Numerical Flexural Strength Analysis of Thermally Stressed Delaminated Composite Structure under Sinusoidal Loading

C K Hirwani[1], S Biswas[2], K Mehar[3] and S K Panda[4]

[1,2,3,4]Department of Mechanical Engineering, National Institute of Technology Rourkela Rourkela, Odisha, India.

Abstract. In this article, we investigate the thermomechanical deflection characteristics of the debonded composite plate structure using an isoparametric type of higher-order finite element model. The present mid-plane kinematic model mainly obsoletes the use of shear correction factor as in the other lower-order theories. The separation between the adjacent layers is modeled via the sub-laminate technique and the intermittent continuity conditions imposed to avoid the mathematical ill conditions. The governing equation of equilibrium of the damaged plate structure under the combined state of loading are obtained using the variational principle and solved numerically to compute the deflection values.

Introduction

Delamination is a separation of two adjacent layers/plies or lamina of a laminated composite.

It is a predominant forms of failure in many laminated composites systems, especially when there is no reinforcement in the thickness direction.

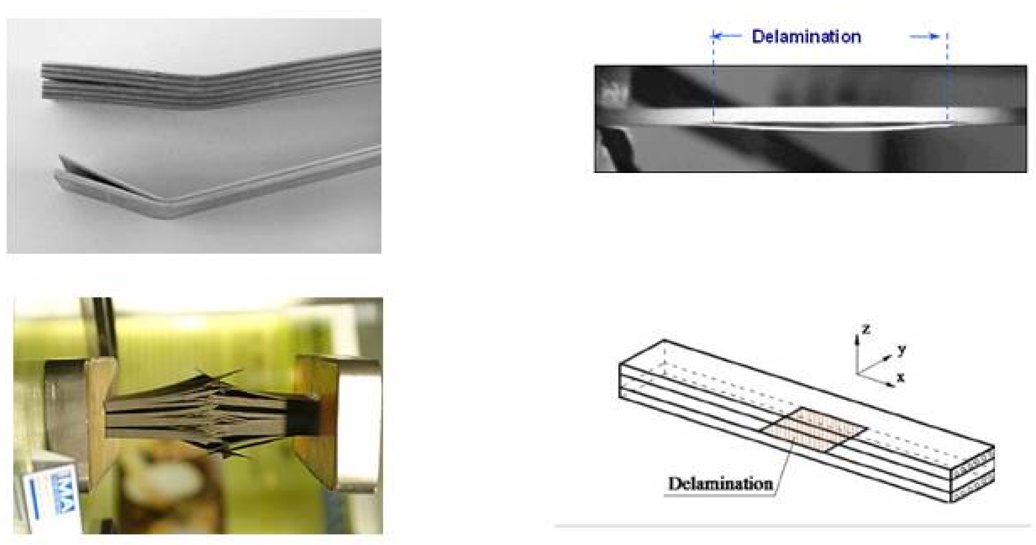


Figure 1. Delamination in laminated composite

Mathematical Formulations

Figure 2 shows a layered composite plate structure consisting a delamination at centre mid-plane whose geometrical parameter is defined as length, *a*; breadth, *b* and height, *h* in ξ_1, ξ_2 and ζ direction, respectively. The laminate is made of *n* number of equally thick laminae oriented at an angle θ.

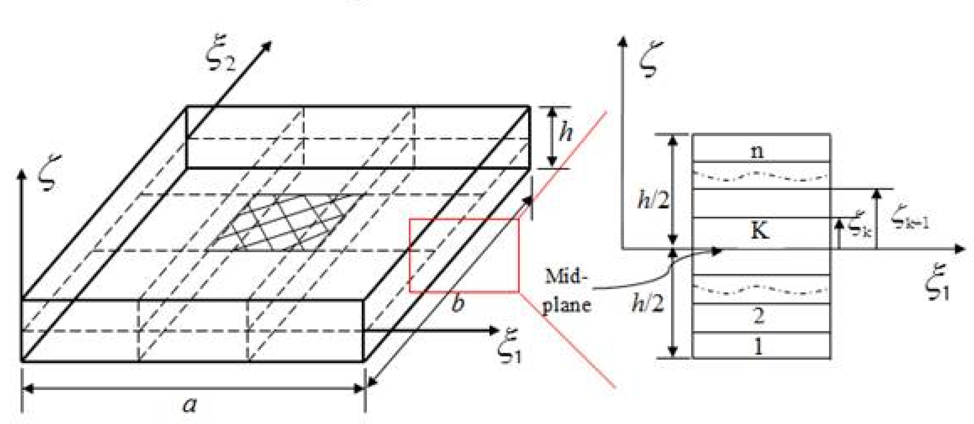


Figure 2. Geometry of the layered composite plate structure

The higher-order displacement kinematics (nine degrees of freedom) as presented in Reddy and Liu [1] is used for the current modeling purpose and shown as:

$$\left.\begin{aligned} \bar{u}(\xi_1,\xi_2,\zeta) &= u_0 + \zeta\phi_1 + \zeta^2\psi_1 + \zeta^3\theta_1 \\ \bar{v}(\xi_1,\xi_2,\zeta) &= v_0 + \zeta\phi_2 + \zeta^2\psi_2 + \zeta^3\theta_2 \\ \bar{w}(\xi_1,\xi_2,\zeta) &= w_0 \end{aligned}\right\} \quad (1)$$

where, $\bar{u}$, $\bar{v}$ and $\bar{w}$ represents the displacement of any point along ξ_1, ξ_2 and ζ directions, respectively. The u_0, v_0 and w_0 denotes the mid-plane displacement whereas ϕ_1 and ϕ_2 are the rotation at (ζ=0) of the norma l tomid-plane about ξ_2 and ξ_1, respectively.Rest of the terms ψ_1, ψ_2, θ_1 and θ_2 are the higher-order term of Taylor series expansion.

The laminate constitutive relations in terms of force and moments can be expressed as [2, 3]:

$$\{F_{ij}\} = [D]\{\{\varepsilon_{ij}\} - \{\alpha_{ij}\}\Delta T\} \quad (2)$$

where, $\{F_{ij}\}$, $[D]$ and $\{\varepsilon_{ij}\}$ are denoted the forces and moment vector,elastic constant matrix, mid-plane strain and curvature matrices, respectively. Further, 'ΔT' is the temperature gradient $\{\alpha_{ij}\} = \{\alpha_{\xi_1\xi_1}\ \alpha_{\xi_2\xi_2}\ 0\ 0\ \alpha_{\xi_1\xi_2}\}'$ coefficient of thermal expansion along the principal material directions.

$$\{F_{ij}\} = \left[N_{\xi_1\xi_1}\ N_{\xi_2\xi_2}\ N_{\xi_2\zeta}\ N_{\xi_1\zeta}\ N_{\xi_1\xi_2}\ M_{\xi_1\xi_1}\ M_{\xi_2\xi_2}\ M_{\xi_2\zeta}\ M_{\xi_1\zeta}\ M_{\xi_1\xi_2}\right]^T \quad (3)$$

$$[D] = \sum_{k=1}^{n} \int_{\zeta_{k-1}}^{\zeta_k} (\bar{Q}_{ij})_k (1, \zeta, \zeta^2 \ldots \zeta^6) d\zeta \quad (4)$$

$$[Q_{ij}]_k = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & Q_{16} \\ Q_{12} & Q_{22} & 0 & 0 & Q_{26} \\ 0 & 0 & Q_{44} & Q_{45} & 0 \\ 0 & 0 & Q_{45} & Q_{55} & 0 \\ Q_{16} & Q_{26} & 0 & 0 & Q_{66} \end{bmatrix}_k \quad i, j = 1, 2, \ldots, 6 \quad (5)$$

$(\bar{Q}_{ij})_k$ in eqation (4) is the transformed reduced stiffness matrix that is derived from the transformation of the reduced stiffness matrix $(Q_{ij})_k$ [4].

Similarly, the necessary strain of Equation (2) is elaborated as:

$$\{\varepsilon_{ij}\} = \left[\varepsilon_{0\xi_1\xi_1}\ \varepsilon_{0\xi_2\xi_2}\ \gamma_{0\xi_2\zeta}\ \gamma_{0\xi_1\zeta}\ \gamma_{0\xi_1\xi_2} K_{\xi_1\xi_1}\ K_{\xi_2\xi_2}\ K_{\xi_2\zeta}\ K_{\xi_1\zeta}\ K_{\xi_1\xi_2}\right]^T \quad (6)$$

The generalised strain-displacement relation whose detail can be seen in [5]

The generalised strain-displacement relation

$$\{\varepsilon_{ij}\} = \begin{Bmatrix} \varepsilon_{\xi_1\xi_1} \\ \varepsilon_{\xi_2\xi_2} \\ \varepsilon_{\xi_2\zeta} \\ \varepsilon_{\xi_1\zeta} \\ \varepsilon_{\xi_1\xi_2} \end{Bmatrix} = \left\{ \left(\frac{\partial\bar{u}}{\partial\xi_1}\right) \left(\frac{\partial\bar{v}}{\partial\xi_2}\right) \left(\frac{\partial\bar{v}}{\partial\zeta} + \frac{\partial\bar{w}}{\partial\xi_2}\right) \ldots\ldots \left(\frac{\partial\bar{u}}{\partial\zeta} + \frac{\partial\bar{w}}{\partial\xi_1}\right) \left(\frac{\partial\bar{u}}{\partial\xi_2} + \frac{\partial\bar{v}}{\partial\xi_1}\right) \right\}^T \quad (7)$$

$$\{\varepsilon_{ij}\} = [H]\{\bar{\varepsilon}\} \quad (8)$$

The displacement vector using FEM concept [6]

$$\{\delta\} = \sum_{i=1}^{9} [N_i]\{\delta_i\} \quad (9)$$

where, $[N_i]$ and $\{\delta_i\}$ are shape functions and the nodal displacement vector, respectively.

$$\{\delta_i\} = \{u_{0_i}\ v_{0_i}\ w_{0_i}\ \phi_{1i}\ \phi_{2i}\ \psi_{1i}\ \psi_{2i}\ \theta_{1i}\ \theta_{2i}\}^T \quad (10)$$

The mid-plane strain vectoris expressedin terms of nodal displacement vector:

$$\{\bar{\varepsilon}\} = [B]\{\delta_i\} \quad (11)$$

Strain energy expression

$$U = \frac{1}{2}\iint\left\{\sum_{k=1}^{n}\int_{\zeta_{k-1}}^{\zeta_k}\{\sigma_{ij}\}\{\varepsilon_{ij}\}d\zeta\right\}d\xi_1 d\xi_2 \quad (12)$$

Work done due to combined thermo-mechanical load

$$W = \int_A \{\delta\}^T\{q^e\}dA \quad (13)$$

$$\{q^e\} = \{q^e_m\} + \{q^e_{th}\} \quad (14)$$

The integral form of the elemental stiffness matrix:

$$[k] = \int_A \left(\sum_{k=1}^{n}\int_{\zeta_{k-1}}^{\zeta_k}[B]^T[D][B]d\zeta\right)dA \quad (15)$$

The governing equation of the deflection analysis evaluation

$$\partial\Pi = \partial U - \partial W = 0 \quad (16)$$

$$[K]\{\delta\} = \{q\} \quad (17)$$

Result and discussion

The sinusoidal form of loading, the nondimensional form of load and central deflection is given in Equation (21), (22) and (23), respectively.

$$q_m = q_0 \sin\left(\frac{\pi\xi_1}{a}\right)\sin\left(\frac{\pi\xi_2}{b}\right) \quad (21)$$

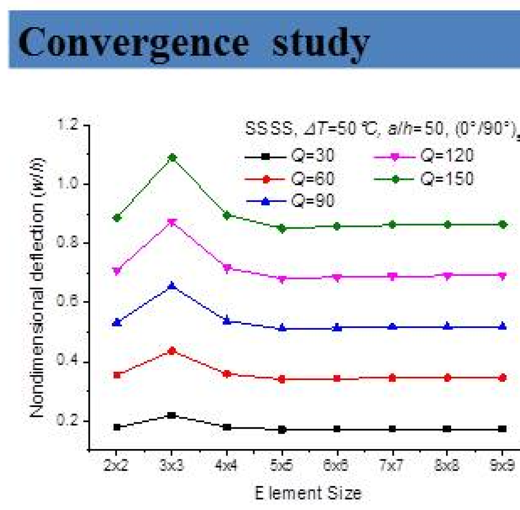
$$Q = q_0 * E_{\xi_2} * (a/h)^4 \quad (22)$$

$$W = w/h \quad (23)$$

Table . Material Properties

$E_{\xi1}$	1181 GPa
$E_{\xi2}=E_{\zeta}$	10.3 GPa
$G_{\xi1\xi2}=G_{\xi1\zeta}$	7.17 GPa
$G_{\xi2\zeta}$	6.21 GPa
$\nu_{\xi1\xi2}=\nu_{\xi1\zeta}=\nu_{\xi2\zeta}$	0.25

Convergence study



Simply-supported, symmetric laminate (0°/90°/90°/0°) thermomechanical load Based on this analysis a mesh size of (8×8) is utilized for further investigation.

Figure 3. Convergence study

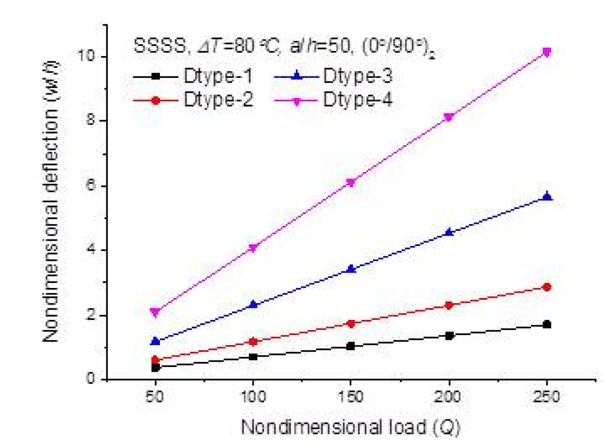
Comparison study

Table . Validation study of nondimensional central deflection responses of the simply supported laminated/delaminated cylindrical shell panel subjected to uniformly distributed load

x/a	Laminate Present	Laminate Nanda [7]	Delaminate (a/4) Present	Delaminate (a/4) Nanda [7]
0	0	0	0	0
0.25	0.4201	0.40915	0.4668	0.41656
0.5	0.5803	0.57273	0.6413	0.58611
0.75	0.4205	0.40928	0.4578	0.41804

Cylindrical panel Eight-layer simply-supported cross-ply laminated and Dtype-2 delaminated composite UDL

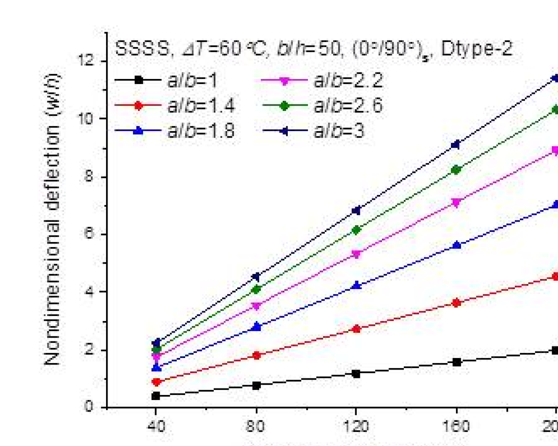
Effect of centrally located delamination size



Simply-supported antisymmetric cross-ply (0°/90°/0°/90°) Dtype-1, Dtype-2, Dtype-3 and Dtype-4 type delamination thermo-mechanical

Figure 4. Effect of delamination size

Influence of aspect ratio



Cross-ply (0°/90°/90°/0°) delaminated (Dtype-2) simply supported edges thermo-mechanical loading

Figure 6. Effect of aspect ratio

Influence of end conditions

Table . Nondimensional central deflection responses of the eight-layer antisymmetric cross-ply delaminated plate.

Load (Q)	Boundary condition CCCC	SCSC	CFCF	SSSS	CFFF
40	0.1074	0.1527	0.1578	0.7052	2.5515
80	0.233	0.3364	0.3453	1.4123	3.697
120	0.3586	0.52	0.5327	2.1193	4.8426
160	0.4842	0.7037	0.7202	2.8264	5.9881
200	0.6098	0.8873	0.9076	3.5334	7.1337

Eight-layer, antisymmetric cross-ply, (0°/90°/0°/90°)2, delaminated (Dtype-3), plate structure

Conclusion

The presence of delamination reduces the overall stiffness and as its size increases, the overall stiffness decreases. The deflection response increases with the increase in aspect ratio and reduces with the increase in Young's modulus and increasing constraint DOF

References

1. Reddy J N and Liu C F 1985 *Int J Eng Sci* **23** 319–330
2. Jones R M 1994 *Taylor & Francis* Philadelphia..
3. Tsai S W and Hahn H T 1980 *Technomic* Westport CT.
4. Reddy J N 2004 *CRC Press* Florida.
5. Mahapatra T R and Panda S K 2015 *J Therm Stresses* **38** 39–68.
6. Cook R D, Malkus D S and Plesha M E 2000 *John Willy and Sons (Asia) Pvt. Ltd* Singapore.
7. Nanda N 2014 *Acta Mechanica* **225** 2893–2901.