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A Low-complexity Suboptimal Algorithm for Joint Resource and Subcarrier Assignment in Downlink OFDMA System

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***Abstract*—Multiuser orthogonal frequency division multiple access (MU-OFDMA) technique has attracted much attention for broadband wireless access in future generation wireless communication systems. To achieve high data rate under constraint conditions, systems require proper resource allocation. This demands, sophisticated signal processing algorithms, which are computationally complex. This paper proposed a low-complexity suboptimal joint algorithm for resource allocation (task-1) and subcarrier assignment (task-2). The power allocation is carried out using the water-filling algorithm. The proposed algorithm is compared with existing Hungarian algorithm (HA) and amplitude craving greedy (ACG) algorithm for spectral efficiency, fairness among the users and computational complexity. The simulations results show that the proposed algorithm performs close to the HA. It maintains high fairness among the users and has reduced computational complexity compare to the existing algorithms.**

***Index Terms*— OFDMA, resource allocation, subcarrier assignment, water-filling algorithm, Hungarian algorithm, fairness ratio.**

I. INTRODUCTION

Orthogonal frequency division multiple access (OFDMA) has been adopted as the promising multiaccess technology in the next generation broadband wireless communication systems, such as worldwide interoperability for microwave access (WiMAX) and the third generation partnership project (3GPP), long term evaluation (LTE) and long term evaluation-advanced (LTE-A) [1, 2]. OFDMA permits multiple users to transmit simultaneously on different subcarriers. Due to orthogonality between subcarriers and by exploiting multiuser (MU) diversity along with adaptive resource allocation (ARA) in term of subcarrier assignment (SA) and power allocation (PA), OFDMA can improve the general system performance [3–5]. Owing to limited number of subcarriers and the transmit power, proper resource allocation (RA) among multiple users is essential to guarantee a fairness in terms of quality of service (QoS) [6, 7]. This requires proper resource allocation algorithms to fulfill the demand under constraint conditions [8, 9].

The extensive studies have been carried out on subcarrier and power allocation in wireless communication [10, 11], and a survey of different adaptive power and bandwidth allocation algorithms for the downlink MU-OFDMA system is presented in [12]. An efficient assignment of a specific set of the subcarrier Ω_k to a particular user k can maximize the overall spectral efficiency of the system. Hence, to maximize the spectral efficiency and system performance; sophisticated resource allocation algorithms are employed [9, 11, 13]. Shen et. al. [7] presented a resource allocation in the MU-orthogonal frequency division multiplexing (OFDM) system to obtained a variable proportional rate constraint. The authors proposed an optimal algorithm to maximize the sum capacity while maintaining the fairness among the users. Kivanc et. al. [8] analyzed the resource allocation problem and considered it as two different closely related tasks. The first task decides number of subcarriers required for each user to fulfill the minimum data rate requirement, which is termed as RA (task-1). Whereas, the second task aims at assigning a specific set of subcarriers (Ω_k) to the user (k), which is termed as SA (task-2). Authors used the bandwidth assignment based on signal to noise ratio (BABS) algorithm for task-1 and amplitude craving greedy (ACG) algorithm for task-2. Lengoumbi et al. [14] presented bandwidth allocation on rate estimation (BARE) algorithm for task-1 and rate profit optimization (RPO) algorithm for task-2.

In this paper, we analyze the resource allocation (task-1) and subcarrier assignment (task-2) problems under total transmit power constraint. Then, we proposed a low-complexity suboptimal algorithm to perform the tasks jointly. The power allocation among subcarriers is carried out using water-filling algorithm. The performance of the proposed algorithm is analyzed for different channel conditions and compared with existing ACG [8] and Hungarian algorithm [9].

Following this introduction rest of this paper is organized as follow, Section II, introduced the OFDMA system model and problem formulation. Section III, proposed the suboptimal algorithm for task-1 and task-2. Simulation results are presented

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in Section IV. Conclusions are drawn in Section V

II. SYSTEM MODEL AND PROBLEM FORMULATION

A cellular network system, which is formed by K uniformly located users in a cell with a single base station (BS) as shown in Figure 1 is considered. The resource allocation and subcarrier assignment problems are examined in the downlink scenario with OFDMA technology.

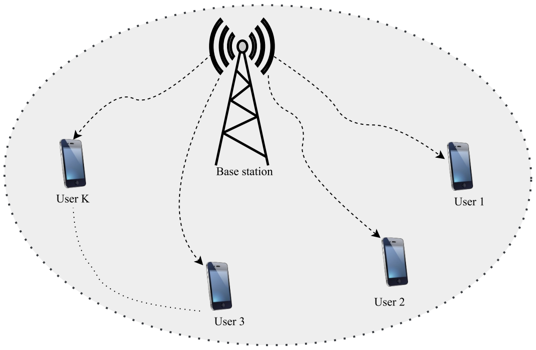

Figure 1. Downlink scenario

The channel consists of N independent parallel narrowband subcarriers which are distributed over the entire bandwidth B. Hence the channel gain $G(k,n)$, for k^{th} user on n^{th} subcarrier, can be expressed as [14]:

$$G(k,n) = C \times d(k)^{-\alpha} \times a_{sh}(k) \times A_f(k,n) \qquad (1)$$

where C is constant for a given environment, $d(k)$ is the distance of user k from the base station, α is path-loss exponent ($2 \leq \alpha \leq 4$). a_{sh} represents the shadowing effect which is log-normal variable with a standard deviation σ_{sh}, ranges from $4dB \leq \sigma_{sh} \leq 12\ dB$. A_f is the small scale fading parameter with Rayleigh distribution. In flat fading, subcarriers suffer due to additive white Gaussian noise (AWGN), which is a zero mean normal distributed random variable with standard deviation σ. The variance and the channel gain to noise ratio (CGNR) for user k on subcarrier n can be presented respectively as:

$$\sigma^2 = N_0 B/N \qquad (2)$$

$$\text{CGNR}(k,n) = \frac{G(k,n)}{\sigma^2} \qquad (3)$$

where N_0 is the thermal noise power spectral density. Since the wireless system can have a finite amount of total transmit power (P_T), so it is necessary to allocate power among the subcarriers subjected to a total power constraint [15]. The CGNR is averaged over the number of users to find the power for each subcarrier

$$\text{CGNR}'_n = \frac{\sum_{k=1}^{K} \text{CGNR}(k,n)}{K}. \qquad (4)$$

The CGNR$'$ along with total transmit power (P_T) can be used to calculate the power for each subcarrier using the water-filling algorithm. Once the power for each subcarrier is calculated, the received signal to noise ratio (SNR) for user k on subcarrier n can be expressed as:

$$\rho_{k,n} = p_n \times \text{CGNR}(k,n) \qquad (5)$$

where p_n is the power for subcarrier n, obtained from water-filling algorithm. Here the power is assumed to be independent of the users and depends on the subcarriers. The total power of a particular user will be the sum of the power allocated to their assigned subcarriers.

If $R_{k,n}$ is the data rate on user k for subcarrier n, the quantities are related as:

$$R_{k,n} = f(p_n \times \text{CGNR}(k,n)) \qquad (6)$$

The function f($\cdot$) is refers to the power-rate function [8]. Using the Shannon's theoretic approach, we can set:

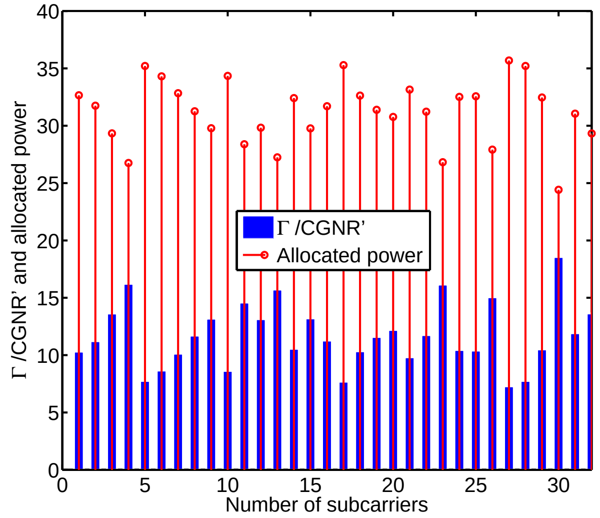

Figure 2. Power allocation using waterfilling algorithm for K=8, N=32, P_T = -30 dBm/Hz

$$R_{k,n} = B * \log_2\left(1 + (p_n * \text{CGNR}(k,n))/\Gamma\right) \qquad (7)$$

where Γ is the SNR gap and can be expressed in terms of bit error rate (BER) [16] as follows:

$$\Gamma = -\ln(5 \times \text{BER})/1.5. \qquad (8)$$

From (5) and (6) it can be observed that $R_{k,n} = f(\rho_{k,n})$, hence for unit bandwidth (7), it can be rewritten as:

$$R_{k,n} = \log_2\left(1 + (\rho_{k,n}/\Gamma)\right). \qquad (9)$$

Maximizing the data rate subjected to the total transmit power constraint namely,

$$\sum_{\text{n=1}}^{\text{N}} p_n \leq P_T \qquad (10)$$

by using Lagrange multiplier method results in following power distribution [15],

$$p_n = \left[\mu - \frac{\Gamma}{\text{CGNR}'_n}\right]^+ \qquad (11)$$

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where μ is a constant selected to satisfy the total transmit power constraint (10) and $[\cdot]^+$ is a function that clamps negative values to zero. This adaptive distribution is called **"spectral water-filling"** [15, 17]. Figure 2 shows the power allocation using the water-filling algorithm among the subcarriers. It represents the noise level and allocated power (mW) for different subcarriers with 8 users, considering -30 dBm/Hz total transmit power in the system.

In this paper, perfect channel state information (CSI) is assumed, i.e., the receiver has complete knowledge of the channel conditions (1). The targeted data rate vector $\mathbf{r}_k^0 = [r_1^0, r_2^0, ...r_K^0]^T$, defines the minimum data rate constraint of each user. However in our case the aim is to maximize the rate r_k, where r_k is the data rate for the set of subcarriers Ω_k allocated to particular user k. The data rate for user k can be expressed as:

$$r_k = \sum_{n\in\Omega_k} R_{k,n}, \tag{12}$$

the power for each user k having assigned subcarriers Ω_k can be calculated as:

$$p_k = \sum_{n\in\Omega_k} p_n \tag{13}$$

Since the power for each user depends on the number of subcarriers assigned to the user, p_k will also satisfy the power constraint condition (10). Therefore, the RA problem can be considered similar to the work by Lengoumbi et. al. [14], where the task is to find:

$$\max \sum_{k=1}^{K} \sum_{n\in\Omega_k} \log_2\left(1+\frac{p_{k,n}G\left(k,n\right)}{N_0\frac{B}{N}}\right), \tag{14}$$

subjected to:

$$p_{k,n} \geq 0 \qquad \text{for all k, n}, \tag{15}$$

$$\sum_{k=1}^{K} \sum_{n\in\Omega_k} p_{k,n} \leq P_T, \tag{16}$$

$$\begin{aligned} &\Omega_k \text{ are disjoint for all k}, \\ &\Omega_1 \cup \Omega_2 \cup \Omega_3 \ldots, \cup \Omega_K \subseteq \{1, 2, 3 \ldots, N\}. \end{aligned} \tag{17}$$

$$\text{Also we have that : } \; p_{k,n} = 0 \;\text{ if }\; x_{k,n} = 0 \quad \forall k, n, \tag{18}$$

In a OFDMA system, a subcarrier can not be shared by more than one user. Let $x_{k,n}$ be the assignment index; where $x_{k,n} = 1$, if n^{th} subcarrier is assigned to k^{th} user otherwise $x_{k,n} = 0$. So we have $\sum_n x_{k,n} = 1$ and $\sum_k x_{k,n} = 1$. The proposed algorithm does not define any minimum rate constraints. It just optimizes the subcarrier assignment by assigning more subcarriers to a user with the lowest rate, this lead to an increase in the fairness of the system.

To understand the system discussed above, the problem is divided into two closely related tasks in a similar way as described by Kivanc et. al. [8]:

A. Resource Allocation: Task 1

In this task, the number of subcarriers ω_k to be allocated to each user k is decided on the basis of minimum data rate requirement of particular user considering their demand. Lengoumbi et. al. [14] presented BARE algorithm, and Kivanc et. al [8] proposed BABS algorithm for task-1. In this work, we have employed the BARE algorithm for this task.

B. Subcarrier Assignment: Task 2

Once the number of subcarriers ω_k for each user k is determined, it is required to choose the best set of subcarriers Ω_k for each user; so that the sum of supported data rates are maximized. This problem is termed as subcarrier assignment. Various algorithms have been investigated for this task. Some of the popular algorithms include, Hungarian [9], ACG, rate craving greedy (RCG) [8], and RPO algorithms [14]. Note that we denote the number of subcarriers in Ω_k by ω_k.

Among the above-discussed subcarrier assignment algorithms, the Hungarian algorithm also known as the Kuhn-Munkres algorithm is most computationally complex. The computational complexity of the algorithm is $O(N^4)$ [18], where N is the number of OFDM subcarriers in the system. The high complexity of the above discussed existing algorithms, motivated us to develop a low-complexity suboptimal joint algorithm for task-1 and as well as for task-2.

III. PROPOSED SUBOPTIMAL ALGORITHM

The existing resource allocation algorithms have high computational complexity. Considering this, in this section, we have proposed a low complexity joint algorithm for task-1 and task-2. The proposed algorithm is described in Algorithm 1.

In the algorithm, $\mathbf{r}$ is the vector containing the data rate of each user. $\mathbf{s}$ is also a vector, used to store the position of the subcarriers already assigned and another vector β is used to store the indexes of the assigned subcarriers. The size of $\mathbf{s}$ and β are $1 \times N$. Here k_{min} represent the index for minimum data rate user in $\mathbf{r}$, and n_{best} indicates the highest data rate (best) subcarrier among remaining unassigned subcarriers.

The algorithm provides the suboptimal assignment of the N subcarriers to K users. It aims to maximize the sum capacity under total power constraint while maintaining the fairness among the users. The proposed algorithm does not assure the minimum rate requirement; it just optimizes the data rate of the users so that the data rate of the poor user can be maximized to maintain the fairness among the users. The algorithm performs both the tasks jointly. Vector $\mathbf{r} = [r_1, r_2, r_3, \cdots, r_K]^T_{K\times 1}$ has the data rate of each user k considering the assigning of a specific set Ω_k of subcarriers as described by (12). s_n takes the value of 1 when the subcarrier n is assigned. β received the column index of the best subcarrier (n_{best}). The sum of the $\mathbf{r}$ will be the total capacity of the system.

IV. SIMULATION RESULTS AND DISCUSSION

The performance of the proposed algorithm is evaluated with an Intel i3-2100 CPU having a 3.1 GHz clock and 4 cores, 4 GB of RAM running in Window 8.1, 64-bit, operating system. The

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Algorithm 1 Proposed suboptimal algorithm.

```
for $k = 0$ to $K - 1$ do
    $r_k \longleftarrow 0$
end for
for $n = 0$ to $N - 1$ do
    $s_n \longleftarrow 0$
    $\beta_n \longleftarrow 0$
end for
$[p_n]$= water-filling($\mathrm{CGNR}', P_T$)
Calculate $R_{k,n}$ from (7)
for $n = 0$ to $N - 1$ do
    $k_{min} \longleftarrow \arg\min_{k}(\mathbf{r})$
    $n_{best} \longleftarrow \arg\max_{n:s_n=0} (R_{k_{min},n})$
    $r_{k_{min}} \longleftarrow r_{k_{min}} + R_{k_{min},n_{best}}$
    $\beta_{n_{\mathrm{best}}} \longleftarrow k_{\mathrm{min}}$
    $s_{n_{best}} \longleftarrow 1$
end for
Total capacity = $\sum_{k=0}^{K-1} r_k$
```

total channel bandwidth was considered as 1 MHz. The power per subcarrier was calculated using the water-filling algorithm and σ_{sh} was fixed to 8 dB. The path loss coefficient and path loss exponent were considered as 10^{-4} and 2.8 respectively, while noise power spectral density was taken as -174 dBm/Hz. The maximum distance of a user was assumed as 5 km. The comparison is carried out in terms of bandwidth power product in which the system bandwidth is fixed and total power of the system varies. Hence the parameter bandwidth power product $(B \times P_T)$ is the multiplication of the system's bandwidth (B) and total transmit power (P_T) of the system.

Here we would like to compare the proposed algorithm with existing algorithms for total capacity with a different number of subcarriers. The effect of σ_{sh} on the capacity of the system was analyzed. The capacity per user having different total transmit power (P_T) and fairness among the users was also be analyzed. Finally, the computational complexity regarding the execution time was evaluated[1]. The BARE algorithm is used for task-1 where as Hungarian, ACG, and Default algorithms are used for task-2. The proposed algorithm work for both the tasks jointly.

First, we will validate the performance of the proposed algorithm for the total capacity. Figure 3, represents the total capacity versus bandwidth power product of the system under consideration. The performance of the proposed algorithm is compared with existing Hungarian, ACG and default algorithms for a different number of subcarriers (N= 64,128). The expanded view for N=128 is also presented to show the proximity of the proposed algorithm with the Hungarian. It can be observed that the proposed algorithm performs close to the Hungarian algorithm in both the cases and has superior performance compare to the ACG and Default algorithms. The performance degradation remains between 0.08 % to 0.10 % of the Hungarian algorithm.

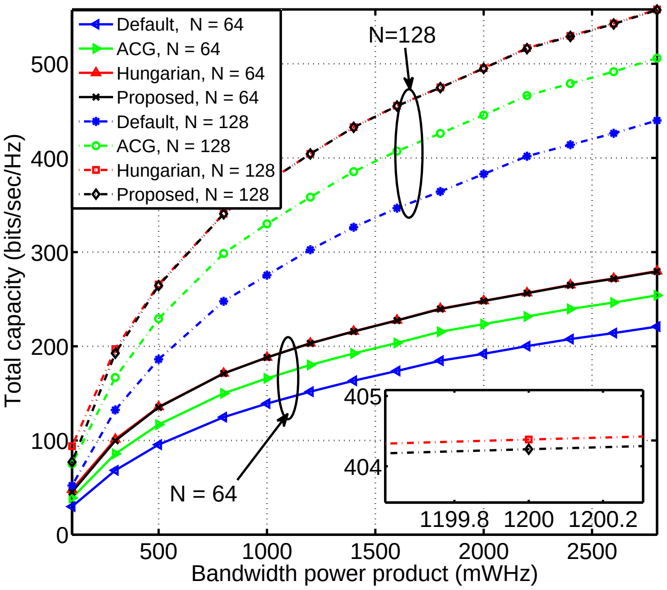

Figure 3. Comparison for the total capacity having K= 8 and N=64, 128

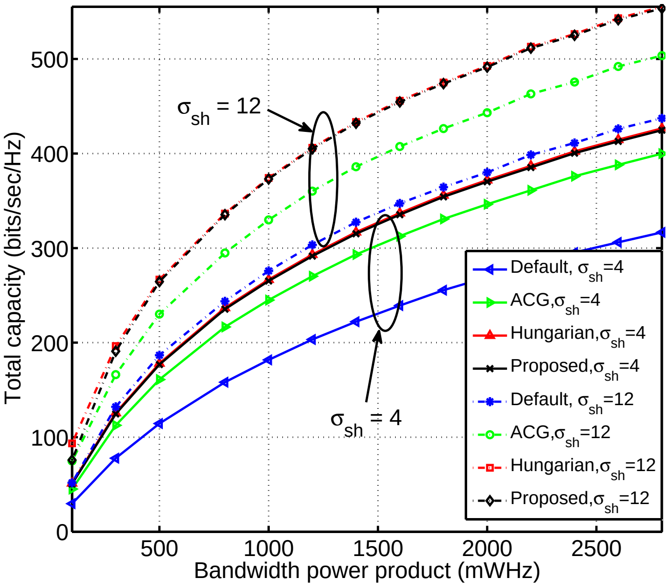

Figure 4. Comparison for the total capacity for σ_{sh}= 4 dB, 12 dB, having K=8 and N= 128

The channel conditions are the key aspects of the wireless system to consider. In Figure 4, we analyzed the performance of the algorithm under different shadowing effect by variating the standard deviation σ_{sh} from 4 dB to 12 dB. It can be noticed that the performance of the algorithm is independent of the channel conditions and has the stable performance in all the channel conditions.

The simulation results for different aggregated transmit power is presented in Figure 5. The figure represents the

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[1] This work uses the default algorithm for task-2, which is just a random assignment of the subcarriers to the users. The default algorithm also uses the BARE algorithm for task-1 and water-filling algorithm for power allocation.

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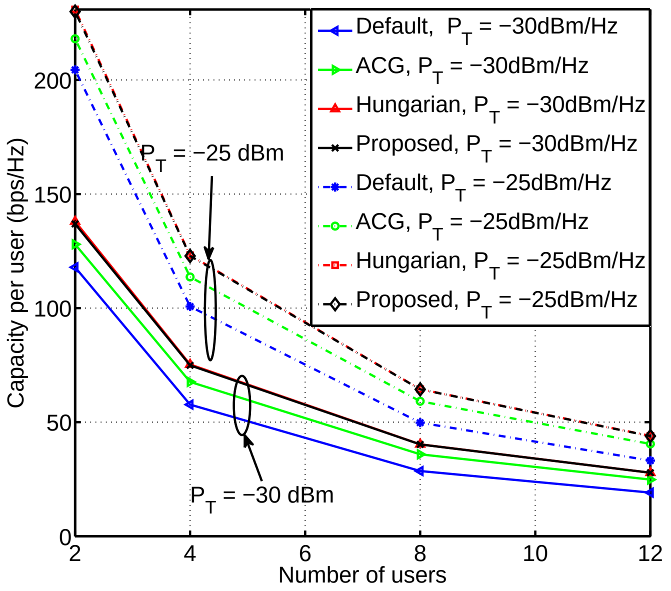

Figure 5. Comparison for capacity per user for P_T= -30 dBm, -25 dBm, having N= 128

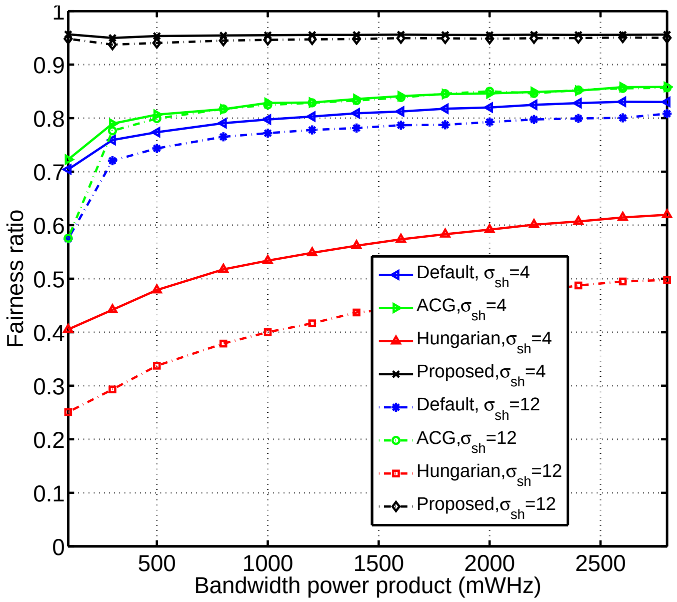

Figure 6. Comparison for fairness among the users, K=8, N=128

capacity per user versus the number of users in the system, under total transmit power -30 dBm and -25 dBm. As per Shannon's capacity theory, total transmit power in the system is approximately exponentially proportional to the system capacity. Hence a larger transmit power, result in a higher system capacity. Since the wireless system can have only some finite amount of transmit power, the power constraint is an important factor to consider. Here the power distribution among the subcarrier is performed using the water-filling algorithm. From the figure, it can be observed that system supports higher capacity per user with increase in P_T. It can be deduced from the figure that, the system capacity per user is reduced by the increasing the number of users in the system. This is because resource are distributed among more users.

Figure 6, shows the comparison of the fairness ratio of proposed algorithm. The fairness ratio is defined by the fairness factor (F). The fairness factor is the ratio of the best user (user with highest data rate) to the worst user (user with lowest data rate). The maximum value of F can be unity, i.e., all users have same data rate. It can be observed that the proposed algorithm has highest fairness factor ($F > 0.95$). Its performance is better than Hungarian algorithm since the Hungarian algorithm optimized the total system capacity, but not the rate of individual user. This results in a worst fairness factor for Hungarian algorithm. Varying the shadowing effect by changing the value of σ_{sh}, the fairness factor of the proposed algorithm degrades slightly. Hence it can be deduced that the proposed algorithm provides a stable fairness among the users even with large shadowing effect.

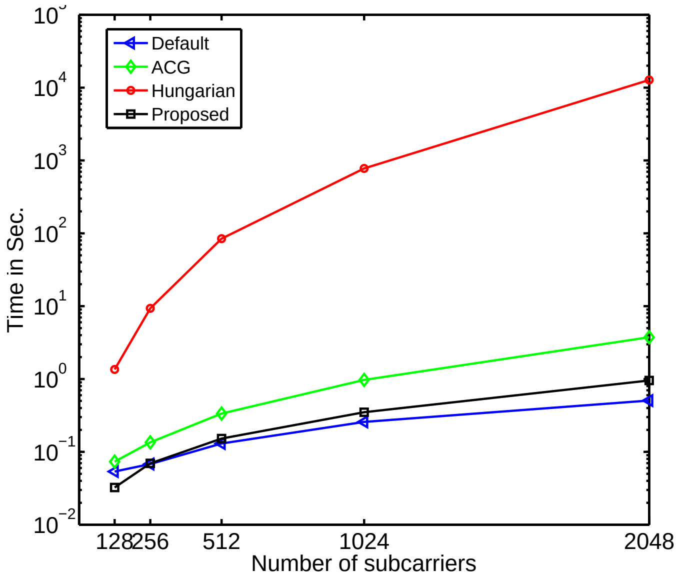

Figure 7. Computational complexity in term of execution time, K=8

A. Computational complexity analysis

The computational complexity of the Hungarian algorithm is $\mathcal{O}(N^4)$ [18]. On the other hand, the computational complexity of the ACG algorithm is $\mathcal{O}(N^2)$. The proposed algorithm has the K operation to find the minimum data rate and N operations to get the best subcarrier. Both of these processes executed N times. Hence the joint worst case computation complexity of the proposed algorithm to perform both tasks is $\mathcal{O}(N+K) \times N$. The worst case complexity of the proposed algorithm can be simplified to $\mathcal{O}(N^2)$ when $N >> K$. The complexity of the default algorithm is $\mathcal{O}(N)$. Figure 7, represents the execution time taken by the algorithms to complete both tasks. Hence the execution time of the BARE algorithm is included in the execution time of Hungarian, ACG and default algorithms. It can be noticed that the proposed algorithm offers

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low complexity in comparison to the existing ‘BARE+ACG’ and ‘BARE+Hungarian” algorithms.

V. CONCLUSION

In this paper, the resource allocation problem is studied in OFDMA wireless system. A suboptimal low-complexity joint algorithm for task-1 and task-2 was proposed. The proposed algorithm performs in the proximity (0.08% to 0.10%) of Hungarian algorithm and outperforms the ACG algorithm in terms of spectral efficiency. It is also verified through simulation results that the proposed algorithm has high fairness factor ($F > 0.95$) even with varying channel conditions. Evaluation of computational complexity of proposed algorithm was carried out and it was observed that it has smaller execution time compared to Hungarian and ACG algorithms.

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