Dynamic Coupling Based Amplitude Death Solution for Stabilizing DC Microgrids

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***Abstract*—Amplitude death (AD) is a coupling induced stabilization of the fixed point (FP) of a dynamical system. This paper applies AD concept, induced by dynamic coupling in order to solve the stabilizing issues in presence of constant power load (CPL) for avoiding the use of separate delay circuitry. This AD method has been demonstrated by numerical simulations as well as the bifurcation analysis.**

I. Introduction

When flexibility comes into the picture, one can say that DC micro-grids (MGs) are better choices than that of the AC MGs. This is because they are more suitable for energy storage and renewable sources as almost all of the loads nowadays are inherently DC [1]. This increased reliability and flexibility of DC MGs have helped engineers to select them in major applications such as telecommunication industries [2], low-power consumer electronics, vehicular technologies [3], industrial power systems [4], naval ships [5], residential homes [6], commercial buildings [7], and so on. Though DC MGs are more stable than AC MGs [8]; but there are some serious stability issues because of the interfacing power electronics for achieving different levels of voltages during the integration of sources, loads, and energy storage devices. In cascaded architecture, point-of-load (POL) converters with a resistive load behaves as an instantaneous CPL [9]. The negative incremental resistance caused by CPL brings the nonlinearity to the systems and results in a limit cycle behavior. This leads to the undesired oscillations [10] in the systems, therefore the state variables can't converge to the fixed values.

It is well accepted that the destabilizing problem of DC power-grid networks needs to be solved for future practical use. Several strategies for enhancing the stability of an operating point have been demonstrated that the use of passive damping [11], the application of a bi-directional DC-DC converter, use of a virtual capacitor [12], feedback controller [9], and control for multiple power sources and loads [13]. These studies have been tackled the destabilization problem from the power electronics viewpoint and mostly follows the small signal linear stability analysis [14]. However, stabilizing problem of such system can not be analyzed by linear system dynamics. It is, therefore, necessary to apply the concept of nonlinear analysis.

AD is a mathematical phenomenon of a nonlinear system by which the FP is stabilized due to the coupling [15]. There are mainly two reasons that can cause AD — strong coupling and sufficiently different natural frequencies of interaction [16]. Recently, the model of coupled systems is reported by Huddy and Skufca [17] where the topology allows application of AD solution to this problem in a pair of DC bus systems. Their work reported the applications of the nonlinear science to the destabilizing problem. In contrast, the present paper proposes the different stabilizing scheme called as dynamic coupling [18], that has its own dynamics and deals with a more fundamental parameter inbuilt in the system, i.e., the shunt capacitor.

The motivations behind this dynamic coupling scheme are that, the dynamic coupling is a rough approximation of the time-delayed coupling for the oscillators with low frequencies and/or short-time delay; RC-ladder coupling [19], which can be approximated by RC wire delay connections in VLSI chips [20]. Hence from the practical viewpoint, the dynamic coupling is reasonable as it can be used for identical as well as non-identical oscillating systems.

II. DC Microgrid and CPL

There is continuous growth in the applications of alternative energy sources such as photovoltaic panels, fuels cells, wind turbines and microturbines. As the output of photovoltaic and fuel cells is DC by nature, it is easier and more efficient to connect them directly to a DC distribution system, or through a controlled DC/DC converter. For example, in traction power systems, DC series motors are employed because of its high starting torque and better voltage regulation characteristics. Another example of DC distribution is the data centers, where the sensitive loads connected to it require uninterrupted power supply even if the main power source is lost. Similarly, in a variety of power system applications based on advanced power-electronics technique such as international space station, spacecraft, electric and hybrid electric cars, telecommunications, terrestrial computer systems, and medical electronics etc., the supplies are mostly of the DC type.

Consider a multi-converter DC bus system as shown in Fig. 1. Note that it could include many LRCs that regulate the main bus voltage, such as the one located between the

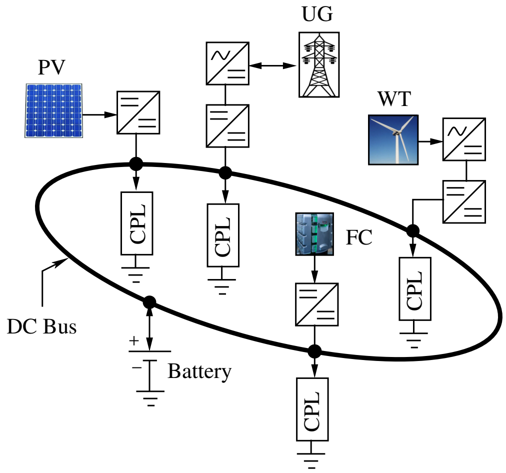

Fig. 1. Conceptual diagram of a DC Bus system (PV: photovoltaic cell, UG: utility grid, WT: wind turbine, FC: fuel cells).

main bus and the local microgrid source converters, which are loaded by other converters. The loads in such systems are the combination of tightly controlled POL converter and a fixed output resistor R_o [21]. Since efficiencies of POL converters are usually very high, this pair can be characterized as an instantaneous CPL, which exhibits negative incremental resistance property. Due to this negative impedance instability, there is a decrease in the voltage stability margin which causes significant oscillations. Since DC microgrids consist many sources and many loads, the entire system is complex, non-linear and coupled. However, DC microgrid stability analysis is constantly improving. Various techniques have been already developed like load shedding, direct connection of energy storage to the main bus, filtering, and control approaches [9] are among the well-known methods. Most studies on CPLs rely on small signal analysis [22]; there is a conclusion is that in constant power loaded dc systems, the equilibrium point/fixed point (FP) of an LRC is unstable. Some other previous studies have large signal analysis [23] method to study the CPL effects on system stability.

An instantaneous CPL can be represented by

$$i = \frac{P_\mathrm{L}}{v} \qquad \forall \quad v \geq \epsilon \tag{1}$$

where i is the input current, P_L is the CPL power, v is the input voltage of the main bus feeding the CPL, ϵ is an arbitrary small positive value. The switch model dynamics for the ideal buck LRC case $(r = 0, R_\mathrm{o} = \infty)$ with a CPL as the only load as given in Fig. 2 is given by

$$\frac{di}{dt} = \frac{qE}{L} - \frac{v}{L}; \quad \frac{dv}{dt} = \frac{i}{C} - \frac{P_\mathrm{L}}{Cv}; \quad i \geq 0, \quad v \geq \epsilon \tag{2}$$

where i and v are the inductor current and capacitor voltage respectively of the buck LRC. The switching function which controls the MOSFET is given by q and its fast average is given by the instantaneous duty cycle d.

During the transient, it's possible that the trajectories of the system can cross the boundary $(i = 0)$. But the converter topology only allows unidirectional current through the inductor $(i \geq 0)$. So it's important to include the discontinuous conduction mode (DCM) operation of the converters [24] for low currents. Hence the average state equations no longer describe the mathematical modeling of the converters and their behavior. A full order model of the converter is represented by,

$$\frac{di}{dt} = \frac{qE}{L} - \frac{2ivf_\mathrm{s}}{q(E-v)}\ ; \quad \frac{dv}{dt} = \frac{i}{C} - \frac{P_\mathrm{L}}{Cv}\ ; \quad i < 0 \tag{3}$$

where f_s is the switching frequency of the converter.

The simulations have been carried out using MATLAB for the system shown in Fig. 2. Fig. 3 shows the voltage oscillations. The experimental validation of the sustained oscillations in voltage waveform is given in Fig. 4.

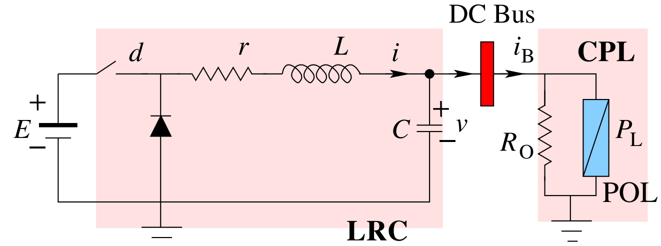

Fig. 2. A simplified buck-based distributed power architecture.

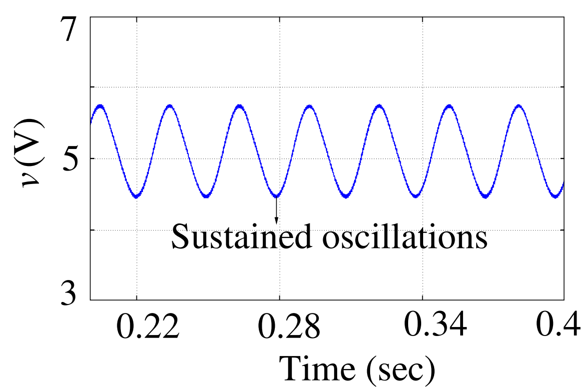

Fig. 3. Simulation result of output voltage of LRC converter.

It is a difficult task to calculate eigenvalues for the system as shown in Fig. 2 described by Eqs. (2) and (3) as it involves different conduction modes of the converters for different conditions. Therefore numerical computation method using XPPAUT [25] is used to determine the eigenvalues and stability of the system. It is observed from the numerical results that the FP[0] is unstable because of the eigenvalues have positive real parts [1]. The phase portrait is shown in Fig. 5. For detailed analysis, readers can refer to [17], [21].

[0]Given by the intersection of x-nullcline (orange) and y-nullcline (green)
[1]$\lambda_1 = 159.84 + j2230.35,\ \lambda_2 = 159.84 - j2230.35$

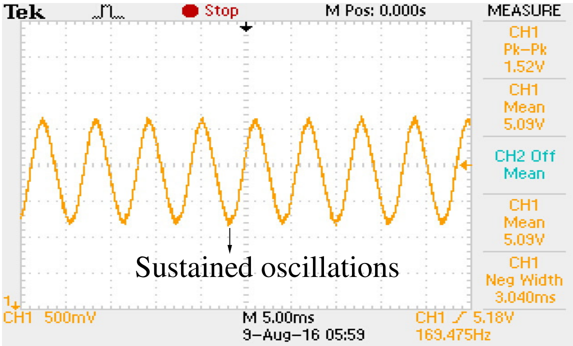

Fig. 4. Oscilloscope traces of sustained oscillations in a buck LRC with CPL. Y- channel: 1 division = 1 V.

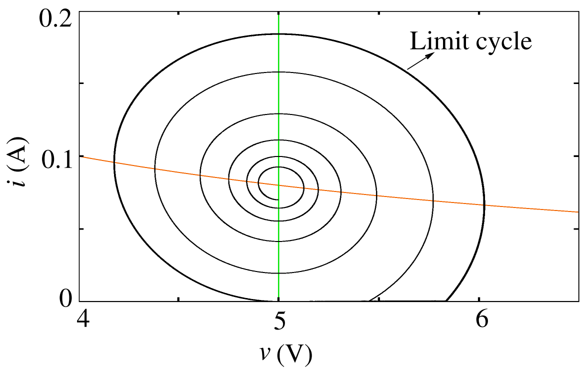

Fig. 5. Stable limit cycle behavior of the destabilised system due to CPL. Numerical results for the ideal buck LRC, with the parameters $P_{\rm L} = 0.4$ W, $E = 10$ V, $L = 4$ mH, $C = 50$ μF, $q = 0.5$, $f_{\rm s} = 20$ KHz.

III. AD Solutions Method by Dynamic Coupling

The dynamic coupling can be applied to two identical systems as shown in Fig. 6, where coupling link consists of a resistor $R_{\rm k}$ and a capacitor $C_{\rm k}$. Here the coupling link has its own set of dynamical equations which bring phase shift between two identical oscillators pulling each other off its limit cycle resulting AD in both the systems. The model dynamics are given by

$$\begin{aligned} \frac{dv_1}{dt} &= \frac{i_1}{C_1} - \frac{P_1}{C_1 v_1} + \frac{(v_{\rm k} - v_1)}{R_{\rm k} C_1} \\ \frac{di_1}{dt} &= \begin{cases} \dfrac{qE - v_1}{L_1} & \text{if } i_1 \geq 0 \\ \dfrac{qE}{L_1} - \dfrac{2 i_1 v_1 f_{\rm s}}{q(E - v_1)} & \text{if } i_1 < 0 \end{cases} \end{aligned} \tag{4}$$

$$\begin{aligned} \frac{dv_2}{dt} &= \frac{i_2}{C_2} - \frac{P_2}{C_2 v_2} + \frac{(v_{\rm k} - v_2)}{R_{\rm k} C_2} \\ \frac{di_2}{dt} &= \begin{cases} \dfrac{qE - v_2}{L_2} & \text{if } i_2 \geq 0 \\ \dfrac{qE}{L_2} - \dfrac{2 i_2 v_2 f_{\rm s}}{q(E - v_2)} & \text{if } i_2 < 0 \end{cases} \end{aligned} \tag{5}$$

$$\frac{dv_{\rm k}}{dt} = \frac{v_1 + v_2 - 2v_{\rm k}}{R_{\rm k} C_{\rm k}} \tag{6}$$

where $v_{\rm k}$ is the voltage across the capacitor $C_{\rm k}$.

Dynamic coupling brings both the systems to steady state by AD phenomena which is illustrated as shown in Fig. 7 and the corresponding experimental result is given in Fig. 8. Numerical analysis indicates that the FP is stable as the eigenvalues have negative real parts [2]. Here all the trajectories converge to the FP which is clear from the phase portrait as shown in Fig.9.

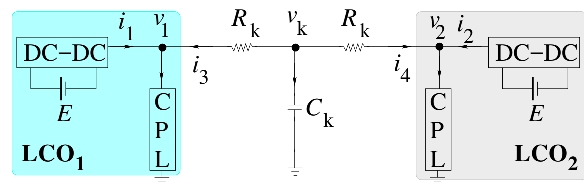

Fig. 6. Block diagram of coupled homogenous LRC systems for dynamic coupling with $R_{\rm k}$ and dynamic element $C_{\rm k}$ as coupling link.

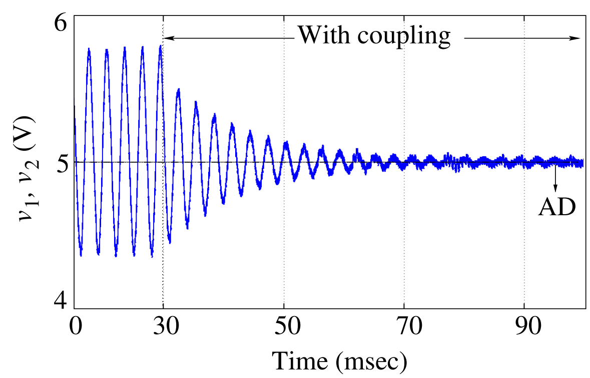

Fig. 7. Simulation results showing sustained oscillations before coupling and AD after coupling using the parameters $P_{\rm L} = 0.4$ KW, $E = 10$ V, $L_1 = L_2 = 4$ mH, $C_1 = C_2 = 50$ μF, $q = 0.5$, $f_{\rm s} = 20$ KHz, $R_{\rm k} = 35$ Ω, $C_{\rm k} = 50$ μF.

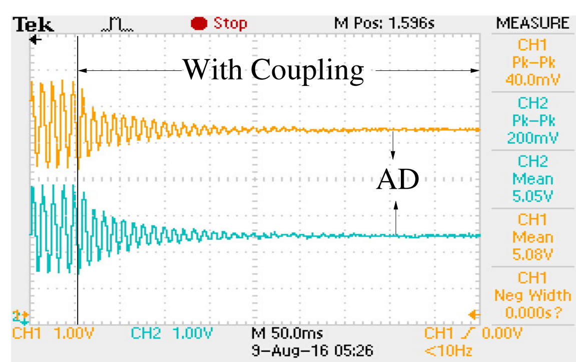

Fig. 8. Oscilloscope traces of dynamically coupled two homogeneous systems. Y- channel: 1 division = 1 V.

[2] $\lambda_1 = -1274.92$, $\lambda_2 = -59.84 + j2116.24$, $\lambda_3 = -59.84 - j2116.24$, $\lambda_4 = -125.874 + j2232.52$, $\lambda_5 = -125.874 - j2232.52$

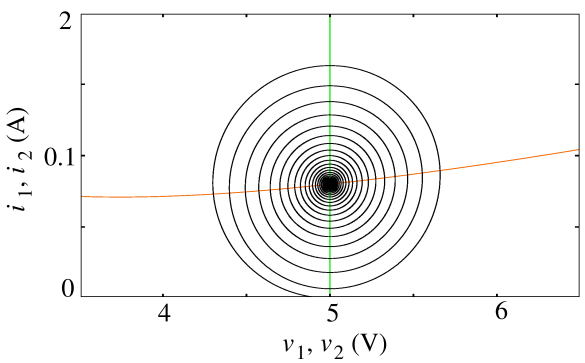

Fig. 9. Phase portrait for dynamically coupled homogeneous systems showing a stable focus.

IV. Bifurcation Analysis

As it's difficult to derive the range of parameters for the AD region, numerical analysis helps us to estimate this range. There are some useful tools for exploring how a dynamical system changes with respect to the variation of parameters. One of such methods is the Maximal Lyapunov exponent (MLE) analysis because it determines a notion of predictability for a dynamical system. The system is chaotic if MLE is positive and is stable if MLE is negative. The most widely observed route to AD is through Hopf bifurcation, where coupling induces stability of the FP of the uncoupled systems. To study this bifurcation a straightforward technique is called continuation bifurcation in which a particular solution (such as FP or limit cycle) is followed as the parameter changes. AUTO provides a number of tools for the automatic detection of bifurcations of FPs and limit cycles.

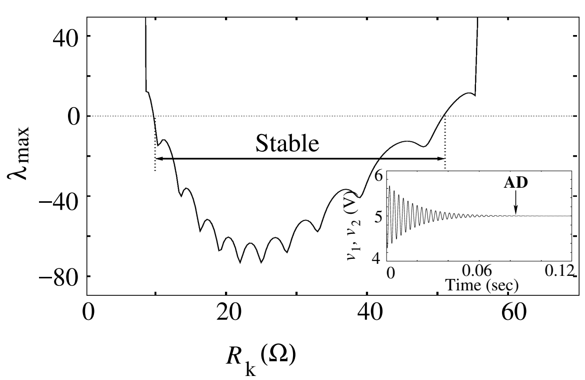

Fig. 10. Maximum Lyapunov exponent as a function of $R_k \in (0, 70)\ \Omega$ for $C_k = 50\ \mu$F. Inset figure: transient trajectories of v_1 and v_2 , as a function of time.

In Fig. 10, the plot gives the MLE for the coupled systems as a function of R_k keeping C_k constant. The area denoted by the arrow shows the AD region or the stability region where the MLE is a negative quantity. The stability analysis shows that AD occurs for the range $8.5\ \Omega < R_k < 56.5\ \Omega$. The same has been verified using continuation bifurcation diagram as shown in Fig. 11, where R_k is taken as the bifurcation parameter. The red line represents stable FPs and the black thin line represents

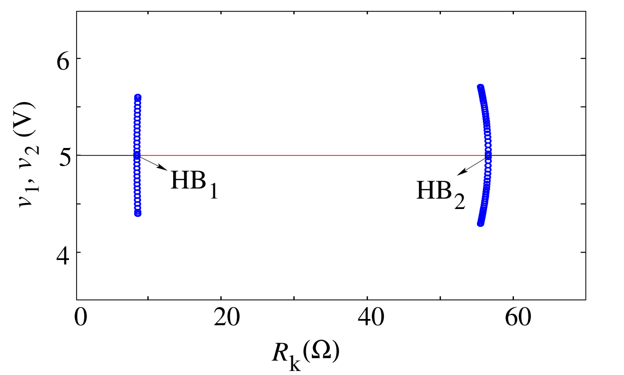

Fig. 11. Bifurcation diagram using $R_k \in (0, 70)\ \Omega$ as the bifurcation parameter for $C_k = 50\ \mu$F.

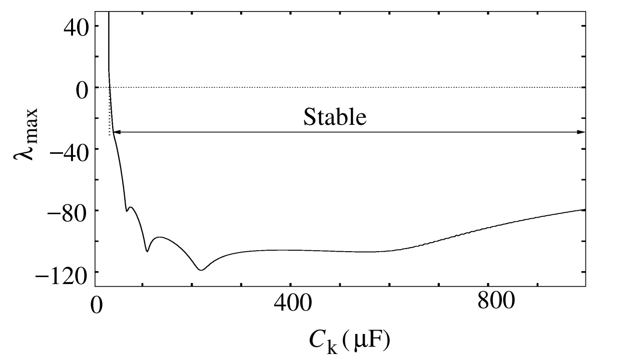

Fig. 12. Maximum Lyapunov exponent as a function of $C_k \in (0, 500)\ \mu$F for $R_k = 35\ \Omega$.

the unstable FPs. Here the two Hopf bifurcation points are at $R_k = 8.484\ \Omega$ and $R_k = 56.5\ \Omega$ giving unstable periodic orbits implied by empty blue circles which are denoted by HB_1 and HB_2 respectively. Stability region exists between these two points HB_1 and HB_2. In Fig. 12, the plot shows that AD happens in the range $C_k > 50\ \mu F$. The two parameters bifurcation diagram is given by Fig. 13 where death island is given by the shaded area of C_k vs R_k plot.

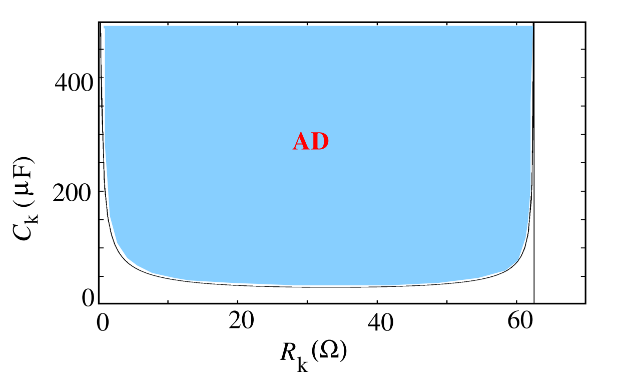

Fig. 13. Two parameters bifurcation diagram using $R_k \in (0, 70)\ \Omega$ and $C_k \in (0, 500)\ \mu$F as the bifurcation parameters.

V. CONCLUSION

The stability issues of DC MGs can be overcome by various feedback control methods. But AD solution method has its uniqueness as it is free from any external controller circuits which increase cost as well as the complexity of the system. Moreover, the dynamic coupling is a replacement of delay coupling scheme where the DC bus need not to be connected with an additional delay circuitry. This coupling scheme can be useful for both homogeneous and heterogeneous systems. Dynamics of the coupling link necessarily helps to cease the oscillations in the constant power loaded converters system and gives the steady DC output voltages. The numerical results based on bifurcation analysis have been done here to highlight the stability region or death island under different parameters values. However, this work can be extended to multiple oscillating systems in a network of DC MGs.

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