COMPARISON OF VPMM, VPMC AND MULTILINEAR MUSKINGUM METHOD APPLIED TO COMPOUND CHANNELS

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ABSTRACT

The VPMM (Variable parameter McCarthy Muskingum) model is derived directly from the Saint Venant equations applied to a gradually varied 1-D unsteady flow in rigid bed channels without considering lateral inflow or outflow. In the VPMM method, the model parameters are determined from the flow and channel characteristics which changes at every time step. The Muskingum-Cunge is a viable alternative to the classical Muskingum method, particularly for the cases where hydrologic data (i.e. streamflow data) are not available, but where hydraulic data (cross-sectional data and channel slopes) can be readily ascertained. The Muskingum-Cunge method matches the numerical diffusion of the discrete model with the physical diffusion of the analytical model. Variable parameter Muskingum-Cunge (VPMC) method and its modified versions conserve more mass than the classical Muskingum-Cunge method. Multilinear Muskingum method is based on time distribution scheme. The methods are being reviewed with varying parameters at each routing time step unlike the then existing multilinear models. The paper reviews VPMM, VPMC and Multilinear Muskingum method, and their applications to some study areas, with their applicability conditions. The study concludes by analyzing their improvements and modifications to the existing models.

Keywords: Flood Routing, VPMM, Variable Parameter Muskingum-Cunge, Multilinear Muskingum method.

1. INTRODUCTION

Flood routing methods are for tracking course of a flood wave in a water body by mathematical means from an upstream point to a downstream point. Over the years simplifying the computation of complex routing methods has been an aspect of great interest for researchers. Flood routing in developing countries like India is a bit difficult due to inadequate number of gauging sites. Hence, simplified methods like variable parameter Muskingum-Cunge model is widely used. However it has mass conservation problem (Ponce and Chaganti, 1994; Perumal and Sahoo, 2007, 2008). To overcome this limitation Perumal (1994 a, b) devised a model using basic Saint Venant equation for routing discharge in prismatic channels with semi-infinite rigid bed. This method is known as the variable parameter Muskingum discharge (VPMD) method. With a similar concept variable parameter Muskingham stage (VPMS) method was developed (Perumal and Ranga Raju, 1998 a, b). Both of these methods perform better than VPMC but still show limitations in mass conservation. Recently, Perumal and Price (2013) developed a physically based routing method. This method is derived from Saint Venant equation and is known as Variable parameter McCarthy Muskingum (VPMM) method. In this method, cross-sectional details of two end sections of the river reach is taken into account. This method conserves mass successfully. VPMM expresses McCarthy's (1938) concept of prism and wedge storages. Two parameters namely, Travel time parameter (K) and weighted parameter (θ) are used and are computed using channel and flow characteristics. Muskingum cunge method is widely used and well established method. In this method the routing parameters are function of grid specifications and channel properties but the results are independent of grid size. The Muskingum –Cunge method is a good alternative to traditional Muskingum method specially where streamflow data is unavailable and cross-sectional data and channel slopes can be used. Most of the models are either models based on solutions of full Saint Venant equation, or their simplifications in the form of non-inertia wave model, kinematic wave model or the approximate convection-diffusion wave model. Ferrick (1985), after analyzing wave types, suggested that use of full Saint Venant equations may not result in accurate wave simulations for all wave types. This argument led to the use of more simplified method like variable parameter Muskingum-Cunge method (VPMC) and its variants. Keefer and McQuivey (1974) pointed out that the assumption that flow variations around a reference discharge used for estimating model parameters is small; is violated when model is linearized about a high discharge, low flows arrive soon and are over damped. And if linearized about low discharge, the peak are late and under-damped. To overcome this multiple input linear models were used, also known as multilinear models. Number of models have been proposed by Keefer and McQuivey (1974), Becker (1976), Kundzewicz (1984) and have attempted to account for non- linear effects of flood wave. Such models though have a few limitations, they can't account for backwater effect because of tides and tributary flows. There were irregularities noted in the peak regions of the hydrograph. Also, there was subjectivity of choosing the number of flow zones for routing of the inflow hydrograph. M. Perumal (1992) presented about Multilinear Muskingum method overcoming these limitations of the then existing multilinear models. The framework of the multilinear method is based on Muskingum method which is used as linear sub-model.

In this paper we have revisited the variable parameter McCarthy Muskingum method, Multilinear Muskingum method, Muskingum cunge method and its suitable modification to these are suggested.

2. THEORITICAL BACKGROUND

2.1 VPMC AND MVPMC ROUTING METHOD

VPMC is a variant of Muskingum-Cunge method which was developed from Muskingum method by Cunge (1969). The routing parameters of Muskingum Cunge method are expressed as Courant and cell Reynolds numbers, by Ponce and Yevjevich (1978). They also developed a model for calculating these parameters. Being nonlinear and calculable, this model was applicable to real world routing problems. A three point method and an iterative four point method were suggested to vary the two variables as function of flow parameters.

Acc. to Ponce and Chaganti (1994) the routing equations of Muskingum Cunge method in four point grid configuration is –

$$Q_{j+1}^{n+1} = C_0 Q_j^{n+1} + C_1 Q_j^{n} + C_2 Q_{j+1}^{n}$$

Where n is temporal index and j is spatial index.

C_0 = (-1+C+D) / (1+C+D)

C_1 = (1+C-D) / (1+C+D)

C_2 = (1-C+D) / (1+C+D)

And routing coefficients are defined as –

C = c (Δt/Δx), D = q / (S_oCΔx)

Where C = Courant number, c = celerity, Δt is time interval, Δx is space interval, D = cell Reynolds number, q = unit width discharge, S_o = bottom slope.

Wave celerity (c) is defined as –

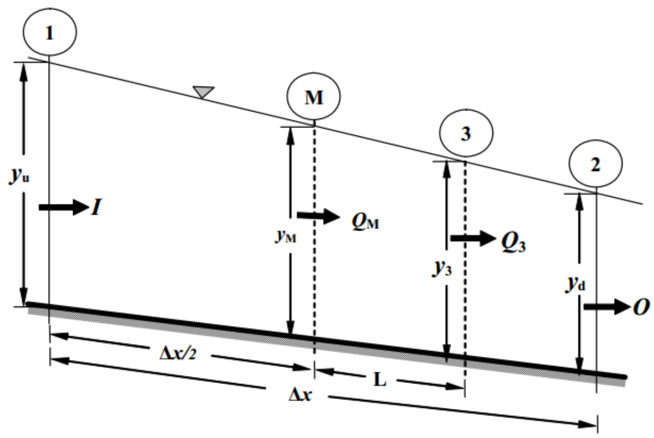
c = β (Q/A) = β (q/d).

β = exponent of rating, A = area of flow, d = flow depth.

The results are independent of grid specification, provided there is minimum numerical dispersion. This can be ensured by keeping Courant number slightly greater than or equal to one. (Cunge, 1969; Ponce and Theurer, 1982; Ferrick et al., 1983)

2.2 VPMM ROUTING METHOD

A simple way of developing a fully mass conserving and physically based variable parameter Muskingum method, taking into account storage concept of McCarthy(1938) , was proposed by Perumal and Price (2013). This method is derived from Saint Venant equations. In VPMM method it is hypothesized that for a steady flow with any shape of prismatic cross-section, the cross-sectional area of flow at one point is uniquely related to discharge at that point, thus defining the steady flow rating curve. However in unsteady flow this relationship is between the stage and corresponding steady discharge at any given instant of time recorded at the downstream section, preceding to the corresponding midsection (steady stage section) of the routing reach. The sketch of routing reach of VPMM method is shown below –



The routing parameters (K and θ) for VPMM are expressed as –

$K = \frac{\Delta x}{v_M}$, $\theta = 0.5 - \frac{L}{2\Delta x}$

Where, v_M = steady flow velocity at mid-section.

The distance from mid-section (L) at which steady discharge occurs is given as-

$$L = \frac{Q_M}{S_0 B_M C_M}\left[1 - \frac{4{F_M}^2}{9}\left(\frac{P}{B}\frac{dR}{dy}\right)_M^2\right]$$

Where, Q_M = steady discharge at mid section (M). y = stage corresponding to steady discharge at mid-section (M). S_0 = channel bed slope. B_M = Top width of water surface. F_M = Froude's number, P = wetted perimeter, R = hydraulic radus, C_M = celerity at mid-section of sub-reach corresponding to stage (y).

The classical Muskingum equation thus derived from VPMM is –

$$Q_{i+1}^{n+1} = C_1 Q_i^{n+1} + C_2 Q_i^n + C_3 Q_{i+1}^n$$

Where, $C_1 = \frac{\Delta t - 2k^{n+1}\theta^{n+1}}{\Delta t + 2k^{n+1}(1-\theta^{n+1})}$, $C_2 = \frac{\Delta t + 2k^n \theta^n}{\Delta t + 2k^{n+1}(1-\theta^{n+1})}$, $C_3 = \frac{-\Delta t + 2k^n(1-\theta^n)}{\Delta t + 2k^{n+1}(1-\theta^{n+1})}$

Where, Q_{i+1}^{n+1} = Discharge at section i+1 at time step n+1.similarly for other variations of Q at sections i and i+1 at time steps n and n+1. It may be noted that unlike VPMC, VPMM does not consider concept of matching numerical diffusion with physical diffusion.

2.3 MULTILINEAR MUSKINGUM FLOOD ROUTING METHOD

Multilinear models of flood wave propogation based on time distribution scheme have some limitations, by incorporating the linear sub model of muskingum, equation an attempt was to overcome them by Perumal(1992). In the proposed method the parameters vary at each routing step, this eliminates the discussion of choosing and deciding the no of flow zones to be used for routing the inflow hydrograph.

In this multilinear model the inflow is divided into blocks of constant duration equal to the routing time interval. Each of these blocks is then routed through muskingum model. This method eliminates the need to decide on no of flow zones and irregularities noted in peak reagions of the hydrograph after simlation.

The equation of Muskingum method used is –

$S = K[\Phi I + \Phi(1-x)]$

Where , S = storage at time t, I= inflow at time t, O = Outflow at time t. K = travel time and x = weighting perimeter.

Where , $K = \Delta x/c$, $\Phi = 0.5 - (Q_0/2Bc\Delta x)$

Δx = reach length, c = celerity corresponding to Q_0 , B= channel width, S_0 = bed slope.

Also,c = 1.67v_0 where, v_0 = velocity corresponding to Q_0. And, $Q_0 = aI$, I = current value of inflow and a = coefficient following $0<a<1$. The discrete unit hydrographs are evaluated on the classical muskingum equations of discharge(Q) with coefficients C_1,C_2,C_3 .

To calculate the ordinates of these discrete unit hydrographs, we can use these equations –

1) $h_1 = C_1$
2) $h_2 = C_2 + C_3 . C_1$
3) $h_3 = C_3(C_2 + C_3 . C_1)$
4) $h_n = C_3^{n-2}(C_2 + C_3 . C_1)$

3. STUDY AREA

3.1 VPMC AND MVPMC ROUTING METHOD

For numerical experiments, Ponce and Chaganti (1994) tested variable parameter Muskingum –Cunge method on Thomas (1934) classical problem. The problem was routing a sinusoidal wave through a prismatic channel 500 miles long. The wave had a 96 hr. period, unit width, baseflow (q_b) was taken as 4.64m^2/s, peak inflow (q_{pi}) was taken as 18.58 m^2/s, bed slope assumed to be 1 ft. /mile, Manning's n was considered 0.0297. Discharge – depth ratio was expressed as, $q = 0.688d^{5/3}$.

Three values of (q_b / q_{pi}) were used – 4, 10, 20.

Two different temporal and spatial set of values in two resolution levels were used by Thomas (1934)-

1) Δx = 25 miles (40.22 km), Δt = 6h.
2) Δx = 12.5 miles (20.11 km), Δt = 3h.

The methods considered for the experiment are as follows –

1) The constant parameter method (CPMC). In this method routing parameters are same for all computations. The avg. discharge (q_a) is used to calculate avg. celerity (c_a).
$$q_a = (q_b + q_{pi})/2$$
2) The three-point variable parameter method (VPMC3), in which routing parameters, C and D are based on celerity and avg. unit width discharge at the three grid points.
$$q_a = (q_j^n + q_{j+1}^n + q_j^{n+1})/3$$
$$c_a = (c_j^n + c_{j+1}^n + c_j^{n+1})/3$$
3) The iterative four-point variable-parameter method (VPMC4), in which routing parameters for each computational cell are based on q_a, c_a on the four grid points.
$$q_a = (q_j^n + q_{j+1}^n + q_j^{n+1} + q_{j+1}^{n+1})/4$$
$$c_a = (c_j^n + c_{j+1}^n + c_j^{n+1} + c_{j+1}^{n+1})/4$$
For improving convergence, c_{j+1}^{n+1} is calculated using q_{j+1}^{n+1}.
4) A modified three – point variable parameter method (MVPMC3), the difference from its previous version is that average celerity is calculated from the discharge computed as —
$$q_a = (q_j^n + q_{j+1}^n + q_j^{n+1})/3$$
5) A modified iterative four-point variable parameter method (MVPMC4) in which routing parameters C and D are based on a q_a at four grid points.
$$q_a = (q_j^n + q_{j+1}^n + q_j^{n+1} + q_{j+1}^{n+1})/4$$

The avg. celerity is computed from the discharge calculated above.

3. 2 VPMM ROUTING METHOD

For numerical experiments ,all three shapes of uniform prismatic channel used by Todni(2007) are used(as shown in fig. below). This includes a rectangular channel with bed width of 50m, a trapezoidal channel with B=15m and a triangular channel, both with slope ratio 1:5(V:H).

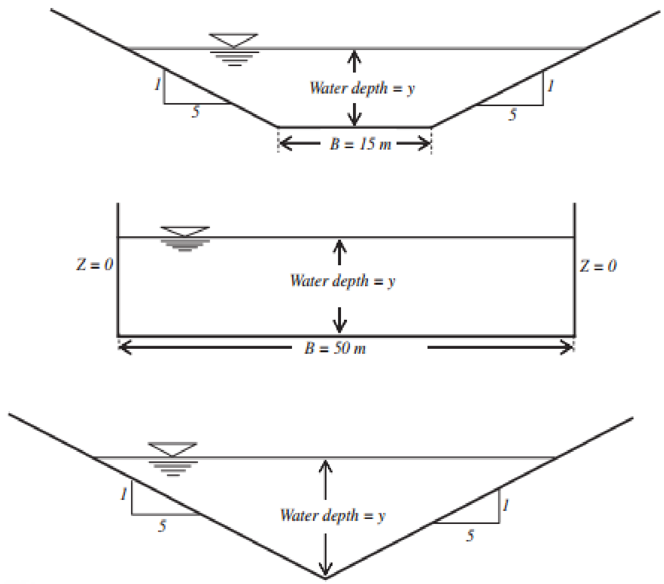


Fig. - prismatic cross-sectional shapes of routing reaches in the numerical experiments. (M. Perumal, 2013)

The VPMM method is routed at 100km from the inflow section. The reach is further divided in to 8 sub reaches ($\Delta x = 12.5$) and 50 sub reaches ($\Delta x = 2$). The inflow hydrograph is expressed as

$$I(x = 0, t) = I_b + (I_p - I_b)\left[\frac{t}{T}\, exp\left(1 - \frac{t}{T}\right)\right]^{\beta}$$

where, I_b = initial steady flow (100m^3/s), I_p = Peak flow(900m^3/s), T = time to peak flow(24hr), β=16(shapefactor).

3.3 MULTILINEAR MUSKINGUM METHOD

Perumal adopted the inflow hydrograph expresed as this equation –

$$\mathrm{I} = \mathrm{I_b} + (\mathrm{I_p} - \mathrm{I_b})\left(\frac{\mathrm{t}}{\mathrm{t_p}}\right)^{1/(\lambda-1)} \exp\left[\frac{1 - \mathrm{t}/\mathrm{t_p}}{\lambda - 1}\right]$$

the inflow hydrograph was defined by a four - parameter Pearson Type III distribution. I_b = initial steady flow (100m3/s) , I_p = peak flow (1000 m3/s) , t_p = time to peak (10h) , λ = skewness factor (1.15).

A channel with 50m width was used. The hydrograph was routed 40 km from the inflow section. Value of the coefficient a is used as 0.4 for computing reference discharge.

Table 1 : Channel configurations

Channel type	Bed slope	n value
1	0.0002	0.04
2	0.0002	0.02
3	0.002	0.04
4	0.002	0.02

4. DISCUSSION

VPMC AND MVPMC ROUTING METHOD

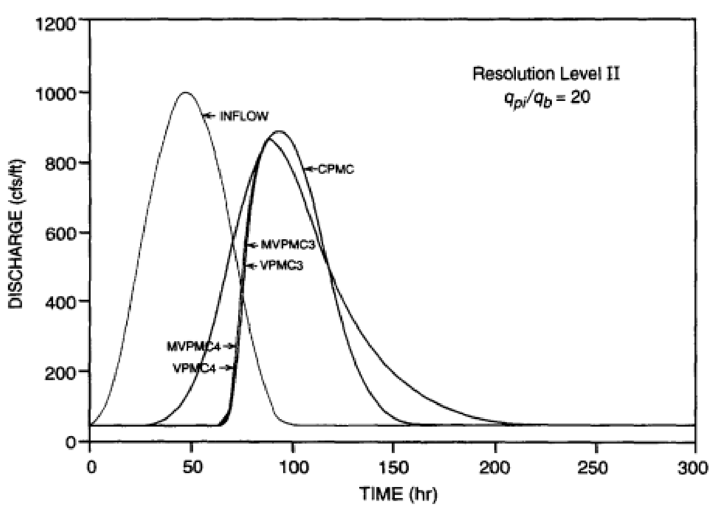


Fig – outflow hydrographs for resolution level II and peak inflow to baseflow ratio, $q_{pi}/q_b = 20$. (Ponce and Chaganti 1994).

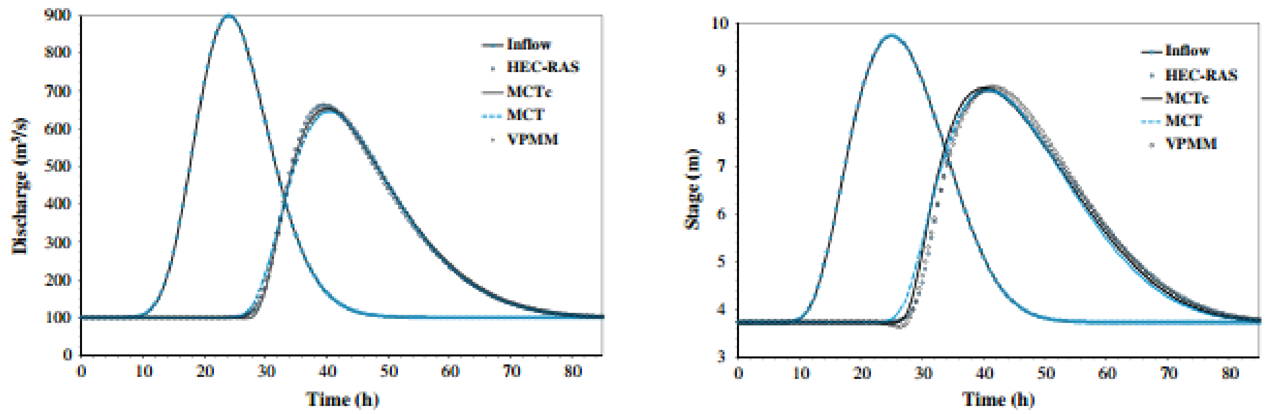
It is seen that CPMC method lacks the nonlinear tendency as there is steepening of the rising limb. We also noted that all the variable parameter methods had steepening of the rising limb followed along with flattening of the receding limb. The CPMC method conserves mass exactly while variable parameter methods are susceptible to loss of mass, though it is small but perceptible. We also saw that ratio of peak outflow to peak inflow is almost same for all peak inflow to baseflow ratios. Which suggests that wave diffusion is independent of peak inflow to baseflow ratio. It is also seen that loss of mass for three point method is greater than four point method. Also, modified methods conserve more mass than conventional VPMC methods.

VPMM ROUTING METHOD

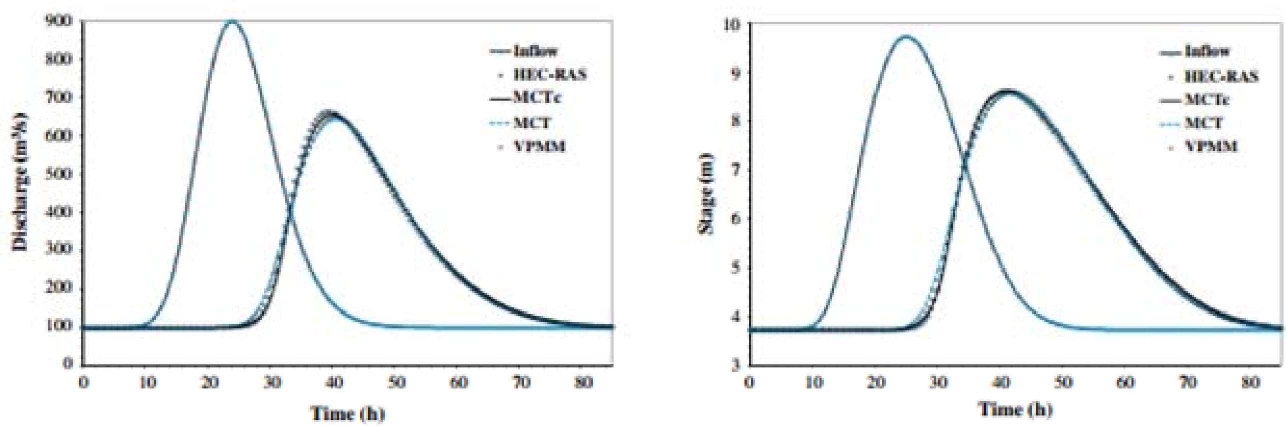
The results I rectangular and triangular channels are qualitatively similar to that of trapezoidal channel. This shows that VPMM method is fully mass conservative and is able to reproduce benchmark solutions.

Following is the comparison in VPMM , MCT(Muskingum-Cunge-Todni), CT_c in discharge and stage hydrographs (for Δx=12.5 km , Δx = 2 km) –

1) Δx=12.5 , Reproduction of the benchmark discharge hydrograph by the VPMM, MCT and MCT_c methods corrseponding to 8 sub reaches.(M. Perumal , 2013)



2) Δx=2 , Reproduction of the benchmark discharge hydrograph by the VPMM, MCT and MCT_c methods corrseponding to 50 sub reaches.(M. Perumal , 2013)



In the discharge hydrograph with Δx = 12.5 km, we can see that both VPMM and MCT_c produce solutions very close to the benchmark solutions of HEC-RAS, which is a river analysis system. However, it was observed that both VPMM and MCT_c performed better than MCT. In the stage-time hydrograph it was observed that VPMM was able to reproduce benchmark stage hydrograph more accurate than MCT and MCT_c methods. The hydrograph of MCT method deviated significantly w.r.t the rising part of the benchmark hydrograph.

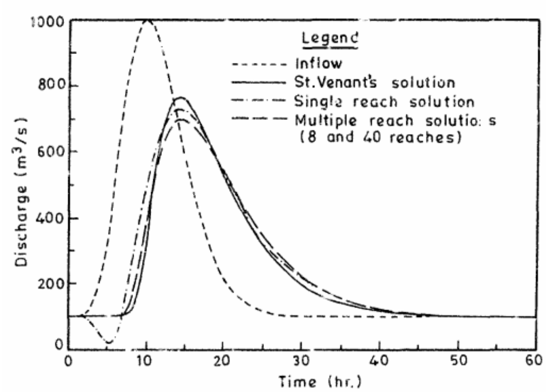
When Δx = 2 km, in discharge hydrograph , MCT_c method reproduces solution more accurate than VPMM , while VPMM performs marginally better than MCT. In the stage hydrograph, MCT_c is observed to produce better simulations than VPMM and MCT. However , VPMM is able to reproduce the rising part of the benchmark hydrogrpah more accurately than MCT methods.

We can draw the inference from the above graphs that VPMM, MCT, MCT_c(MCT with Cappelaere correction) all produce discharge and stage hydrographs very close to the benchmark solutions of HEC-RAS.

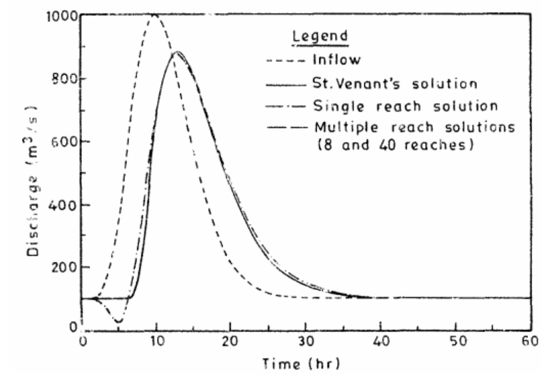
Thus, the paper have successfully established VPMM method as a full mass conservative flood method, derived from Saint Venant equations for flood routing of prismatic channels of any shape of cross-section. VPMM method gives stage hydrograph for corresponding discharge hydrograph. Also, the approach of the method justifies the heuristic assumptions made by McCarthy (1938) regarding the storage of channel reach expressed as prism and wedge storage. The results of study reveal that VPMM method provides better insight into Muskingum method and is recommended for routing floods in river channels and floodplains.

MULTILINEAR MUSKINGUM METHOD

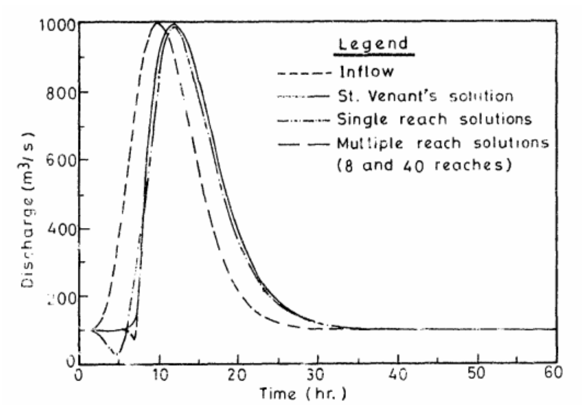
Following are the few routing result at 40km of diff. channel types (Perumal M.,1992) –



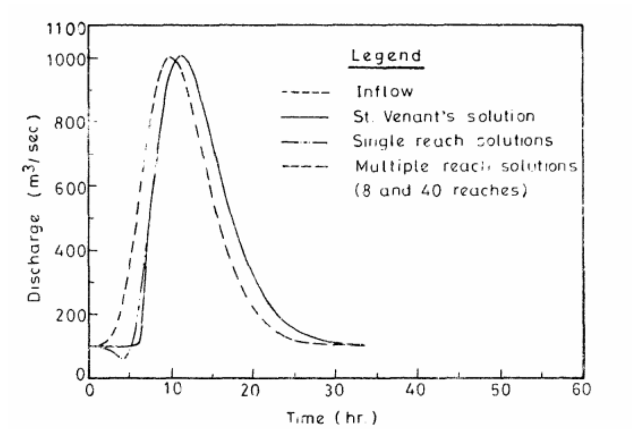
Channel type 1



Channel type 2



Channel type 3



Channel type 4

We can see from the graphs of channel type 1 that Saint Venant equations are better reproduced by multiple reach routing rather than single reach routing. Same conclusion can be made for other channel types too. Although overall results of channel type 2 are better than that of 1, while that of channel type 3 and 4 are better than them. The reproduction of Saint Venant equations is seen better in channel type 3 and 4. It was also observed that when no. of sub-divisions of the channel reach increasses, magnitude of reduced outflow is minimized (Harley, 1967).

It was concluded that this multilinear model can account for non-linearity in flood wave movement better than the existing multilinear methods. The method eliminates the subjectivity involved in deciding flow ranges to further divide inflow hydrographs into zones. Variation of travel time (K) and weighting parameter (Φ) are were also noted for hydrographs with wide loop rating curves. The principle of conservation of volume holds good for the described method.

CONCLUSION

From the present study, review of the three important Muskingum methods i.e. VPMM, VPMC and Multilinear Muskingum method has been done and necessary improvements have been suggested. The following conclusions are made.

1. In VPMC, loss of mass for four point method is least among the other modified VPMC methods. The modified methods conserve more mass than conventional VPMC methods, and thus are desired.

2. In VPMC and MVMPC methods, ratio of peak outflow to peak inflow is almost same for all peak inflow to baseflow ratios.

3. In VPMM, the approach of the method justifies the storage of channel reach expressed as prism and wedge storage.

4. VPMM is fully mass conserved flood routing method and perform at par with the benchmark methods and determines the stage hydrograph corresponding to the routed discharge hydrograph. It provides better insight into Muskingum method and is recommended for routing floods in river channels and floodplains.

5. The principle of conservation of volume holds good for the Multilinear model.

6. This Multilinear model can account for non-linearity in flood wave movement better than the then existing multilinear methods. The method eliminates the subjectivity involved in deciding flow ranges to further divide inflow hydrographs into zones.

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