Higher Order Eigenvalue-Moment-Ratio based Blind Spectrum Sensing : Application to Cognitive Radio

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***Abstract*—This paper presents the application of higher order eigenvalue moment ratio based blind spectrum sensing to the cognitive radio. It starts with an investigation on recently proposed eigenvalue based blind spectrum sensing techniques, that work under sample starving environment. A modified version of eigenvalue moment ratio (EMR) spectrum sensing based on random matrix theory (RMT) is proposed. EMR technique is considered to provide superior performance in small sample environment, where the number of samples received by secondary is comparable to number of antennas. Previous works on EMR has been limited to second order moment, where as proposed technique is extended to fourth order moment. The asymptotic test statistic distribution of received signal is derived and an analytical expression for detection probability is presented. Results are validated using receiver operating characteristic curves and are compared with state-of-art techniques like AGM, SLE and EMR.**

***Index terms*—** Cognitive radio (CR), Random Matrix Theory (RMT), Eigenvalue, Blind Spectrum sensing, AGM, SLE, EMR, ROC

I. Introduction

The radio spectrum is a most precious natural resource in current era of development in wireless technologies. However on observing radio spectrum, it is observed that there is inefficient utilization of the radio spectrum. Some of the bands are heavily used most of the time, whereas some are underutilized most of the time. The current spectrum utilization varies between 10%-90% [1, 2]. This brings in the requirement of the improved spectrum utilization. Spectrum sensing is a technique to determine underutilized spectrum, which is termed as Spectrum Holes[3].

In conventional spectrum sensing techniques such as energy detector and matched filter detection, full or partial information of the primary user (PU) signal characteristics and noise variance information are required at the cognitive radio receiver [4]. But often such information is not available in real time environment. Under such condition, blind spectrum sensing technique which does not require primary signal knowledge or noise variance are useful. This method usually utilizes the correlation structure present in the received signal. Some of the blind detection techniques include eigenvalue arithmetic to geometric mean(AGM)[9, 10, 16], Scaled largest Eigenvalue(SLE)[11, 15, 17], Eigenvalue Moment Ratio (EMR)[12–14] .

Eigenvalue Moment Ratio (EMR) method proposed by Huang *et al.* in [14] have shown that it provides superior performance comparison to other maximum likelihood (ML) estimation theory dependent blind algorithms like AGM, GLR and SLE. Second-order EMR algorithm is considered to be equivalent to Johan's detector, one of the powerful test that performs under small sample[19]. Previous work on EMR suggest that moving towards higher order has potential to improve performance in terms of detection quality. Performance improvement have been generally shown by simulation.

In this paper a fourth order eigenvalue moment ratio based on random matrix theory(RMT) detector is considered. The Asymptotic distribution test statistic for the higher order EMR is derived in presence of primary signal. The theoretical probability of detection is also derived. Theoretical and Monte-Carlo simulation of receiver operating characteristic (ROC) shows close similarity between them. Computation of decision statistic of the proposed method is obtained by using matrix trace and internal Frobenius product operation, instead of eigenvalue decomposition(EVD) technique. This in turn offers superior computational efficiency. Fourth order EMR performs better in terms probability of detection in comparison to second order EMR and other techniques such as AGM, SLE, when number of samples are small. This has been validated using simulation. The relative error of asymptotically derived threshold and simulated threshold of higher oder EMR detection smaller compared to other techniques. The results of relative error show superior performance for IEEE 802.22 cognitive radio standard false alarm settings.

Following this introduction, paper is organized as follows: In section II, System model based on binary hypothesis test is described and widely considered conventional blind sensing algorithms are also analyzed. In section III, Fourth order EMR is proposed, expression for test statistic and decision threshold are derived. Following this section IV presents, simulation results to validate proposed method.This section also provides analysis of results . Finally section V providing concluding remarks.

II. Problem Formulation and Eigenvalue based Blind Spectrum Sensing Techniques

A. System Model

The scenario considered here is a single-input-multiple-output (SIMO) cognitive radio model, in which secondary user (SU) has n receive antennas. Receiver signal at the front end of SU can be given as, $y_k(k \in \{1, 2, ..., m\})$. Primary detection by SU is formulated as binary hypothesis problem. Hypothesis H_0 denotes absence of PU and hypothesis H_1 denotes presence of PU. Two hypotheses can be written as[14]:

$$H_0 \;: y_k = w_k \quad (1)$$

$$H_1 \;: y_k = \mathbf{H}s_k + w_k \quad (2)$$

where $\mathbf{H} \in \mathcal{C}^{n\times 1}$ denotes the SIMO channel coefficients between the primary user and secondary user. The value of $y_k = [y_1(k), ..., y_n(k)]^T$ represent observation vector, $s_k = [s_1(k), ..., s_n(k)]^T$ represent the signal vector and $w_k = [w_1(k), ..., w_n(k)]^T$ represent noise vectors at the kth sampling instant. Here, $(.)^T$ denotes the matrix transpose and $y_i(k)(i = 1, ..., n)$ represent the output of the ith antenna and s_k primary signal which has complex Gaussian distribution having zero mean and unknown variance σ_s^2, $w_i(k)(i = 1, ..., n)$ represent additive noise at the ith antenna with i.i.d. complex Gaussian random process with mean zero and unknown variance τ [14]. In this system model noise is assumed to be uncorrelated to signal. Since received signal at secondary front end y_k assumed to be Gaussian distributed with mean zero, covariance matrix is sufficient to extract the statistical properties. $R = E[y_k y_k^H]$ represents the population covariance matrix, which under the two hypothesis, can be written as [14]

$$H_0 \;: R = \tau I_n \quad (3)$$

$$H_1 \;: R = \mathbf{H}R_s\mathbf{H}^H + \tau I_n \quad (4)$$

Where the Hermition of matrix is denoted by $(\cdot)^H$, I_n is the $n \times n$ identity matrix, the primary signal covariance matrix is represented by $R_s = \mathrm{E}[s_k s_k^H]$, and the mathematical expectation is represented as $\mathrm{E}[\cdot]$.

B. Eigenvalue based blind Spectrum Sensing Techniques

This subsection presents some of recently proposed eigenvalue (EV) based blind sensing techniques. These EV based techniques which work better under small sample environment have comparable performance to higher oder EMR are considered. These include arithmetic to geometric mean (AGM) of eigenvalue, Scaled Largest Eigenvalue (SLE) and Eigenvalue Moment Ratio (EMR). Previous work on EMR considers multiple primary user model and estimating the number of primary user. But in this work, single primary user is considered to give larger emphasize on higher oder EMR.

1) Arithmetic to Geometric Mean (AGM): AGM is blind EV techniques that is derived in framework of generalized likelihood ratio (GLR) test, that depend on maximum likelihood estimation theory. It is based on estimation of arithmetic to geometric mean of EV of sample covariance matrix (SCM). The Test statistic for AGM is given as[14]:

$$\xi_{AGM} = 2(m-1)log\left(\frac{\frac{1}{n}\sum_{i=1}^{n} l_i}{\left(\prod_{i=1}^{n} l_i\right)^{1/n}}\right)^n \quad (5)$$

where $l_1 \geq l_2 \geq ... \geq l_n$ are sample eigen values in decreasing order of $\hat{R}$.
The threshold value for AGM is given as:

$$\gamma_{AGM} = \frac{2}{c_1}\bar{\Gamma}^{-1}(1-\epsilon, n^2-1) \quad (6)$$

where $c_1 = \frac{1-(2n^2+1)}{6mn}$ and the inverse of incomplete gamma function is represented by $\bar{\Gamma}^{-1}$.
Threshold formula for AGM is derived by assuming fixed number of antennas n and number samples $m \to \infty$, though threshold obtained is optimal but it suffer from an error for finite number of samples and antennas. So method works better in large sample size [9, 10, 14, 15].

2) Scaled Largest Eigenvalue(SLE): SLE detector is insensitive to the noise estimation error [10, 11]. SLE is formulated as ratio of largest EV of SCM to the trace of SCM obtained at the secondary. The Test statistic for SLE is given as[11, 14]:

$$\xi_{SLE} = \frac{l_1}{\frac{1}{n}\sum_{i=1}^{n} l_i} \quad (7)$$

where $l_1 \geq l_2 \geq ... \geq l_n$ are the decreasing sample eigen values of $\hat{R}$.
The threshold value for SLE is given as:

$$\gamma_{SLE} = \left(1+\sqrt{\frac{n+0.5}{m+0.5}}\right)^2 + \frac{\left(\sqrt{n+0.5}+\sqrt{m+0.5}\right)^{4/3}}{m\left[(n+0.5)(m+0.5)\right]^{1/6}} F_2^{-1}(1-\epsilon) \quad (8)$$

where $F_2(.)$ is the CDF of the Tracy-Widom distribution of order two. This method has exact analytical CDF and threshold for finite number of samples and antennas as threshold is derived in the regime where $n, m \to \infty$ and $n/m \to c$. Though its performance with large number of sample size compared to antennas is comparable to EMR. But threshold and CDF calculation are computationally intensive. Its performance deteriorates as the number samples size equates small number of antennas at the secondary [14].

3) Eigenvalue Moment Ratio(EMR): EMR is defined as the ratio of the jth moment of the sample EVs calculated from an $n \times m$ signal-free observation matrix to the j th power of the first moment of sample eigenvalues almost surely (a.s) converges to a deterministic value as $n, m \to \infty$ and $n/m \to c \in (0, \infty)$ [14]. Here, j is an integer larger than or equal to 2. Derivation of EMR algorithm is done with perspective of random matrix theory, analyzed thoroughly in [14]. The Test Statistic is given as[12–14]:

$$\xi_{EMR} = \hat{M}_2/(\hat{M}_1)^2. \tag{9}$$

where the first moment is

$$\hat{M}_1 = \frac{1}{n} tr(\hat{R}),$$

and the second moment is:

$$\hat{M}_2 = \frac{1}{n} \|\hat{R}\|_F^2$$

where $\hat{R}$ is the estimated SCM of signal free observation matrix. The threshold value for EMR is given as:

$$\gamma_{EMR} = 1 + c + \frac{\sqrt{2} c Q_{-1}(\epsilon)}{n} \tag{10}$$

Where $n/m \to c \in (0, \infty)$ and ϵ is the probability of false alarm, where $Q^1(\cdot)$ is inverse of the Gaussian complementary distribution function. Theoretical decision threshold calculated for EMR is is very accurate for finite small number samples and finite antennas. And EMR technique reformulated in [14] by using Frobenius inner product and matrix trace operations so it reduces the computational cost and indirectly it works better with small sample environment. In previous work of *Huang et al.* [14] restricts there analyses to $j = 2$ and they only provide the numerical analyses for $j > 2$. So our proposed work extends analysis of EMR to $j = 4$. And show that theoretical and numerical detection of higher-order EMR is superior than previously proposed algorithms.

III. Higher Order Eigenvalue Moment Ratio: HO-EMR

In this paper we propose the modified version of EMR, in which jth power is enhanced to four. Analysis is done using random matrix theory (RMT) concept as similar to EMR[14]. Here theoretical asymptotic test statistic distribution of proposed technique is obtained in presence of primary signal.The theoretical probability of detection is also derived.

A. The Test Statistic Expression

The expression for jth moment of eigenvalue of sample covariance matrix is given as[14]:

$$\hat{M}_j = \tau^j \sum_{k=0}^{j-1} \frac{c^k}{k+1} C_j^k C_{j-1}^k \triangleq \tau^j \eta(j) \left(\hat{M}_j\right)^2 = (\tau_j \eta(j))^2 \tag{11}$$

where $\eta(j) = \sum_{k=0}^{j-1} \left(c^k/(k+1)\right) C_j^k C_{j-1}$ and C_j^k denotes the total number of k combinations of j numbers.

Suppose if we take the value of $j = 2$ the above expression can be given as below

$$\begin{aligned} \eta(2) &= \sum_{k=0}^{1} \frac{c^k}{k+1} C_j^k C_{j-1}^k \\ &= \frac{c^0}{1} C_2^0 C_1^0 + \frac{c^1}{2} C_2^1 C_1^1 \\ &= \frac{2!}{2!0!} \frac{1!}{1!0!} + \frac{c}{2} \frac{2!}{1!1!} \frac{1!}{1!0!} \\ &= 1 + c \end{aligned} \tag{12}$$

$\hat{M}_j$ asymptotically converges to the jth moment of the population eigenvalues associated with R[14, 18].

$$\hat{M}_j \xrightarrow{a.s.} M_j \triangleq \int t^j dF^R(t) \tag{13}$$

where $dF^R(t)$ is the Marcenko-Pastur density.

Now M_j is calculated as:

$$M_j = \tau^j \eta(j)$$

$$\hat{M}_j = \frac{1}{m} \sum_{i=1}^{m} l_i^j$$

$$\hat{M}_1 = \frac{1}{m} \sum_{i=1}^{m} l_i \xrightarrow{a.s.} \tau \tag{14}$$

$$\hat{M}_2 = \frac{1}{m} \sum_{i=1}^{m} l_i^j \xrightarrow{a.s.} \tau^j \eta(j) \tag{15}$$

Now the test statistic can be given as the ratio of square of the second moment to the first moment raised to the power of 4. As this ratio yield a constant value independent of noise variance. Hence from equation (14) and ()15) the test statistic is given as:

$$\begin{aligned} \text{Test Statistic } (\xi_p) &= \frac{\left(\hat{M}_2\right)^2}{\left(\hat{M}_1\right)^4} \\ &= \frac{\left(\tau^j \eta(j)\right)^2}{\tau^4} \\ &= \frac{\left(\tau^2 \eta(2)\right)^2}{\tau^4} \\ &= \eta^2(2) \\ &= (1+c)^2 \\ &= 1 + 2c + c^2 \end{aligned} \tag{16}$$

So, test statistic is independent of τ (noise variance) and converges to a constant, hence called blind detection.

B. Theoretical Decision Threshold

In order to obtain constant false alarm value we must find a threshold value which should be independent of τ [22]. The expression for sample covariance matrix is obtained as:

$$\text{SCM } \hat{R} = \left(\frac{1}{m}\right) \sum_{k=1}^{m} y_k y_k^H \tag{17}$$

$\hat{R}$ satisfies the hypothesis of ξ_p. The statistic

$$\zeta \triangleq n[\xi_p - (1+c)^2] \tag{18}$$

converges in distribution to a Gaussian process with mean zero and variance $2c^2$ as $m, n \to \infty$, $n/m \to c \in (0, \infty)$, i.e.,

$$\zeta \xrightarrow{D} \mathcal{N}(0, 2c^2) \tag{19}$$

The false alarm probability for the new proposed method is calculated as

$$\begin{aligned} P_{fa} &= P(\xi_p > \gamma_p \mid H_0) \\ &= P\left(\frac{n[\xi_p - (1+c)^2]}{\sqrt{2}c} > \frac{n[\gamma_p - (1+c)^2]}{\sqrt{2}c} \mid H_0\right) \\ &= \int_{n\left[\frac{\gamma_p-(1+c)^2}{\sqrt{2}c}\right]}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}t^2\right) dt \\ &= Q\left(\frac{n[\gamma_p - (1+c)^2]}{\sqrt{2}c}\right) \end{aligned} \tag{20}$$

where $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} \exp\left(-\frac{t^2}{2}\right)$

For a particular false alarm value ϵ, the threshold can be calculated as

$$\begin{aligned} \epsilon &= Q\left(\frac{n[\gamma_p - (1+c)^2]}{\sqrt{2}c}\right) \\ \Rightarrow \quad Q^{-1}(\epsilon) &= \frac{n[\gamma_p - (1+c)^2]}{\sqrt{2}c} \\ \Rightarrow \quad \frac{\sqrt{2}cQ^{-1}(\epsilon)}{n} &= \gamma_p - (1+c)^2 \\ \Rightarrow \quad \gamma_p &= (1+c)^2 + \frac{\sqrt{2}cQ^{-1}(\epsilon)}{n} \\ \Rightarrow \quad \gamma_p &= 1 + 2c + c^2 + \frac{\sqrt{2}cQ^{-1}(\epsilon)}{n} \end{aligned} \tag{21}$$

C. The Theoretical Detection Probability:

In order to find the expression for detection probability[22], we must know the distribution of ξ_p under H_1 hypothesis. An approximate analytical expression for the distribution of ξ_p based on RMT is derived below[14]. First we assume that for distribution of ξ_p in presence of p "asymptotically identifiable" signals whose population eigenvalues are above the asymptotic limit of detection.The signal eigenvalues of $\psi = HR_sH^H$ is given by $\lambda_1 \geq \lambda_2 \geq ... \geq \lambda_p$. Assume

$$\lambda_p > \lambda_{DET} \triangleq \sqrt{c}\tau \tag{22}$$

where λ_{DET} is the asymptotic limit of detection. As $n, m \to \infty$ and $n/m \to c \in (0, \infty)$, we have

$$\xi_p \xrightarrow{D} \mathcal{N}(\mu_\epsilon, \sigma_\epsilon^2) \tag{23}$$

where

$$\mu_\epsilon = \frac{n[v_2] + (n-p)\tau^2(1+c)^2}{[v_1 + (n-p)\tau]^2} \tag{24}$$

$$\sigma_\epsilon^2 = \nabla^T D \nabla \tag{25}$$

with

$$\begin{aligned} v_1 &= \sum_{i=1}^{p} (\lambda_i + \tau)\left(1 + \frac{c\tau}{\lambda_i}\right) \\ v_2 &= \sum_{i=1}^{p} (\lambda_i + \tau)^2 \left(1 + \frac{c\tau}{\lambda_i}\right)^2 \\ D &= \tau^2 c \begin{bmatrix} 1 & 2\tau(1+c)^2 \\ 2\tau(1+c)^2 & 2\tau^2(2c^2 + 5c + 2) \end{bmatrix} \\ \nabla &= \frac{n}{[v_1 + (n-p)\tau]^3} \begin{bmatrix} -1\left[v_2 + (n-p)\tau^2(1+c)^2\right] \\ v_1 + (n-p)\tau \end{bmatrix} \end{aligned}$$

$\therefore$ The detection probability is given as

$$\begin{aligned} P_d &= P\left(\xi_p > \gamma_p \mid H_1\right) \\ &= P\left(\frac{\xi_p - \mu_\epsilon}{\sigma_\epsilon} > \frac{\gamma_p - \mu_\epsilon}{\sigma_\epsilon} \mid H_1\right) \\ &= Q\left(\frac{\gamma_p - \mu_\epsilon}{\sigma_\epsilon}\right) \end{aligned} \tag{26}$$

D. The Proposed Detection Algorithm:

The Higher order based eigenvalue moment ratio method algorithm can be given as below:

Step 1 : Compute the SCM by: $\hat{R} = (1/m)\sum_{k=1}^{m} y_k y_k^H$

Step 2 : Calculate the Test Statistic ξ_p.

Step 3 : Determine the presence of primary signals by comparing ξ_p with the predetermined threshold γ_p. If $\xi_p > \gamma_p$, the signal is present; otherwise, the signal does not exist.

E. Advantages

a) Suitable in low SNR environment.
b) Prior knowledge regarding PU and noise variance is not required.
c) Noise uncertainty does not effect.
d) Computational complexity is less.

F. Disdvantages

a) Hardware complexity.

IV. Simulation Results

Simulation was conducted to evaluate the performance of the proposed method with other detectors such as AGM, SLE and EMR. The simulation result represents an average of 50000 Monte Carlo trials. The cognitive radio network consists of primary user having a single antenna.The primary message signal used is a DSB-SC modulated signal, with message signal frequency $f_m = 16Hz$, carrier frequency used is $f_c = 1KHz$ and the sampling frequency is $f_s = 4KHz$. The PU data set scenario is similar to the once considered by Bhargavi *et al.* [7]. The Rayleigh channel coefficient is randomly generated at each run. Figure-1 show the P_d versus SNR plot for the new proposed method and it can be observed that the simulated results matches exactly with the theoretical value.

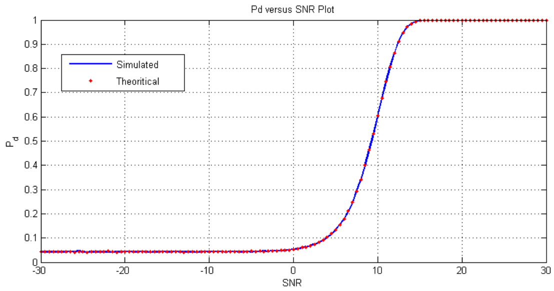

Fig. 1. P_d Versus SNR plot for New Proposed method

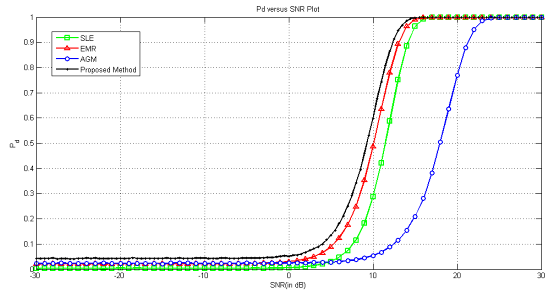

Fig. 2. P_d Versus SNR plot for n=7,m=8,$P_{fa} = 0.01$ and $p = 1$

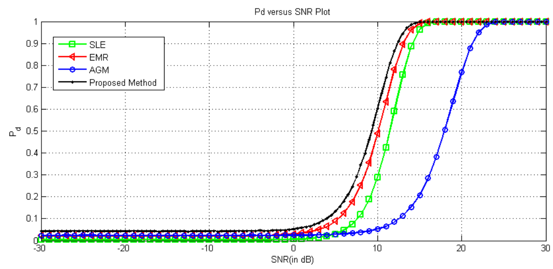

Fig. 3. P_d Versus SNR plot for n=12,m=15,$P_{fa} = 0.01$ and $p = 1$

A CR network which has n antennas to receive the signal at secondary user side and m is the number of samples collected at each antenna is used. Figure-2 shows the P_d versus SNR plot for cognitive radio network for $n = 7$ and $m = 8$ and it can be observed that the proposed method has better performance than existing blind detectors such as AGM, SLE and EMR detectors. Figure-3 presents the performance for $n = 12$ and $m = 15$ and here also the proposed technique shows better performance than existing blind detectors in low SNR and in data limited environment. Relative error of asymptotically derived threshold and simulated threshold of EMR and higher oder EMR detection is presented in Figure-4. Relative error formula and comparative analysis with other detection techniques have been analyzed by Huang *et al.* [14]. Plot shows that with low number of samples fourth-order EMR has small threshold error compared to EMR detection at low false alarm rate of 0.01. So fourth-order EMR is suitable candidate for IEEE 802.22 cognitive radio standard

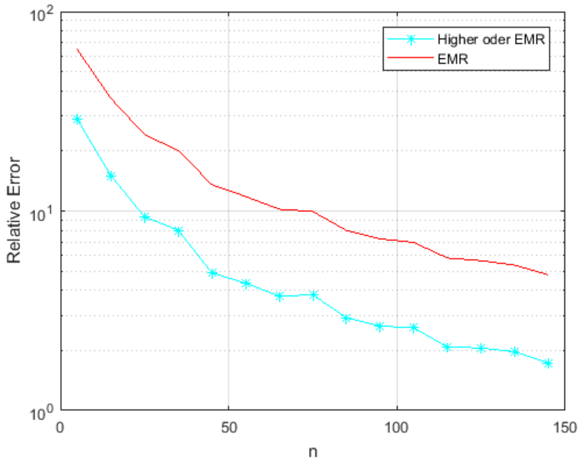

Fig. 4. Relative error of asymptotic theoretical threshold vesus number of samples for both methods. m/n=0.08 and $P_{fa} = 0.01$

that expects false alarm rate to be 0.1

V. Conclusion

In this paper a higer order EMR spectrum sensing is proposed, which provides desired performance in small sample environment. As the proposed method utilizes all the signal eigenvalues for detection, it can be considered superior to other blind detection techniques such as AGM, SLE and EMR. Asymptotic test statistic distribution of proposed technique is obtained in presence of primary signal. The theoretical probability of detection is also derived. It is shown that as order of EMR increases performance detection improves. Results presented are for the single primary user and fourth order moment show the superiority of result. In future work, limits on order of EMR and computational complexity issues will be discussed.

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