39.9287	39.9287	71.1090
Lee *et al.* [17]	(23.329)	39.712	39.712	(71.263)

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No. of Table-1

Table 2 Comparison of non-dimensional fundamental frequencies of a simply- supported SSSS square plate with a cutout.

$$a/b = 1, b/h = 100, \nu = 0.3$$

Non dimensional frequency, $\omega = \bar{\omega}a^2\sqrt{(\frac{\rho h}{D})}$

c/a	Non-dimensional frequencies					
	Present	PACFAC [16]	Ref [17]	Ref [16]	Ref [14]	Ref [33]
0	19.7336	19.752	19.739	19.740	19.7392	19.57
0.2	19.1339	19.120	18.901	18.762	20.1933	19.16
0.4	20.7387	20.732	20.556	20.785	-	-
0.5	23.4218	23.235	23.329	23.664	24.243	23.650
0.6	28.3068	28.241	28.491	28.844	-	-
0.8	56.9487	57.452	58.847	58.062	58.3585	58.25

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Table 3 Comparison of frequencies in Hz for the CCCC cylindrical curved panel with and without cutout.

$$a=b=500mm, h=2mm, \nu=0.3$$

$$E=7020\ kg/mm^2, \rho = 2720\ kg/m^3$$

R	Mode No	c/a=0.0		c/a=0.5	
		Present FEM	Siva *et al.* [27]	Present FEM	Siva *et al.* [27]
Plate	1	69.76	69.2	126.73	125.7
	2	142.27	140.9	147.92	147.3
	3	142.27	140.9	147.92	147.3
	4	209.79	206.9	199.04	199.2
2000	1	215.23	213.9	184.73	185.3
	2	245.23	243.2	187.52	188.2
	3	328.34	326.1	295.68	295.9
	4	336.83	333.3	310.30	309.7

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No. of Table-3
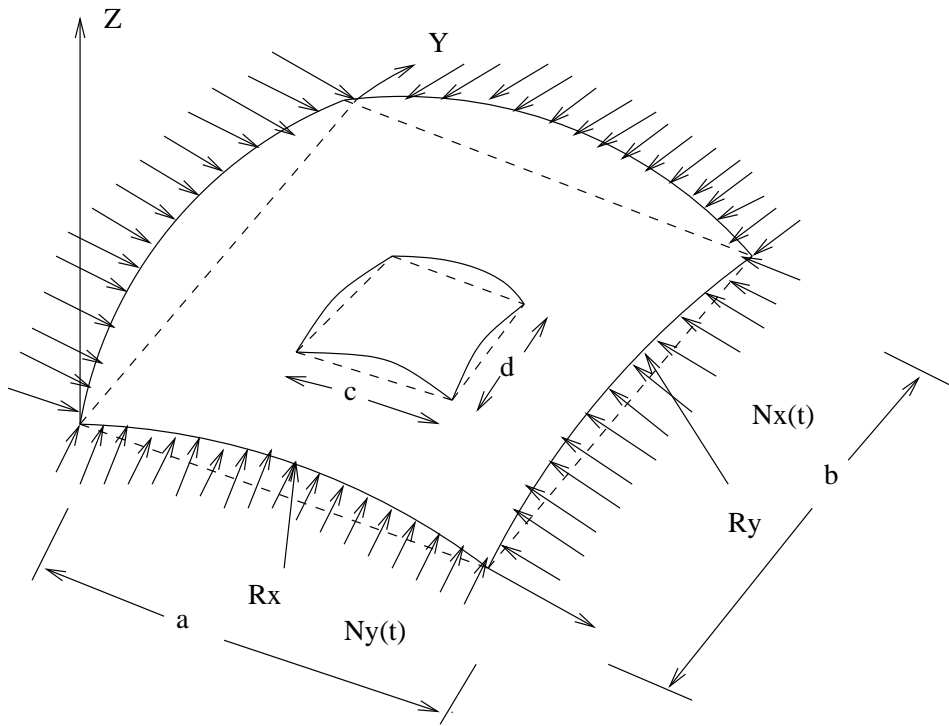
Doubly Curved Panel With Cutout

Names of authors: Sri S. K. Sahu
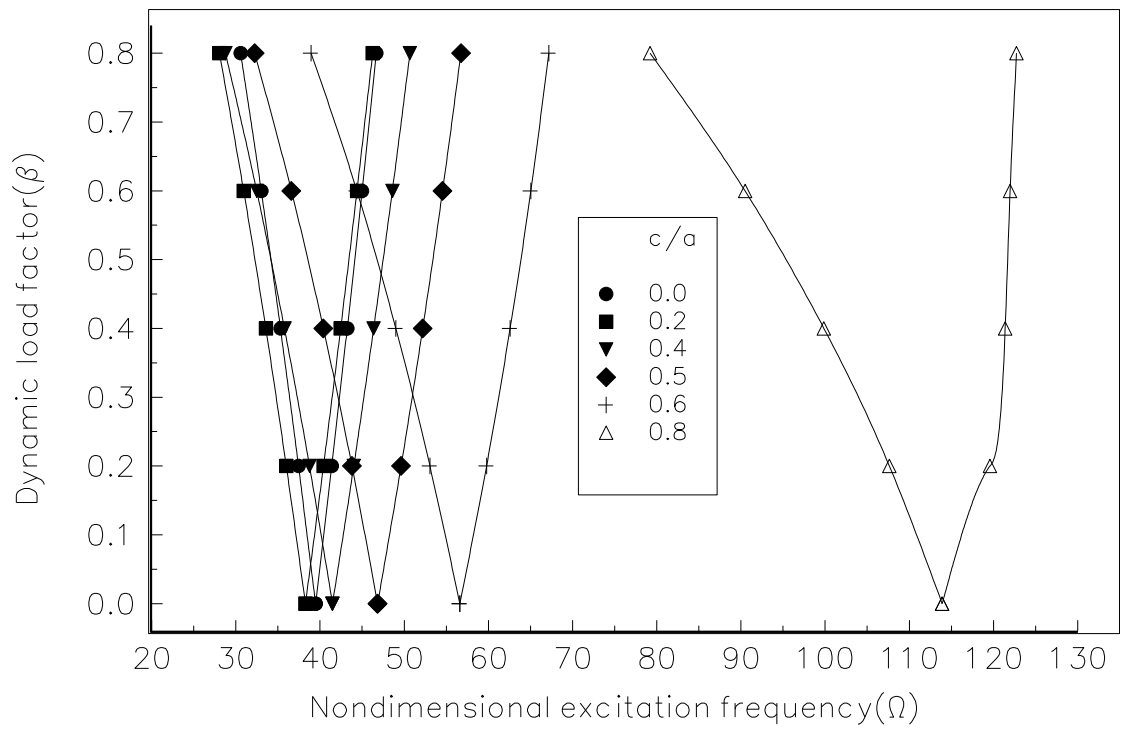
Prof. P. K. Datta

No. of illustration-1
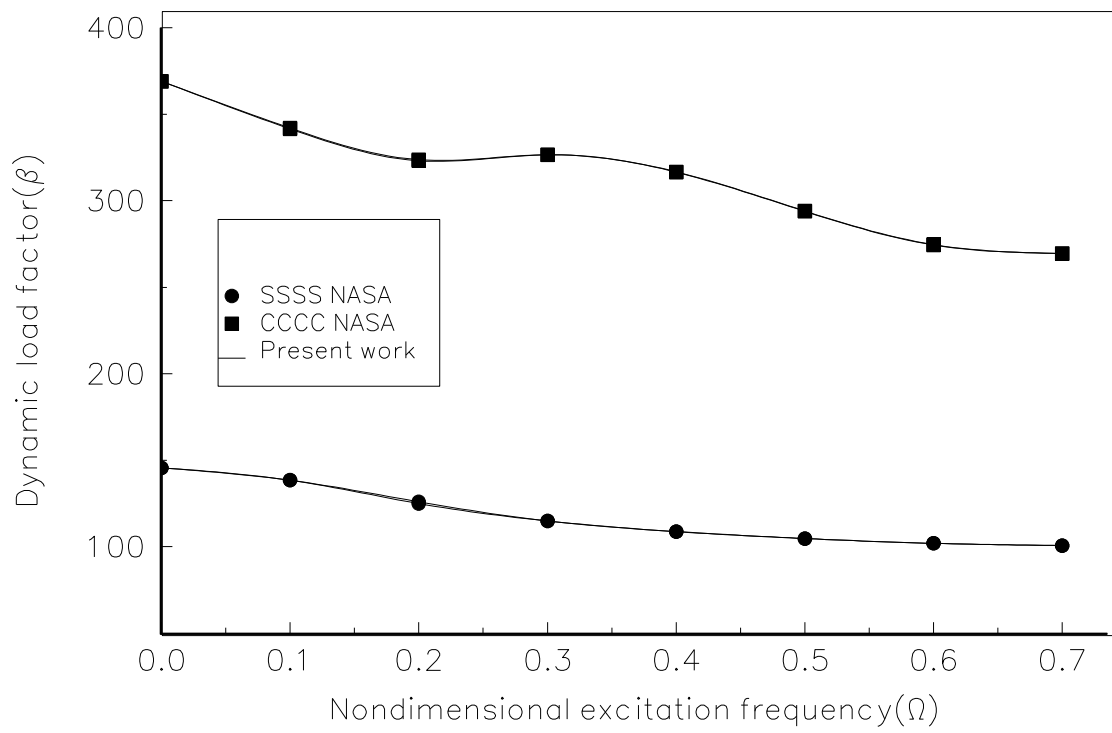
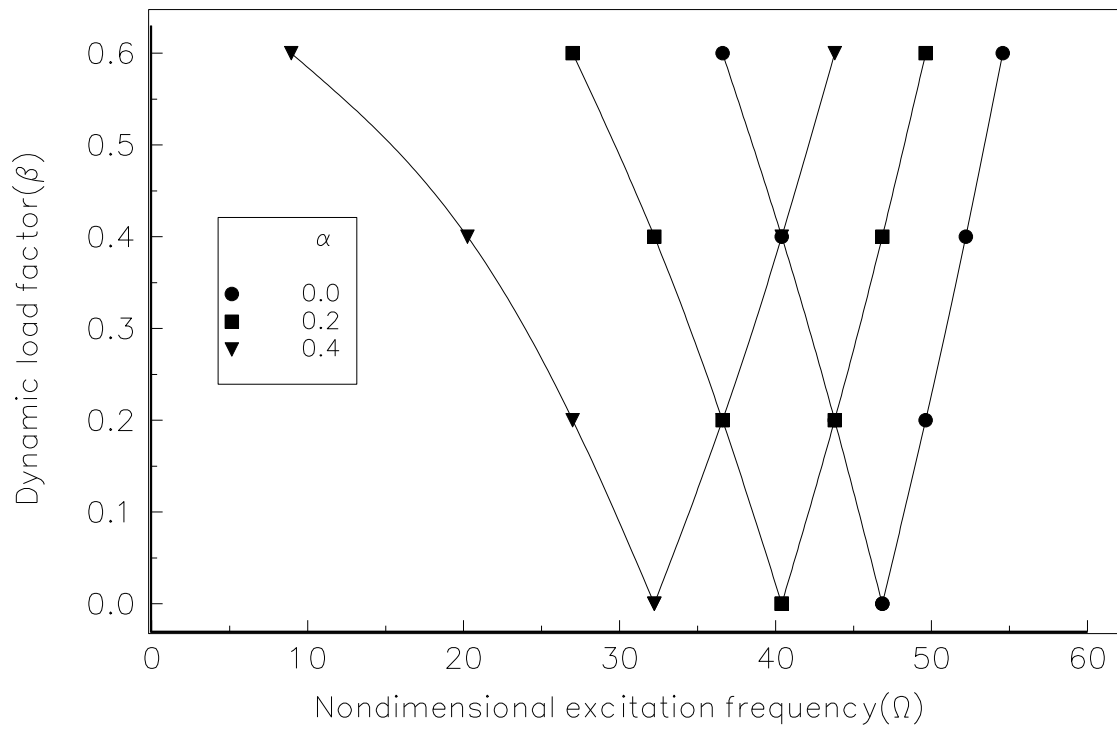Names of authors: Sri S. K. Sahu

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No. of illustration-2

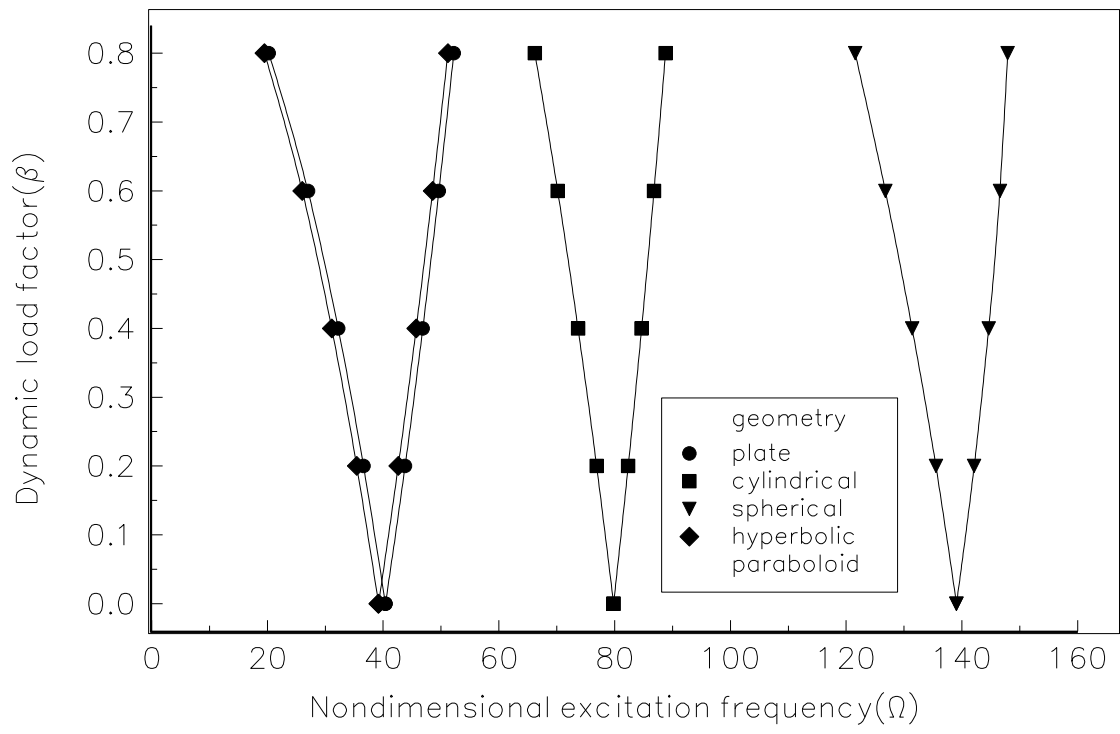Names of authors: Sri S. K. Sahu

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No. of illustration-3

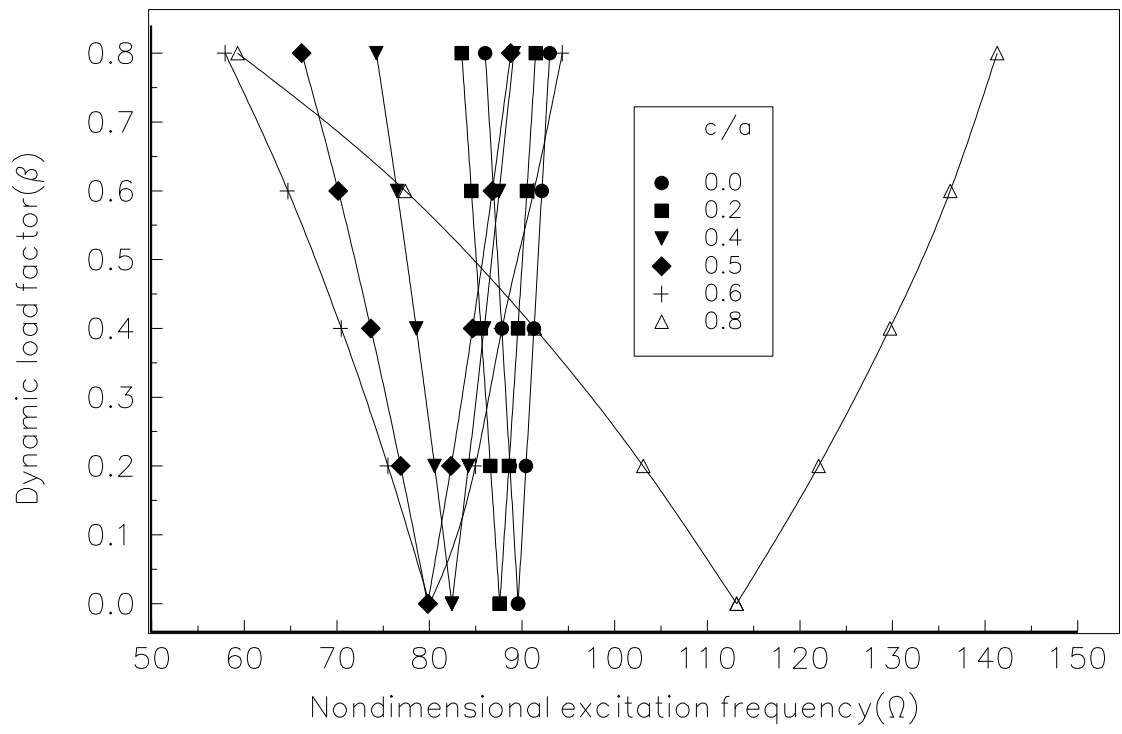Names of authors: Sri S. K. Sahu

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No. of illustration-4

Names of authors: Sri S. K. Sahu

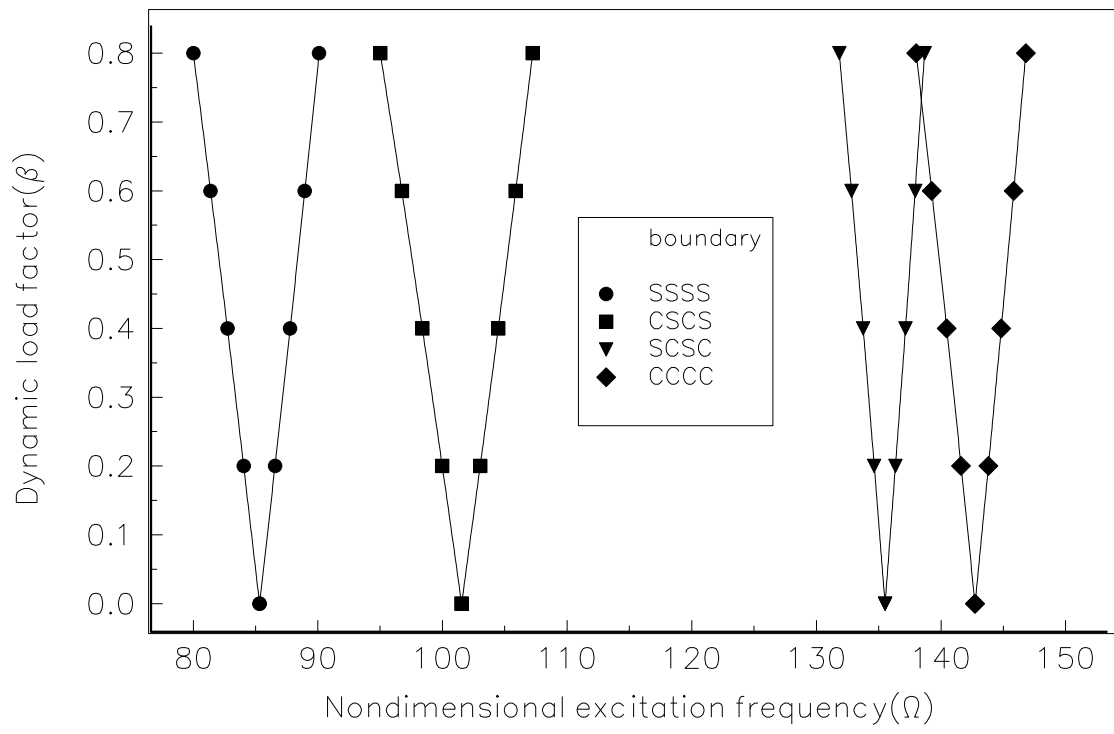Prof. P. K. Datta

No. of illustration-5

Names of authors: Sri S. K. Sahu
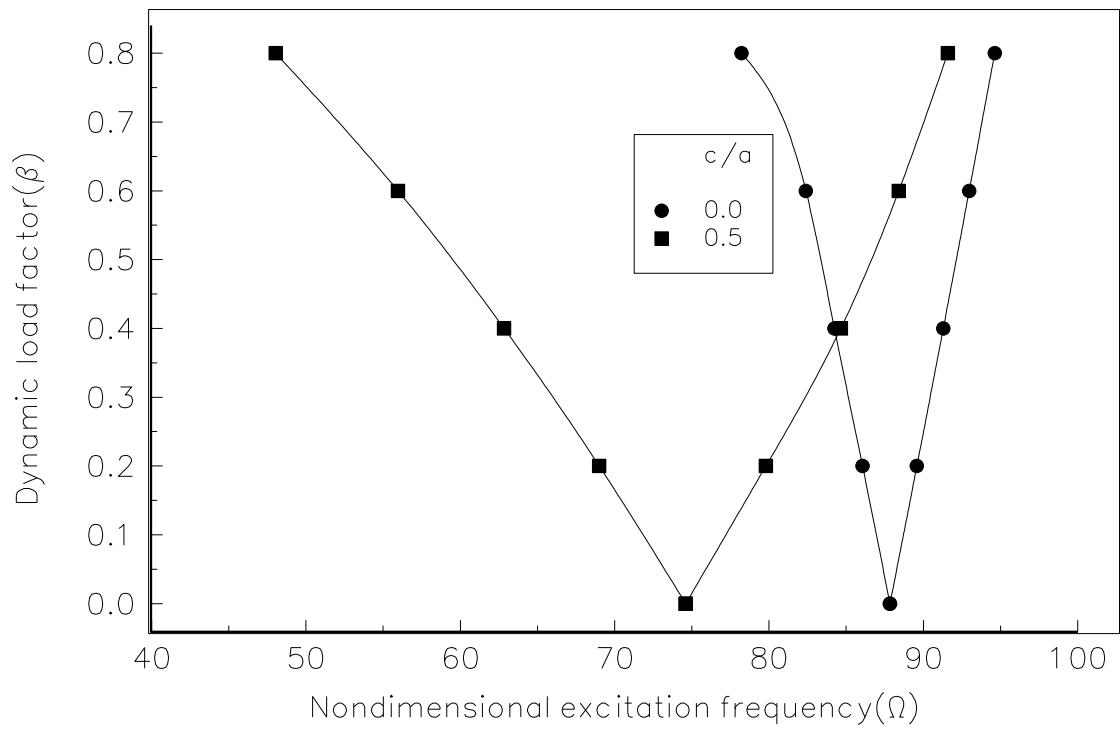
             Prof. P. K. Datta

No. of illustration-6

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        Prof. P. K. Datta

No. of illustration-7

Names of authors: Sri S. K. Sahu

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No. of illustration-8

NOTATIONS

a, b Dimensions of shell

R_x, R_y Radii of curvatures

c, d Dimensions of the cutout

E Young's modulus

ν Poisson's ratio

ρ Mass density

G   shear modulus

$[K]$ Stiffness matrix

$[K_g]$ Geometric stiffness matrix

$[M]$ Mass matrix

$\{q\}$ Vector of generalized cordinates

w Deflection of mid-plane of shell

θ_x, θ_y Rotations about axes

Ω, ω Frequency of forcing function and transverse vibration

α, β Static and dynamic load factors

N_{cr} Critical buckling load

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Figure 1 Geometry and co-ordinate systems of a doubly curved panel with cutout

Figure 2 Comparison of buckling loads of plates with different sizes of cutouts.

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Figure 5 Effect of cutout on instability region of different curved panels: flat panel ($a/R_x = b/R_y = 0$), Cylindrical ($a/R_x = 0, b/R_y = 0.25$), Spherical ($a/R_x = b/R_y = 0.25$), Hyperbolic paraboloid ($a/R_x = -0.25, b/R_y = 0.25$) for a/b=1, c/a=0.5 and $\alpha = 0.2$

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Figure 7 Effect of boundary conditions (SSSS, CSCS, SCSC, CCCC) on instability region of the curved panel for a/b=1, $a/R_x = 0.0, b/R_y = 0.25$, c/a=0.3 and $\alpha = 0.2$

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