ML Based Velocity Estimator via Gamma Distributed Handover Counts in HetNets

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Abstract*—In this paper, we present a maximum likelihood (ML) based approach to estimate velocity of mobile users in Heterogeneous Networks (HetNets). HetNets by architecture are hierarchal combination of randomly deployed base stations (BSs) with varied transmit power levels and hence have non-uniform coverage area. Further, in order to improve network capacity, BS density in HetNets is much higher in comparison to traditional cellular networks. The increased BS densification in HetNets results in frequent handovers which if not managed properly may leads to service failures. One of the fundamental challenge in effective handover management is to accurately estimate the velocity of mobile users. Thus, we propose a velocity estimation strategy based on handover count samples which is in accordance with Release-8 of LTE specifications. We analyze the handover statistics by modeling BS location via stochastic geometry and coverage area by Poisson-Voronoi tessellation. The probability mass function (PMF) of handover count is approximated via Gamma distribution as it has very small approximation error compared to Gaussian distribution. Using the approximated PMF, we first derive maximum likelihood (ML) base velocity estimator for the respective mobile user in the network. In addition, we also derive the Cramer-Rao lower bound (CRLB). We validate our proposed estimation approach via numerical results in which we observe tight closeness between asymptotic variance of estimated velocity and CRLB. Our results also demonstrate that velocity estimation error decreases individually with increase in BS density and time duration specified for handover count measurements.

***Keywords*—Cramer-Rao lower bound (CRLB), Maximum likelihood (ML) estimator, Heterogeneous networks (HetNets), Velocity estimation, Handover count, Gamma distribution.**

I. Introduction

Over the past few decades, cellular network have gone through a remarkable development while catering to increased demand of data rate [1]. Network densification appears to be one of the promising solution as it contributes towards 1000-fold times capacity improvement as needed for 5G cellular networks [2]. Deploying small cells inside the macro cell to support the above requirement of high data rate, is the main idea of heterogeneous networks (HetNets). HetNets includes a variety of the base stations (BSs) supported by diverse radio access technology with different power levels [3]. This network densification reduces the load served by individual BSs, thereby increasing per user throughput in the overall network. The promising solution comes at the cost of increased handover count [4]. This is due to mobility of users in the network which aggressively change their connectivity with the nearby BS's in order to achieve best Quality of Service (QoS).

In HetNets, densification of BSs results in reduction of respective cell coverage areas which leads to small travel time of high velocity mobile user in the cell [5]. In absence of adequate mobility management, the small travel time may result in increased service failures. Mobility management is a process of handover of ongoing connectivity from serving BS to a new BS when mobile user passes through cells of neighboring BSs. In this process user velocity and BS density act as vital parameters for successful transfer of connectivity. Knowledge of BS density in a geographical area is a network parameter and can be assumed to be known. However, computing accurate velocity of mobile users is a challenging task. The current generation smart devices are equipped which with different sensors along with global positioning system (GPS) and Wi-Fi network can help in immediate velocity measurement. However, these devices come with the limited battery capacity. Further, services like GPS are not ubiquitous, as it gives a weak signal strength in the dense urban area. Similarly Wi-Fi signals are not available in rural area [6]. Therefore, these features are insufficient for accurate computation of mobile user velocity.

In Release-8 of long-term evaluation (LTE) specification, handover count based regulation for mobility state detection has been made. Estimation of mobile user velocity and mobility state detection based on handover count are introduced in [7], [8]. In these works, it is shown that probability mass function (PMF) of handover count can be approximated by Gamma and Gaussian distribution functions. It is further identified that mean square error for Gamma approximation is comparatively very less compared to Gaussian approximation. However, due to computation complexity authors of [7], [8] have designed velocity estimator based on Gaussian approximation of PMF for handover count. On the basis of our limited literature survey, we have not found any practical velocity estimator based on Gamma distribution approximated PMF for handover count.

In this paper, we exploit PMF of handover count to design a maximum likelihood (ML) based velocity estimation approach in HetNets. ML estimator is a most prominent approach to design practical estimator as it is an alternative in the situation where minimum variance unbiased (MVU) estimator doesn't exists, or is very difficult to find even it exists [9]. The PMF of

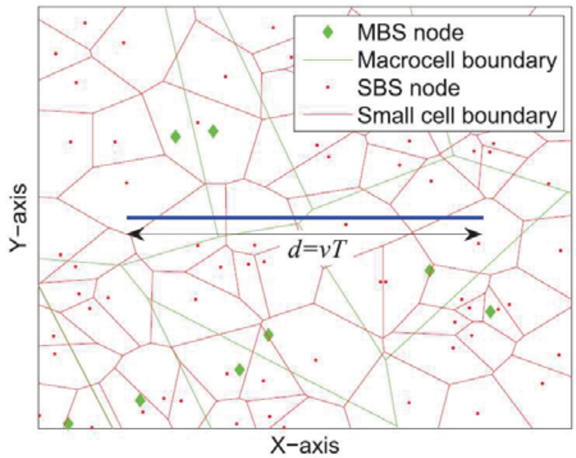

Fig. 1: Coverage of heterogeneous cellular network

handover count in our approach is approximated by Gamma distribution. We have also derived corresponding Cramer-Rao lower bound (CRLB) to characterize the accuracy of proposed estimator for a given BS density. Since service provider has the information of the number of BSs in a particular geographical area, the BS density in that area can be calculated and broad-casted as part of system information in next generation networks.

Rest of the paper is organized as follows. The system model used for deploying small cell BSs inside the macro cells using stochastic geometry is explained in Section II. Next, we determined the approximated PMF of handover count using Gamma distribution in Section III. In section IV, a ML estimator based on approximated PMF of handover count using Gamma distribution is derived. In section V, CRLB for velocity estimator based on handover count is calculated. The proposed velocity estimation approach is validated via numerical results in Section VI. Section VII carries the conclusion of the paper.

II. System model

Consider a geographical area in which small cells (eg. femto-cells, pico-cells) are deployed inside a traditional macro cell network. Since footprint of small cell BSs varies according to the location and radiated power level, we model respective coverage area via Poisson-Voronoi tessellation as shown in Fig. 1. Further, we exploits stochastic geometry to model the random deployment of BS in the considered geographical area. Here we denote, the small cell BS density by λ in BSs/km^2. In addition, for simplicity in analysis we consider mobile user's travel in straight path trajectory. However, other movement trajectories of mobile users can be accounted by analyzing them as sum of piecewise linear trajectories. Due to movement of mobile user, BS have to perform handover process in order to maintain desired QoS. We define handover count as the total number of handovers that takes place in a predefined time period (T), or it is the number of intersections between the user path and the coverage area boundaries of BSs [8], [10].

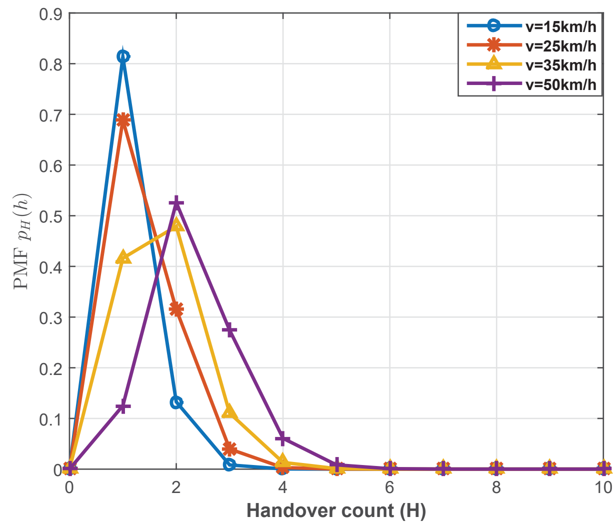

Fig. 2: PMF of handoff count for $\lambda = 100BS/km^2$

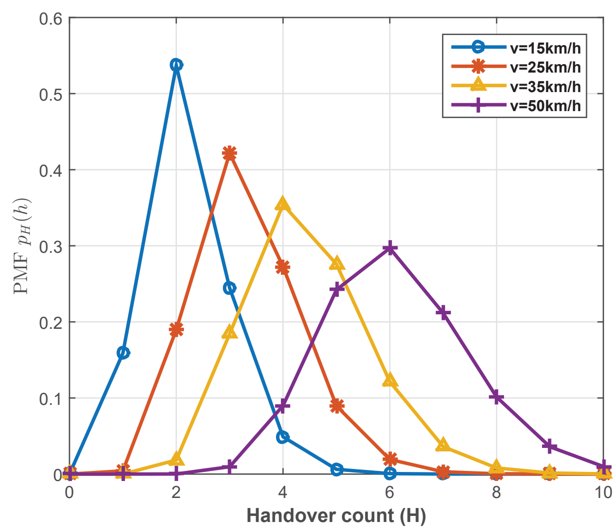

Fig. 3: PMF of handoff count for $\lambda = 1000BS/km^2$

Since the geometrical location of small cell BS and its coverage area is considered as stochastic in nature, the handover count is also a random value. However, the statistics of handover count is assume to remain same irrespective of the change in the direction of the mobile user. The computation of exact density function of handover count has been found to be very complex and mathematically intractable [10]; therefore, we perform Monte Carlo simulation to plot the probability mass function (PMF) for handover count as a function of velocity v and BS density λ. The graphs of PMF against handover count for different user velocities and for $\lambda = 100$ and 1000 are shown in Fig. 2 and Fig. 3, respectively. The specified time (T) for taking handover count sample is assumed to be $10s$ so that estimator can provide fast results.

From Fig. 2, we can be observe that, when BS density is small, there is overlapped in PMFs for different velocity. Thus, we can expect a smaller accuracy of estimated velocity. On the other hand, for higher BS density as shown in Fig. 3 , the PMFs are separated fairly apart, and thus we can expect better accuracy in velocity estimation. Further, we can also observe that with increase in user velocity there is the higher variance of handover count, which will results in lower accuracy of velocity estimation. Here, it is also necessary to notice that with an increase in BS density (λ) and velocity, PMF of handover count resembles the Gamma distribution. These observations have also be verified in some previous research work [7].

III. Handover count PMF approximation

As explained in the introduction section, objective of this work is to estimate the user velocity on the basis of number of handovers in a fixed time span. Since exact expression for PMF of handover count is not available in the literature, we approximate the PMF using Gamma distribution. This approach is validated in [8]. Gamma distribution is generally used for approximating the statistical distribution of the parameters related to Poisson point process (PPP) such as area, edges etc. It can also be effectively exploited to statistically approximate handover count PMF. The expression for Gamma probability density function (PDF) for random variable x can be stated as,

$$p\left(x\right) = \frac{\beta^{\alpha}}{\Gamma\left(\alpha\right)} x^{\alpha-1} e^{-\beta x}, \qquad for\ x \in \left(0, \infty\right) \tag{1}$$

where, $\Gamma\left(\alpha\right) = \int\limits_{0}^{\infty} t^{\alpha-1} e^{-t} dt$ is the Gamma function and α, β are respective shape and scale parameter. We can not directly map the Gamma PDF to handover count PMF as Gamma PDF is continuous while handover count PMF is a discrete function. Therefore, we fit the sections of Gamma PDF to the PMF of handover count. The Gamma PDF sections are obtained by integrating Gamma PDF between integer values of x. For example, integrating the gamma PDF for the values of x lying between 0 and 1 gives the PMF value for $h = 0$; similarly, for x lying between 1 and 2 gives the PMF value for $h = 1$, and so on. Thus, the PMF of handover count h for a given velocity v and BS density λ can be expressed as,

$$p_H^G\left(h; v\right) = \int\limits_{h}^{h+1} p\left(x\right) dx,\ \ for\ h\ \in\ \ \left(0, 1, 2,\ \ \ldots\right). \tag{2}$$

By substituting the value of $p(x)$ from equation (1) to (2) we get,

$$p_H^G\left(h; v\right) = \frac{\beta^{\alpha}}{\Gamma\left(\alpha\right)} \int\limits_{h}^{h+1} x^{\alpha-1} e^{-\beta x} dx, for\ h \in \left(0, 1,\ \ \ldots\right). \tag{3}$$

By transform of variable, let $r = x\beta$; replacing x by r equation (3) we get,

$$p_H^G\left(h; v\right) = \frac{1}{\Gamma\left(\alpha\right)} \int\limits_{\beta(h)}^{\beta(h+1)} r^{\alpha-1} e^{-r} dr \tag{4}$$

$$= \frac{\Gamma\left(\alpha, \beta h, \beta\left(h+1\right)\right)}{\Gamma\left(\alpha\right)} \tag{5}$$

where, $\Gamma\left(\alpha, \beta h, \beta\left(h+1\right)\right) = \Gamma\left(\alpha, \beta h\right) - \Gamma\left(\alpha, \beta\left(h+1\right)\right) = \int\limits_{\beta(h)}^{\beta(h+1)} r^{\alpha-1} e^{-r} dr$ is the incomplete Gamma function.

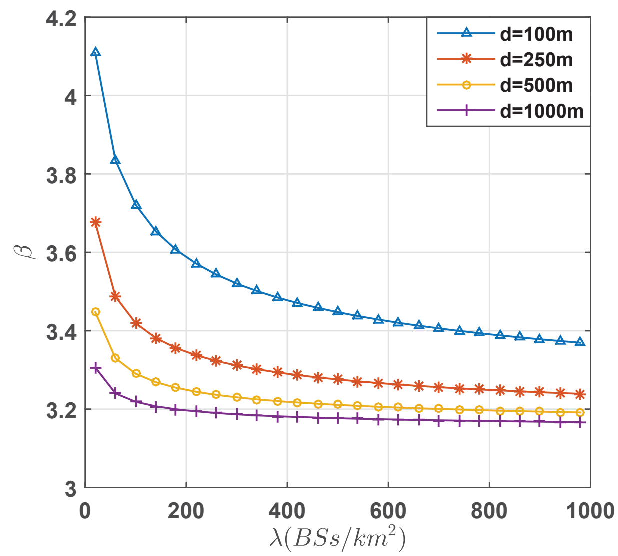

Fig. 4: β parameter for PMF of handover count versus BS density

The shape and scale parameters for Gamma distribution based approximation are selected such that the mean square error (MSE) between simulated and approximated Gamma distribution is minimized. The α and β parameters for approximated Gamma distributed based PMF of handover count is analyzed in [7], and can be stated here as,

$$\alpha = 2.7 + 4vT\sqrt{\lambda} \tag{6}$$

$$\beta = \pi + \frac{0.8}{0.38 + vT\sqrt{\lambda}}. \tag{7}$$

Here, we have assumed that the distance traveled by the mobile user with velocity (v) is $d = vT$ during the specified time period (T). For Gamma distribution the variance of random variable is inversely proportional to the β parameter. In Fig. 4, we plot the variation in β parameter against BS density for specified duration $T = 10s$. It can be verified from equation (7) as well as from Fig. 4, that with increase in BS density the β parameter decreases resulting in increase in variance of handover count. Further, we can notice that with an increase in velocity v, β decreases, resulting in increase in the variance of handover count and thus implying lower estimation accuracy. These observations are analogous to our discussion in Section II, and thus satisfying the Gamma distribution based approximation of PMF for handover count.

The PMF of handover count can also be approximated using Gaussian distribution as presented in [7] and is stated here as,

$$p_H^g(h;v) = \frac{1}{\sqrt{2\pi\sigma^2(v)}} e^{-\frac{(h-\mu(v))^2}{2\sigma^2(v)}}, \quad for\ h \in \{0,1,2,\ldots.\} \tag{8}$$

in which the approximate values of $\mu(v)$ and $\sigma^2(v)$ are given by,

$$\mu(v) = \frac{4vT\sqrt{\lambda}}{\pi}$$

$$\sigma^2(v) = 0.07 + 0.41vT\sqrt{\lambda}$$

The accuracy of Gamma and Gaussian approximated PMF for handover count can be quantified by computing the MSE between approximated and simulated PMFs. The MSE can be expressed as,

$$MSE = \frac{1}{N}\sum_{h=1}^{N} \left[p_H^c(h) - p_H(h)\right]^2, \ for\ c \in \ (G, g) \tag{9}$$

where N is the number of points in PMF. By comparing the MSE between approximated and simulated PMFs, it has been found that approximated Gamma distributed PMF provides nearly ten times smaller MSE than Gaussian distribution [7]. Hence, we consider Gamma distribution based approximation for PMF of handover count in following analysis.

IV. ML VELOCITY ESTIMATOR

The ML estimator is based on principle of maximizing the likelihood function. It is a suitable approach for obtaining practical estimator, and can be used to solve complicated estimation problems [9]. The performance of ML estimator relies on the property that it is asymptotically efficient. In order to find ML estimator, we consider the approximated PMF using Gamma distribution as described in equation (5); taking logarithm on both sides, we get,

$$ln\left(p_H^G(h;v)\right) = \ \log_e\Gamma(\alpha, \beta h, \beta(h+1)) - \log_e\Gamma(\alpha) \tag{10}$$

Differentiating equation (10) with respect to v, the log-likelihood function can be expressed as,

$$\frac{\partial}{\partial v} ln\left(p_H^G(h;v)\right) = \ \frac{\partial}{\partial v}\log_e\Gamma(\alpha, \beta h, \beta(h+1)) - \frac{\partial}{\partial v}\log_e\Gamma(\alpha) \tag{11}$$

Analyzing the first term in the right hand side (RHS) of equation (11),

$$\frac{\partial}{\partial v}\log_e\Gamma(\alpha, \beta h, \beta(h+1)) = \frac{1}{\Gamma(\alpha, \beta h, \beta(h+1))} \times \frac{\partial}{\partial v}\Gamma(\alpha, \beta h, \beta(h+1)) \tag{12}$$

let $m_1 = \beta h$ and $m_2 = \beta(h+1)$; the modified Equation (12) can be restated as,

$$\frac{\partial}{\partial v}\log_e\Gamma(\alpha, m_1, m_2) = \frac{1}{\Gamma(\alpha, m_1, m_2)} \times \left(\frac{\partial}{\partial \alpha}\Gamma(\alpha, m_1, m_2)\frac{d\alpha}{dv} + \frac{\partial}{\partial m_1}\Gamma(\alpha, m_1, m_2)\frac{dm_1}{dv} + \frac{\partial}{\partial m_2}\Gamma(\alpha, m_1, m_2)\frac{dm_2}{dv}\right). \tag{13}$$

Each term of the equation (13) can be simplified as,

$$\frac{\partial}{\partial \alpha}\Gamma(\alpha, m_1, m_2) = \frac{{}_2F_2(\alpha, \alpha; \alpha+1, \alpha+1; -m_1)\, m_1^\alpha}{\alpha^2} - \frac{{}_2F_2(\alpha, \alpha; \alpha+1, \alpha+1; -m_2)\, m_2^\alpha}{\alpha^2} - \gamma(\alpha, m_1)\log m_1 + \gamma(\alpha, m_2)\log m_2$$

$$\frac{\partial}{\partial m_1}\Gamma(\alpha, m_1, m_2) = -e^{-m_1}{m_1}^{\alpha-1}$$

$$\frac{\partial}{\partial m_2}\Gamma(\alpha, m_1, m_2) = -e^{-m_2}{m_2}^{\alpha-1}$$

$$\frac{d\alpha}{dv} = 4T\sqrt{\lambda}$$

$$\frac{dm_1}{dv} = \frac{-0.8\bar{h}T\sqrt{\lambda}}{\left(0.38 + vT\sqrt{\lambda}\right)^2}$$

$$\frac{dm_2}{dv} = \frac{-0.8\left(\bar{h}+1\right)T\sqrt{\lambda}}{\left(0.38 + vT\sqrt{\lambda}\right)^2}$$

where, $\gamma(\alpha, m_1) = \int\limits_0^{m_1} r^{\alpha-1}e^{-r}dr$ is lower incomplete Gamma function. The generalized hyper-geometric function can be expressed as,

$${}_2F_2(a_1, a_2; b_1, b_2; m) = \sum_{k=0}^{\infty}\frac{(a_1)_k (a_2)_k}{(b_1)_k (b_2)_k}\frac{m^k}{k!} \tag{14}$$

where, $(a)_k = a(a+1)(a+2)\ldots(a+k-1), for\ k \geq 1$. Next, we analyze the second derivative term in the RHS of equation (11),

$$\frac{\partial}{\partial v}\log_e\Gamma(\alpha) = \psi(\alpha)\frac{\partial \alpha}{\partial v} = 4T\sqrt{\lambda}\psi(\alpha). \tag{15}$$

Substituting the values from equations (13)-(15) in (11), the derivative of log-likelihood function can be expressed as,

$$\frac{\partial}{\partial v} ln\left(p_H^G(h;v)\right) = \frac{4T\sqrt{\lambda}\beta^\alpha}{\alpha^2\Gamma(\alpha, \beta h, \beta(h+1))} \Big({}_2F_2(\alpha, \alpha; \alpha+1, \alpha+1; -\beta h)\, h^\alpha - {}_2F_2(\alpha, \alpha; \alpha+1, \alpha+1; -\beta(h+1))(h+1)^\alpha - \frac{\alpha^2}{\beta^\alpha}\left(\gamma(\alpha, \beta h)\log(\beta h) - \gamma(\alpha, \beta(h+1))\log(\beta(h+1))\right)\Big) + \frac{0.8T\sqrt{\lambda}\beta^{\alpha-1}e^{-\beta h}\left[h^\alpha - e^{-\beta}(h+1)^\alpha\right]}{\Gamma(\alpha, \beta h, \beta(h+1))\left(0.38 + vT\sqrt{\lambda}\right)^2} - 4T\sqrt{\lambda}\psi(\alpha). \tag{16}$$

For finding ML estimator, we equate the derivative of log-likelihood function to zero i.e. $\frac{\partial}{\partial v} ln\left(p_H^G\left(h;v\right)\right) = 0$. Thus, equating equation (16) to zero results in,

$$\begin{aligned}&\beta^{\alpha}\big({}_2F_2\left(\alpha,\alpha;\alpha+1,\alpha+1;-\beta h\right)h^{\alpha}\\ &\quad -{}_2F_2\left(\alpha,\alpha;\alpha+1,\alpha+1;-\beta\left(h+1\right)\right)\left(h+1\right)^{\alpha}\big)\\ &-\alpha^2\left(\gamma\left(\alpha,\beta h\right)\log\left(\beta h\right)-\gamma\left(\alpha,\beta\left(h+1\right)\right)\log\left(\beta\left(h+1\right)\right)\right)\\ &\quad+\frac{0.2\alpha^2\beta^{\alpha-1}e^{-\beta h}\left[h^{\alpha}-e^{-\beta}\left(h+1\right)^{\alpha}\right]}{\left(0.38+vT\sqrt{\lambda}\right)^2}\\ &\quad\quad-\alpha^2\Gamma\left(\alpha,\beta h,\beta\left(h+1\right)\right)\psi\left(\alpha\right)=0.\end{aligned} \tag{17}$$

Due to complexity of expression in (17), we use numerical approximation to gets the roots of $\hat{v}$. Thus, the ML based velocity estimator as a function of random handover count h can be expressed as,

$$\hat{v} = \frac{\pi}{4T\sqrt{\lambda}}\left(-0.3784+\sqrt{h^2+\frac{8\times 0.07}{0.41\pi}h+0.0732}\right) \tag{18}$$

The proposed ML estimator $\hat{v}$ is a nonlinear function of handover count. Hence, to determine the mean and variance of $\hat{v}$, we use the statistical linearization argument. The efficiency of estimator thus will be proved for linear approximation only; however, if handover count samples are large enough then the efficiency will also be applicable to nonlinear estimator [9]. Defining function g such that,

$$\hat{v} = g\left(w\right) \tag{19}$$

where $w = h^2 + \frac{8\times 0.07}{0.41\pi}h$, then,

$$g\left(w\right) = \frac{\pi}{4T\sqrt{\lambda}}\left(-0.3784+\sqrt{w+0.0732}\right) \tag{20}$$

Linearizing about,

$$w_0 = E\left(w\right) = E\left[h^2\right] + \frac{8\times 0.07}{0.41\pi}E\left[h\right]$$

we get,

$$g\left(w\right) \approx g\left(w_0\right) + \left.\frac{dg\left(w\right)}{dw}\right|_{w=w_0}\left(w-w_0\right) \tag{21}$$

let $h = \left[h_n : n = 0,1,2\ldots\ldots,\ N-1\right]$ be a vector of N handover count samples accumulated over a past service time; then the sample mean and second moment of handover count can be expressed as, $E\left[h\right] = \frac{1}{N}\sum_{n=0}^{N-1} E\left[h_n\right]$ and $E\left[h^2\right] = \frac{1}{N}\sum_{n=0}^{N-1} E\left[h_n^2\right]$ respectively. Substituting sample mean and second moment in equation (21), the linearized estimate of velocity can be expressed as,

$$\begin{aligned}\hat{v} \approx v + \frac{\pi^2}{32T\sqrt{\lambda}\left(vT\sqrt{\lambda}+0.2971\right)}\Bigg(h^2+\frac{8\times 0.07}{0.41\pi}h-\\ \left(\frac{1}{N}\sum_{n=0}^{N-1}E\left[h_n\right]+\frac{8\times 0.07}{0.41\pi N}\sum_{n=0}^{N-1}E\left[h_n^2\right]\right)\Bigg)\end{aligned} \tag{22}$$

To determine the biasness we take expectation of proposed ML estimator i.e. $E(\hat{v})$. The velocity estimator expressed in equation (22) is an asymptotically unbiased as $E\left[\hat{v}\right] \rightarrow v$ for $N \rightarrow \infty$. Additionally, the asymptotic variance of ML estimator becomes,

$$var_{ML}^G = \frac{0.25291v}{T\sqrt{\lambda}} \tag{23}$$

V. CRLB for velocity estimation

The Cramer-Rao lower bound (CRLB), is the lower bound in the variance for an unbiased estimator. If the estimator gives on an average the true value of an unknown parameter, then it is termed as an unbiased estimator. The efficient estimators are those whose variance achieves CRLB. However, if it is not possible to calculate an efficient estimator than the estimator which gives the lowest variance is called minimum variance unbiased (MVU) estimator [9]. Consider the approximated PMF for handover count using Gamma distribution expressed in equation (16); the CRLB for velocity estimation can be computed as,

$$var_{CRLB}^G\left(\hat{v}\right) \geq \frac{1}{\mathrm{E}\left[\left(\frac{\partial \ln p_H^G\left(h;v\right)}{\partial v}\right)^2\right]} \tag{24}$$

Substituting the derivative term of equation (16) in (24), we numerically evaluates the CRLB as,

$$var_{CRLB}^G\left(\hat{v}\right) \geq \frac{1}{\frac{T\sqrt{\lambda}}{0.25291v}+\left(\frac{0.28284T\sqrt{\lambda}}{0.068+0.41vT\sqrt{\lambda}}\right)^2} \tag{25}$$

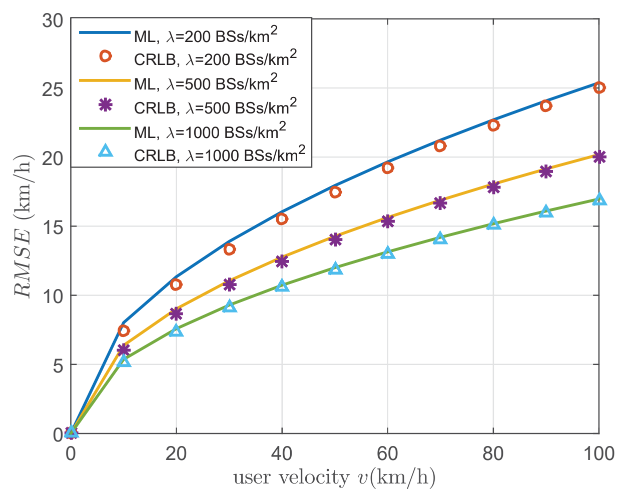

Fig. 5: RMSE versus user velocity, ($T = 10s$)

VI. Numerical results

In this section, we present the performance of proposed ML estimator and compare its variance with the CRLB obtained from approximated Gamma distributed PMF of handover count. The performance metric is the square root of variance which is equivalent to root mean square error (RMSE). The CRLB plot is obtained using equation (25), while the

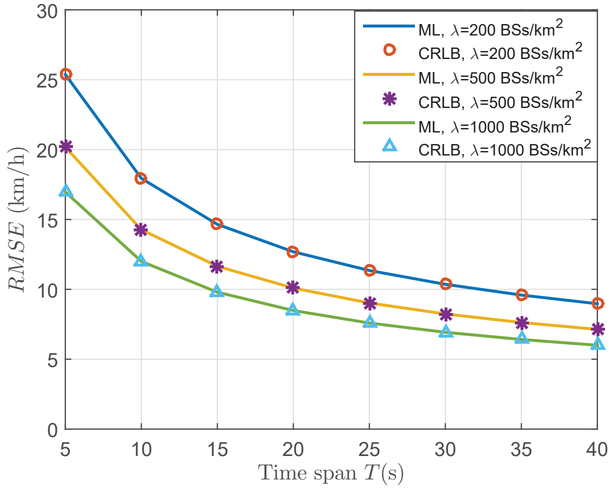

Fig. 6: RMSE versus time span, ($v = 50km/h$)

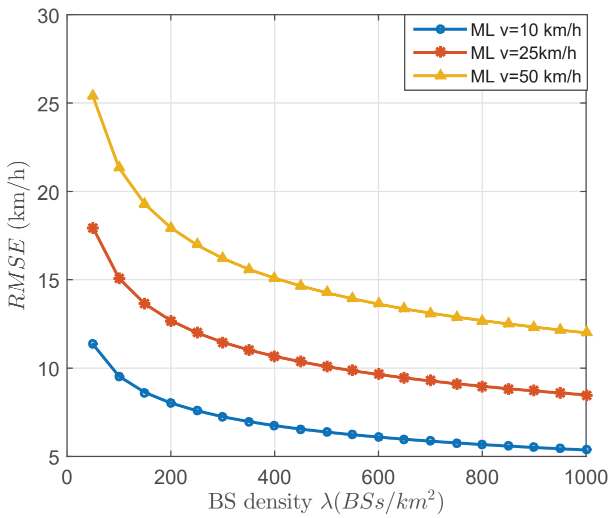

Fig. 7: RMSE versus BS density, ($T = 10s$)

variance plot for ML estimator is obtained through numerical computation of equation (23). The RMSE plot of proposed ML estimator with increase in user velocity for various BS densities $\lambda = 200, 500, 1000$ is shown in Fig. 5. Here we have assumed time span for handover count measurement to be $T = 10s$, so that estimator can provide fast results. It can be observed from the plot that with increase in user velocity, CRLB increases. It can also be observed that the variance of proposed ML estimator tight matches with CRLB with increase in BS density. This validates our approach as our estimator is asymptotic efficient.

The time period used for handover count measurement depends on service provider strategy; therefore the variation of RMSE for proposed ML estimator with time span is also investigated and is shown in Fig. 6. Consider the scenario when user is moving with velocity $v = 50km/h$ and BS density $\lambda = 500BSs/km^2$. The service provider uses time span $T = 10s$ and $T = 40s$ for handover count measurement. The RMSE of ML based velocity estimator are 14 and 7 km/h respectively. Thus, we can note notice that the variance of ML estimator decreases with increase in time span. Thus, we can conclude that longer time span increases the accuracy of ML velocity estimation. However, increase in time span slow downs the response of the estimator. So, there exist a the trade-off between accuracy and response time of the proposed estimator. Finally, in Fig. 7, we plot RMSE against BSs density λ. Once again consider the scenario when user is moving with velocity $v = 50km/h$ and time span $T = 10s$. The RMSE of ML based velocity estimator for BS density $\lambda = 200BSs/km^2$ and $\lambda = 1000BSs/km^2$ are observed as 16 and 7 km/h, respectively. Thus, we can be conclude that the variance of velocity estimator decreases with increase in BS density λ, which facilitate more accurate velocity estimation in the hyper-dense network.

VII. Conclusion

In this paper, we proposed a ML based velocity estimator exploiting handover count samples for HetNets. Since computation of exact PMF of handover count is mathematically intractable, we consider an approximate PMF of handover count using Gamma distribution. Next, we derived an ML estimator for velocity estimation in HetNets and compared it with the CRLB. The results show tight closeness of ML estimator asymptotic variance with CRLB. Further, we observed that variance of proposed ML estimator decreases with increases in time span used for handover count samples. Thus, there exists a trade-off between the accuracy and response of estimator. Also, the variance of velocity estimator decreases with increase in BS density, and hence facilitates accurate velocity estimation in the hyper-dense network.

References

[1] J. Ma, L. Song, and Y. Li, "Cost efficiency for economical mobile data traffic management from users x2019; perspective," *IEEE Transactions on Wireless Communications*, vol. 16, no. 1, pp. 362–375, Jan 2017.

[2] X. Ge, S. Tu, G. Mao, C. X. Wang, and T. Han, "5g ultra-dense cellular networks," *IEEE Wireless Communications*, vol. 23, no. 1, pp. 72–79, February 2016.

[3] H. S. Dhillon, R. K. Ganti, F. Baccelli, and J. G. Andrews, "Modeling and analysis of k-tier downlink heterogeneous cellular networks," *IEEE Journal on Selected Areas in Communications*, vol. 30, no. 3, pp. 550–560, 2012.

[4] D. Lopez-Perez, I. Guvenc, and X. Chu, "Mobility management challenges in 3gpp heterogeneous networks," *IEEE Communications Magazine*, vol. 50, no. 12, pp. 70–78, 2012.

[5] X. Yan, Y. A. Şekercioğlu, and S. Narayanan, "A survey of vertical handover decision algorithms in fourth generation heterogeneous wireless networks," *Computer networks*, vol. 54, no. 11, pp. 1848–1863, 2010.

[6] C.-H. Chen, C.-A. Lee, and C.-C. Lo, "Vehicle localization and velocity estimation based on mobile phone sensing," *IEEE Access*, vol. 4, pp. 803–817, 2016.

[7] A. Merwaday and . Gven, "Handover count based velocity estimation and mobility state detection in dense hetnets," *IEEE Transactions on Wireless Communications*, vol. 15, no. 7, pp. 4673–4688, July 2016.

[8] A. Merwaday and I. Guvenc, "Handover count based ue velocity estimation in hyper-dense heterogeneous wireless networks," in *2015 IEEE Globecom Workshops (GC Wkshps)*. IEEE, 2015, pp. 1–6.

[9] S. M. Kay, "Fundamentals of statistical signal processing, volume i: estimation theory," 1993.

[10] J. Moller, *Lectures on random Voronoi tessellations*. Springer Science & Business Media, 2012, vol. 87.