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Parametric Instability of Composite Curved Panel Subjected to Concentrated Edge Loading

S. K. Sahu, *Department of Civil Engineering. REC, Rourkela,Orissa-769008,*
email: sksahu@nitrkl.ac.in

P. K. Datta, *Department of Aerospace Engineering, I.I.T., Kharagpur-721302*
email- pkdatta@aero.iitkgp.ernet.in

Abstract -The parametric resonance characteristics of laminated composite curved panels subjected to various in-plane periodic concentrated edge loadings are studied using finite element analysis. The first order shear deformation theory is used to model the curved panels, considering the effects of transverse shear deformation and rotary inertia. The effects of number of layers, static load factor, boundary conditions and ply orientations for different positions of loading on the principal instability regions of curved panels are studied in detail using Bolotin's method. Quantitative results are presented to show the effects of shell geometry, lamination details and load parameters on the stability boundaries.

Key words-Parametric resonance, composite, concentrated loading, instability region

1. Introduction

Structural elements subjected to in-plane periodic forces may lead to parametric resonance, due to certain combinations of the values of load parameters. The instability may occur below the critical load of the structure under compressive loads over wide ranges of excitation frequencies. Thus the parametric resonance characteristics are of great importance for understanding the dynamic systems under periodic loads. Parametric resonance in shell structures under periodic loads has been of considerable interest since the subject was more elaborately introduced by Bolotin (1964) and Yao (1965). The parametric instability of thick orthotropic cylindrical shells was studied analytically by Bert and Birman (1988). The dynamic instability of composite simply-supported circular cylindrical shell was analysed by the Method of Multiple Scale (MMS) by Cederbaum (1992). A perturbation technique was employed by Argento and Scott (1993) to study the instability regions subjected to axial and torsional loading. The dynamic instability of laminated composite circular cylindrical shells was studied by Ganapathi and Balamurugan (1998) using a C^0 shear flexible two nodded axi-symmetric shell element. The dynamic stability of cross-ply laminated composite cylindrical shells under combined static and periodic axial force was investigated by Ng, Lam and Reddy (1998) using Love's classical theory of thin shells. Most of the above mentioned investigators studied the dynamic stability of uniformly loaded closed cylindrical shells with a simply supported boundary condition. The practical importance of stability analysis of curved panels/open shells has been increased in structural, aerospace (skin panels in wings, fuselage etc.), submarine hulls and mechanical applications. The vibration and buckling stresses were studied for cylindrical panels [(Baharlou and Leissa 1987), (Ye and Soldatos 1995)]. The study of the parametric instability behaviour of curved panels is new. Recently the effects of curvature and aspect ratio on dynamic instability for a uniformly loaded simply-supported laminated composite thick cylindrical panel were studied by Ganapathi *et al*.(1994). But the applied load and boundaries are seldom uniform in practice. The application of non-uniform loading and boundary conditions on the structural component will alter the global quantities such as free vibration frequency, buckling load and dynamic instability region(DIR). The buckling characteristics of flat panel [(Leissa and Ayub 1988), (Leissa and Ayub 1989)] and closed cylindrical shell [(Libai and Durban 1977), (Durban and Ore 1999)] due to concentrated loading were investigated.

In the present study, the parametric instability characteristics of curved composite panels subjected to various in-plane concentrated loads are investigated. The influences of various parameters like effects of number of layers, static & dynamic load factors, various boundary condition, ply orientations for different positions of loads on the instability behaviour of curved panels have been examined. The present formulation of the problem is made general to accommodate a curved panel with finite curvatures in both the directions having arbitrary load and boundary conditions.

2. Theory and Formulations

The basic configuration of the problem considered here is a doubly curved panel subjected to various forms of concentrated in-plane edge loading and is shown in Fig.1.

Governing equations

The equation of equilibrium for free vibration of a structure subjected to periodic loads can be expressed in the form:

$$[M]\{\ddot{q}\}+[[K_e]-P[K_g]]\{q\}=0 \qquad (1)$$

The in-plane load P is periodic and can be expressed in the form:

$$P(t)=P_s+P_t Cos\Omega t \qquad (2)$$

where P_s is the static portion of load P(t). P_t is the amplitude and Ω is the frequency of the dynamic portion of P(t). The stress distribution in the plate will be non-uniform and periodic . Taking

$$P_s=\alpha P_{cr}\ ,\ \ P_t=\beta P_{cr} \qquad (3)$$

and using Eq. (3), the equation of motion in matrix form is obtained as:

$$[M]\{\ddot{q}\}+[[K_e]-\alpha P_{cr}[K_g]-\beta P_{cr}[K_g]Cos\Omega t]\{q\}=0 \qquad (4)$$

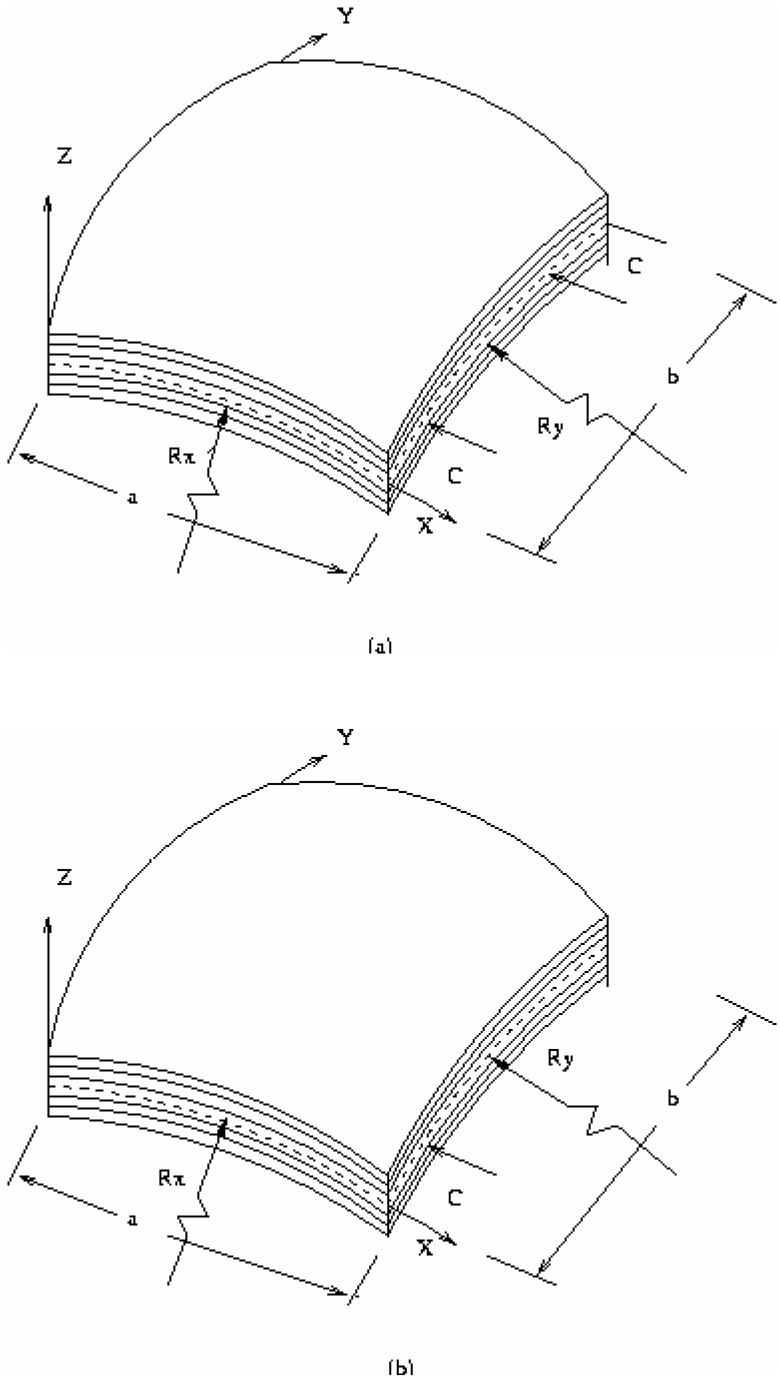

Fig. 1 Geometry and co-ordinate systems of doubly curved panel subjected to concentrated loads: a) load from both ends, b) load from one end.

Eq.(4) represents a system of second order differential equations with periodic coefficients of the Mathieu-Hill type. The boundaries of the dynamic instability regions are formed by the periodic solutions of period T and 2T, where $T=2\pi/\omega$. The boundaries of the primary instability regions with period 2T are of practical importance [Bolotin 1964] and the solution can be achieved in the form of the trigonometric series

$$q(t) = \sum_{k=1,3,5,..}^{\infty} [\{a_k\} Sin(k\theta t/2) + \{b_k\} Cos(k\theta t/2)] \qquad (5)$$

Putting this in Eq.(4) and if only first term of the series is considered, equating coefficients of Sin θt/2 and Cos θt/2 , the equation (4) reduces to

$$[[K_e] - \alpha P_{cr}[K_g] \pm \frac{1}{2}\beta P_{cr}[K_g] - \frac{\Omega^2}{4}[M]]\{q\} = 0 \qquad (6)$$

Eq.(6) represents an eigenvalue problem for known values of α,β and P_{cr}. The two conditions under a plus and minus sign correspond to two boundaries of the dynamic instability region (DIR). The eigenvalues are Ω, which give the boundary frequencies of the instability regions for given values of α and β. In this analysis, the computed static buckling load of the panel is considered as the reference load. An eight nodded curved isoparametric element is employed in the present analysis with five degrees of freedom u, v, w, θ_x and θ_y per node. First order shear deformation theory is employed and the displacement field assumes that mid-plane normal remains straight before and after deformation, but not necessarily normal after deformation, so that

$$\bar{u}(x,y,z) = u(x,y) + z\theta_x(x,y)$$
$$\bar{v}(x,y,z) = v(x,y) + z\theta_y(x,y)$$
$$\bar{w}(x,y,z) = w(x,y)$$

where,

$\bar{u}$, $\bar{v}$, $\bar{w}$ and *u*, *v* and *w* are displacement components in the x, y, z directions at any point and at the mid surface respectively. The constitutive relationships for the shell becomes:

$$\begin{Bmatrix} N_i \\ M_i \\ Q_i \end{Bmatrix} = \begin{bmatrix} A_{ij} ....B_{ij}....0 \\ B_{ij}....D_{ij}....0 \\ 0......0....S_{ij} \end{bmatrix} \begin{Bmatrix} \varepsilon_j \\ k_j \\ \gamma_m \end{Bmatrix}$$

3. Results and Discussions

Numerical results are presented for composite curved panels with different combinations of boundary conditions. S and C denote a simply-supported and clamped edge respectively. The notation SCSC identifies a curved panel with the edges simply supported at x=0, a and clamped at y=0, b. The following boundary conditions for simply supported (S), clamped (C) are assumed for the first order shear deformation theory:

Cross-ply

S: v=w= θ_y =0 at x=0,a and u=w= θ_x =0 at y=0, b

C: u=v=w= $\theta_x = \theta_y$ =0 at any edge

Angle-ply
S: u=w= θy=0 at x=0, a and v=w=θx=0 at y=0,b
C: u=v=w=θx=θy=0 at any edge.

Convergence study

The convergence studies are made for non-dimensional fundamental frequencies of vibration of doubly curved shells and are omitted for sake of brevity. From the above convergence study, 10 ×10 mesh has been employed to idealise the panel in the subsequent analysis. The present formulation is validated for free vibration analysis of a composite curved panels. The results are presented in Table 1, showing good comparison with the literature. To validate the formulation further, the buckling results of singly curved shells/panels, subjected to uniform loading are compared in Table 2 with those available in the literature. The above studies indicate good agreement between the present study and those from the literature. Once the free vibration and buckling results are validated, the dynamic instability studies are made.

Table 1 Non-dimensional fundamental frequencies for the various simply supported spherical shells/panels.
a/b=1, h/ R_x =1/500, a/h=100, E_{11}/E_{22} =25, G_{23} =0.2 E_{22} , ν_{12} = 0.25
Non dimensional frequency, $\varpi = \omega a^2 \sqrt{(\rho / E_{22} h^2)}$

Laminations	Non-dimensional frequencies	
	Reddy (1984)	Present FEM
0/90/0	30.963	30.9940
$(0/90)_{\bar{s}}$	31.050	31.0793
0/45/0	41.748	41.6659
$(0/45)_s$	45.535	45.3005
0/90	28.778	28.8255
$(0/90)_2$	30.506	30.5368
45/-45	49.289	49.1690
$(45/-45)_2$	58.94	58.8263

Table 2 Non-dimensional buckling loads for the simply supported singly curved cylindrical panel. a=0.25m, b=0.25m, h=2.5mm,

$a/R_x = 0, E_{11} = 2.07\times10^{11} N/mm^2, E_{22} = 5.2\times10^{09} N/mm^2, G_{12} = 2.7\times10^{9} N/mm^2, \nu_{12} = 0.25$

Non-dimensional buckling load, $\lambda = N_x b^2 / E_{22} h^3$

b/R_y	Orientation	Non-dimensional buckling loads	
		Present FEM	Baharlou and Leissa (1987)
0	90/0	12.63	12.63
	0/90	12.63	12.63
0.1	90/0	17.629	17.51
	0/90	17.612	17.49
0.2	90/0	32.565	32.06
	0/90	32.5027	32.17
0.3	90/0	57.28	56.28
	0/90	57.117	56.62

Parametric instability studies

The parametric instability regions are plotted for a uniaxially loaded curved panels with/without static component to consider the effect of various parameters. The geometrical and material properties of the simply supported cylindrical panel are:

a=b=500 mm, h=2 mm, Ry=2000 mm, ρ=1580kg/m ^{3}.

$E_{11} = 141.0 Gpa, E_{22} = 9.23 Gpa, G_{12} = G_{13} = 5.95 Gpa, G_{23} = 2.96 Gpa, \nu_{12} = 0.313$

The non-dimensional excitation frequency $\Omega = \overline{\Omega} a^2 \sqrt{(\rho / E_{22} h^2)}$ is used throughout the dynamic instability studies, (unless otherwise mentioned) where $\overline{\Omega}$ is the excitation frequency in radian/second. The dynamic instability regions have been plotted for simply-supported 2, 4, 8 and 12 layer anti-symmetric angle-ply curved panel. As shown in Fig.2, the excitation frequency increase along with narrow width of instability regions with increase of number of layers. It is seen that the effect of number of layers does not vary much beyond 8 layers and the further parametric studies are carried out for 8 layer shell. The computed buckling load of this simply-supported eight layer anti-symmetric angle ply panel is taken as the reference load for all further computations. The effect of static component of load for α=0.0, 0.2, 0.4 and 0.6 on the instability region is shown in Fig.3. Due to increase of static component, the instability regions tend to shift to lower frequencies and become wider. The effect of lamination angle has been studied for uniform loading with static component. As observed in Fig.4, the instability region is greatly influenced by the ply orientation. The instability regions are smaller and starts at higher frequencies with greater the lamination angle for this range of thickness ratio b/h and material properties for uniform loaded shell.

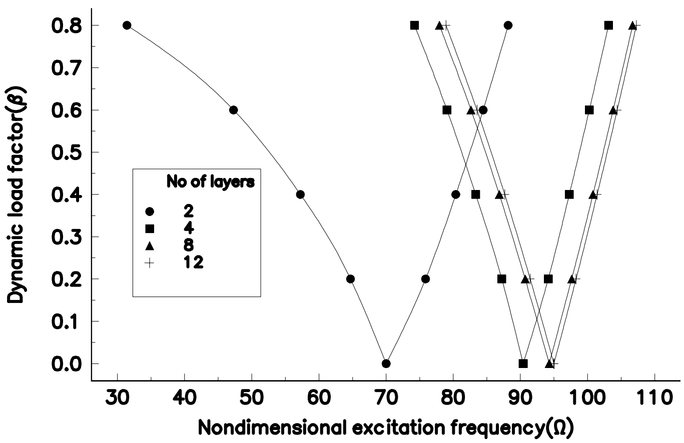

Fig.2 Effect of number of layers on instability region of a uniform loaded simply-supported anti-symmetric angle-ply $\left(\pm 45^0\right)_4$ shell: a=b=0.5m, h=2mm, a/R_x =0.0, b/R_y =0.25, α=0.2, N=2, 4, 8, 12

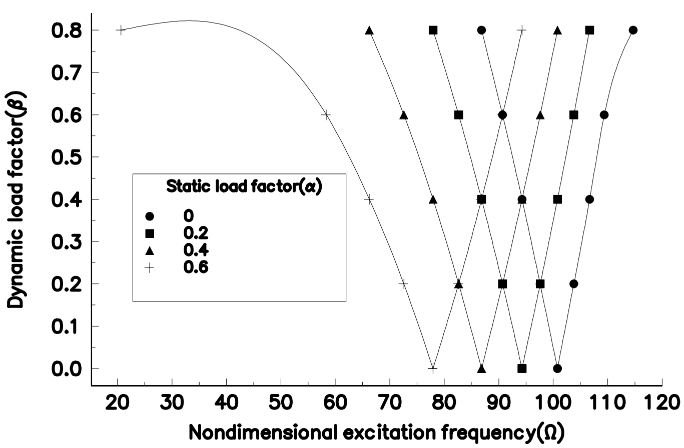

Fig.3 Effect of static load factor on instability region of a uniform loaded simply-supported anti-symmetric angle-ply $\left(\pm 45^0\right)_4$ shell: a=b=0.5m, h=2mm, a/R_x =0.0, b/R_y =0.25, α=0.0, 0.2, 0.4, 0.6

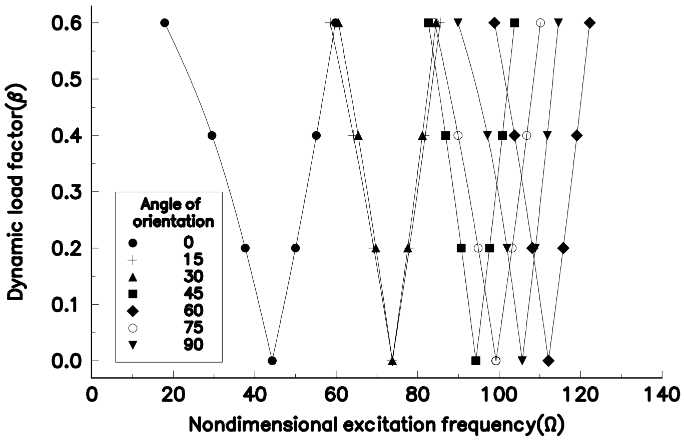

Fig.4 Effect of different ply orientations on instability region of a uniform loaded simply-supported anti-symmetric angle-ply $(\pm\theta^0)_4$ cylindrical panel: a=b=0.5m, h= 2mm, $a/R_x = 0.0, b/R_y = 0.25, \alpha = 0.2$

The ply orientation of 60^0 seems to be a preferential orientation for this type of panel. For this orientation, the instability region shifts to higher frequencies and becomes narrower in comparison with other orientations. Similar trends are also observed by Loy *et al.* (1999) based on a study on free vibration characteristics of shells. The investigation has been extended to instability behaviour under various concentrated loadings. For this, various concentrated edge loadings are considered. The instability behaviour is also affected by the positions of concentrated loading. As can be seen from Fig.5, the instability in general occurs at higher excitation frequencies with increase of distance from the edges (c/b) for types of load for this panel. The panel with a double pair of concentrated loading near the edges shows higher stiffness than a single pair of concentrated loads. The influence of different boundaries on the instability regions of the curved panel under concentrated loading (load case1(a), c/b=0) is shown in Fig.6. The boundaries considered are: simply supported on all sides(SSSS), simply supported on curved surfaces and clamped boundary conditions on straight edges (SCSC), clamped on curved edges & simply-supported on straight edges (CSCS), all sides clamped (CCCC) on the principal instability regions. As expected the excitation frequencies increase from simply supported to clamped edges due to the restraint at the edges. The curved panels supported under CSCS shows more stiffness than the same panel under SCSC condition.The effect of ply orientation has been studied for the curved panel under concentrated loading (load case 1(a), c/b=0.1) with static component.

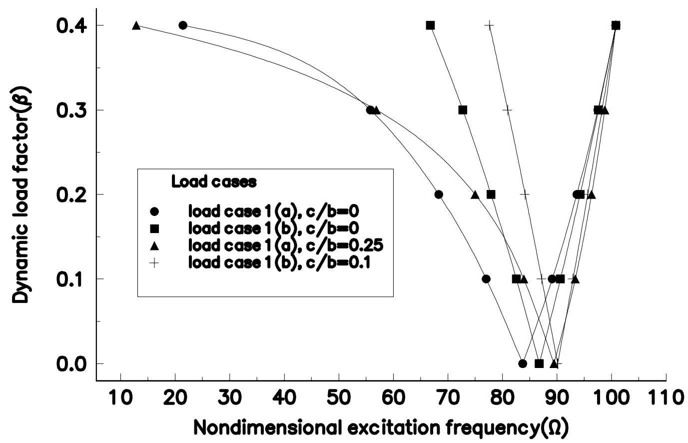

Fig.5 Effect of position of concentrated load on instability region of a simply-supported cylindrical anti-symmetric angle-ply $(\pm 45^0)_4$ shell, a=b=0.5m, h=2mm, a/R_x=0.0 b/R_y=0.25 and Load case 1(a), c/b=0, 0.25 and load case 1(b), c/b=0.0, 0.1, α=0.2

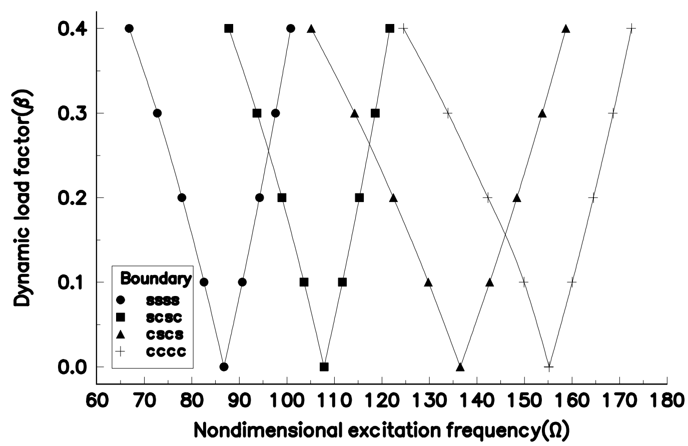

Fig.6 Effect of boundary conditions (SSSS, SCSC, CSCS, CCCC) on instability region of the curved panel for a=b=0.5m, h=2mm, a/R_x=0.0, b/R_y=0.25 and α=0.2

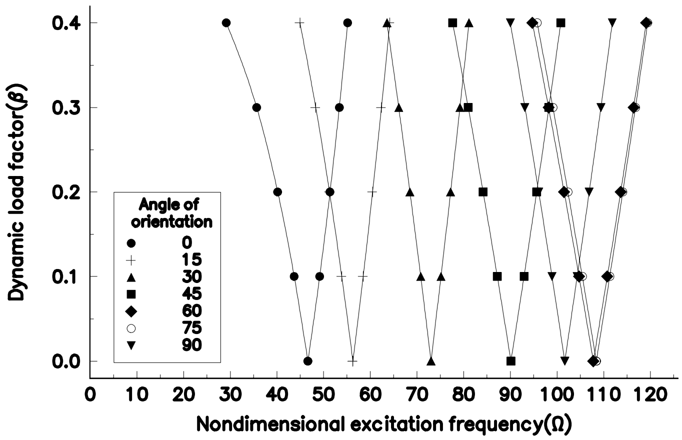

Fig.7 Effect of different ply orientations on instability region of a point loaded simply-supported anti-symmetric angle-ply $(\pm\theta^0)_4$ cylindrical panel: a=b=0.5m, h= 2mm, a/R_x =0.0, b/R_y =0.25, α=0.2, load case 1(b), c/b=0.1

As shown in Fig.7, The instability characteristics are significantly affected by ply orientations. The ply orientation of 60^0 and 75^0 showed higher excitation frequencies and 75^0 seems to be a preferential orientation for this type of panel under this type of load.

4. Conclusion

The results of the stability studies of the shells can be summarised as follows:

1. The laminated composite shells become more stiff with more number of layers.
2. Due to static component, the instability regions tend to shift to lower frequencies with wide instability regions showing destabilizing effect on the dynamic stability behaviour of the curved panel.
3. The instability regions have been influenced due to the restraint provided at the edges.
4. The ply orientations greatly affect the instability behaviour of uniaxially loaded angle-ply curved panels.
5. The onset of instability occurs at a higher excitation frequencies for concentrated loads near the edges but at lower frequencies for concentrated loads than uniform loaded edges.

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