Comprehensive study on a functionally graded Carbon Nanotube reinforced dielectric elastomeric nanocomposite microbeam resonator

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Abstract: Microelectro mechanical systems have widespread applications as sensing elements in manufacturing operations. Small scale, self-energizing sensors are very much required in the modern machining industries. This work presents use of functional graded nanopolymer composites as a microbeam sensing elements in milling application towards prediction of workpiece displacement. A model to investigate the dynamic-pull-in characteristics of a functionally carbon nanotube (FG-CNT) reinforced polymer composite microbeam is developed. Based Eular-Burnoulli theory the dynamic governing equation of an electrostatically actuated micro resonator is derived. The material properties of the FG-CNT composite microbeam are estimated using modified Halpin-Tsai model and rule of mixture. A squeeze film damping is accounted along with electrostatic actuation. The influences of voltage effect, volume fraction, and distribution of CNTs and initial amplitude on dynamic-pull-in behaviors of the microbeam are discussed. A dynamic model of machining process is considered to illustrate the relative motion sensed by the microbeam resonator.

Keywords: Microbeam resonator; Functionally graded material; Electrostatic actuation; Halpin-Tsai model.

1. Introduction

Micro-Electro-Mechanical-Systems (MEMS) technology resulted in high performance components in applications as mass sensors, gyroscopes, accelerometers and others. Modern manufacturing operations make use of MEMS/NEMS technology for efficient production management systems. MEMS sensors and actuators are replacing the conventional systems due to their high sensitivity and effective operating ranges. Recently, in MEMS technology, nanocomposite materials are finding widespread applications in comparison with Silicon based systems due to their several advantages. As one of the important nanocomposite materials, the carbon nanotube (CNT)-reinforced polymers are gaining more attention. Due to their dramatic mechanical, thermal and electrical properties with high stiffness and strength, light weight and high aspect ratio, small quantities these nano-reinforcements are used in matrix materials like polymer, ceramic and metals to achieve the desired properties. Such nanocomposite materials have applications in various domains like as wind-turbine blades, foundation dampers, microbeam resonators as well as in energy-harvesting devices. Microbeams made-up of CNT nanocomposites show superior dynamical actuation or sensing capabilities. Functional grading of carbon nanotubes in polymers results in effective dynamic behavior at microscale structures.

Over the past two decades, MEMS received much attention due to small sizes, high durability, low energy consumption and weight[1–3]. In MEMS, sensing or actuation phenomena takes place differently. There are three common techniques often come into picture: (i) piezoelectric (ii) electromagnetic and (iii) electrostatic systems [4]. Among them electrostatic actuation is well stabilized and preferable due to its efficiency and simplicity. Typical MEMS resonator has two conductive electrodes, in which one is fixed and other is movable. An electric load consists of a DC and AC polarization is applied across the electrodes to deflect the beam by DC component and driven to vibrate by the AC harmonic load. A pull-in phenomenon usually occurs in MEMS resonators due to electrostatic actuation when the applied electrostatic force exceeds the system restoring force. This causes large defection of moveable electrode which results into collapse of the MEMS flexible structure and finally failure of system occurs. The corresponding pull-in voltage is a measure of stable operating margin of the system [5–8].

The increasing applications functionally graded materials in engineering led to the foundation study of micro scaled functionally graded (FG) beam systems. This makes them superior candidate for application such as micro/nano electromechanical system (MEMS/NEMS) [9], atomic force microscopes [10], micro resonator [11], micro/nano switches [12,13] and micro mass sensor [14] due to their high sensitivity and desired performance. These wide range of applications of micro scaled FG structures resulted in this area as a recent research topic. A good number of works have been found in literature to investigate static and dynamic behavior of these structures and identify the intrinsic differences from bulk FG structures [15–18]. As classical continuum theories are incapable to analyze the micro scale structures, a formulation based on non-classical theory (i.e. strain gradient theory) of size dependent FG Eular-Bernoulli beam is developed by Kahrobaiyan et al. [19]. They observed that non-classical FG beams shows stiffer behavior compared to classical one and difference in results of non-classical and classical theory increases with decrease in thickness to length scale parameter of FG-microbeam. Salamat-talab et al. [20] conducted static and dynamic investigation of FG microbeam using modified couple stress theory and third order shear deformation model. They have suggested that size effect is significant for higher vibrational modes. Few other studies related to static bending [21]; vibration [22]; buckling and post-buckling [23] and thermal effect [24] on analysis of FG microbeam are found in recent years.

Microbeam resonators are used as sensors in manufacturing applications. Displacement of the microbeam resonator platform mounted beneath the workpiece due to machining forces can be predicting in terms of electrostatic forces induced in the circuit. Some of the early work in this regard is cited below. Li et al. [25] proposed a three component force sensor design for milling process applications. Sensor structure with 8 parallel elastic beams was considered and Wheatstone bridges were used to convert the displacements into potential differences. Liang et al. [26] presented a review on the microbeam sensors for various fields like biomedical, materials science, industrial automation, dimension measurements in microcomponents and nanomanufacturing engineering. Various principles of force and moment sensing were also described. Application works of microbeams as sensors in machining operations are still open areas and very limited work is available in this line.

Present work deals with the design of an electro-statically excited FG-CNT reinforced polymer composite microbeam resonator for its application in sensing the workpiece deflections during machining operations. During the design of resonator, initially the dynamic-pull-in voltage as a function of microbeam length and other parameter is presented. FG microbeam analysis with electrostatic actuation is first illustrated with lumped-parameter model. Effect of squeeze-film damping between the beam and substrate surface and fringing field around the electrode surfaces are considered. Further, modeling of the microbeam resonator mounted with workpiece under the action of machining forces is presented with spring-mass flexible system. Practical realization of the system is described in latter part of the paper.

2. Design of FG microbeam resonator

Figure 1 shows the schematic of the microbeam resonator subjected to electrostatic force across fixed substrate and moving beam. The resonator is considered as a long and thin microbeam with the length *L* and width *b*, and *h* respectively. The initial gap between the electrodes is g. The coordinate system is located at the neutral axis middle-left end of the resonator, where *x* and *z* represent the horizontal and perpendicular directions, respectively.

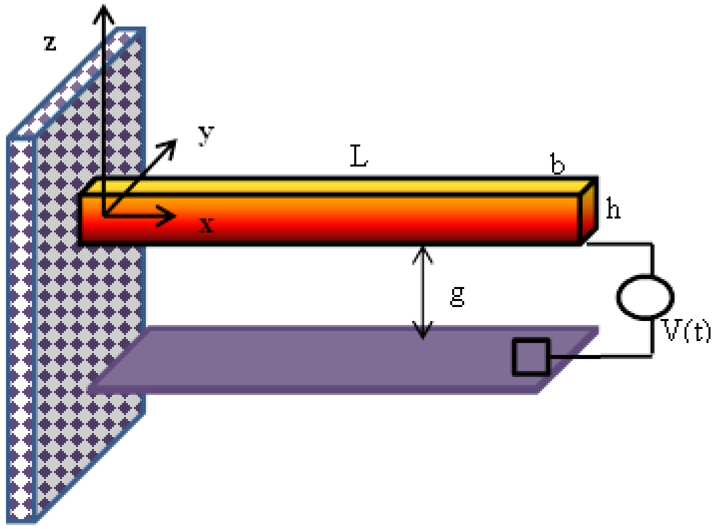


Fig. 1 A schematic representation of microbeam resonator

The governing equation of motion of vibrating micro-actuator is expressed as:

$$m\frac{d^2w}{dt^2}+(EI)_{eq}\frac{\partial^4 w}{\partial x^4}+F_{danp}=F_{es} \tag{1}$$

Where m is mass per unit length of beam, F_{ex} denotes the electrostatic force, Fdamp represents the squeeze-film damping at the air-gap. In order to describe the electrostatic actuation force, a more realistic situation including 'fringing field' modification field is taken into account. The electrostatic force per unit length is given as:

$$F_{es}=\frac{1}{2}\frac{b\varepsilon V^2(t)}{(g-w)^2}\left(1+0.65\frac{(g-w)}{b}\right) \tag{2}$$

where $\in$=8.854×10^{-12} F/m is the vacuum permittivity and $V(t)=V_{dc}+V_{ac}\cos\Omega t$ is input voltage.

The effective flexural rigidity $(EI)_{eq}$ of FG microbeam can be expressed as:

$$(EI)_{eq}=b\int_{-h/2}^{h/2} z^2\left[E(z)\right]dz \tag{3}$$

where *E(z)* is calculated in terms of volume fraction distributions from the Halpin-Tsai empirical relation as follows:

$$E(z)=\left\{\frac{3}{8}\left(\frac{1+\xi_L\eta_L V_{cnt}}{1-\eta_L V_{cnt}}\right)+\frac{5}{8}\left(\frac{1+\xi_T\eta_T V_{cnt}}{1-\eta_T V_{cnt}}\right)\right\}E_m \tag{4}$$

Here, $\xi_L=\frac{2L_{cnt}}{D_{cnt}}$, $\xi_T=2$, $\eta_L=\frac{(E_{cnt}/E_m)-1}{(E_{cnt}/E_m)+\xi_L}$ and $\eta_T=\frac{(E_{cnt}/E_m)-1}{(E_{cnt}/E_m)+\xi_T}$ are longitudinal and transverse efficiency parameters. Also, the volume fraction for FG beam is distributed in thickness direction according to any of the following types:

$$V_{cnt}(z) = V_{cnt}^{*} \qquad \text{for uniform distribution} \tag{5}$$

$$= \left(1+\frac{2z}{h}\right)V_{cnt}^{*} \quad \text{for A-type distribution}$$

$$= \left(2-4\frac{|z|}{h}\right)V_{cnt}^{*} \quad \text{for ◊-type distribution}$$

$$= 4\frac{|z|}{h}V_{cnt}^{*} \quad \text{for X-type distribution}$$

with $Vcnt^*$ as volume fraction of CNT fibers. Viscous damping of microstructures vibrating in air in the narrow gap between two electrodes is dominated by squeeze film damping. Damping strongly affects the dynamics, control, performance and design of MEMS. The effects of damping on the dynamics of MEMS depend on their design and operating conditions. In order to model squeezed film damping, F_{damp} is calculated from two dimensional Reynolds equation for the fluid flow given by

$$\frac{\partial}{\partial x}\left(\frac{\rho_a g^3}{12\mu}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{\rho_a g^3}{12\mu}\frac{\partial p}{\partial y}\right)=\frac{\partial(\rho_a g)}{\partial t} \tag{6}$$

where μ, p, ρ_a and g indicates the effective air viscosity, pressure, air density and air-film thickness respectively. Blench model analytically solves the Reyond's equation with trivial boundary conditions. The squeeze film damping coefficient is expressed by considering first term of Blench series as follows:

$$c_s = \frac{768\mu A^2}{\pi^6 (g-w)^3}\frac{2}{\left[4+\left(\frac{\sigma}{\pi^2}\right)^2\right]} \tag{7}$$

where

$$\sigma = \frac{12A\Omega\mu}{p_a (g-w)^2} \tag{8}$$

The effective dynamic viscosity is obtained using Veijola theory as:

$$\mu = \frac{\mu_0}{\left(1+9.638K_n^{\ 1.159}\right)} \tag{9}$$

where $\mu_0=18.3\times10^{-6}$ m^2/s, viscosity of air. K_n is a dimensionless measure of the relative magnitudes of the gas mean free path and flow characteristics length, called as Knudsen number given by

$$K_n = \frac{\lambda}{g-w} \tag{10}$$

and is equal to λ=0.064 microns in usual calculations.

As an illustration, the MEMS structure considered has 200×80×4.5 μm dimensions and 3 μm gap between substrate and beam. The following material data is considered: E=169 GPa and ρ=2331 kg/m^3 for pure silicon and E_p=2.5 GPa, ρ_p=1180 kg/m^3, E_{cnt}=1000 GPa, ρ_{cnt}=1300 kg/m^3, L_{cnt}=1 μm, D_{cnt}=1 nm for polymer and CNT reinforcement respectively.

First, static pull-in voltage is estimated for pure silicon micro beam resonator and a comparison is made with theoretical values as shown in Figure 2. Both approaches give a pull-in voltage approximately 138 volt and confirm the applicability of present model. In Figure 3, static pull-in voltage obtained for CNT reinforced polymer micro beam is depicted for different volume fraction distributions. In this analysis volume fraction of CNT is kept constant, $V_{cnt}=5\%$. It is observed that X-type distribution results in higher pull-in voltage.

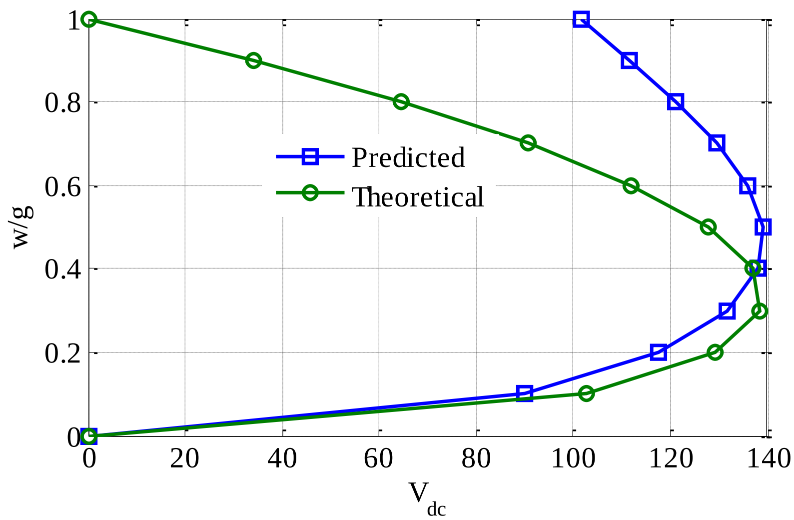


Fig. 2 Static pull-in comparison for present approach with theoretical results

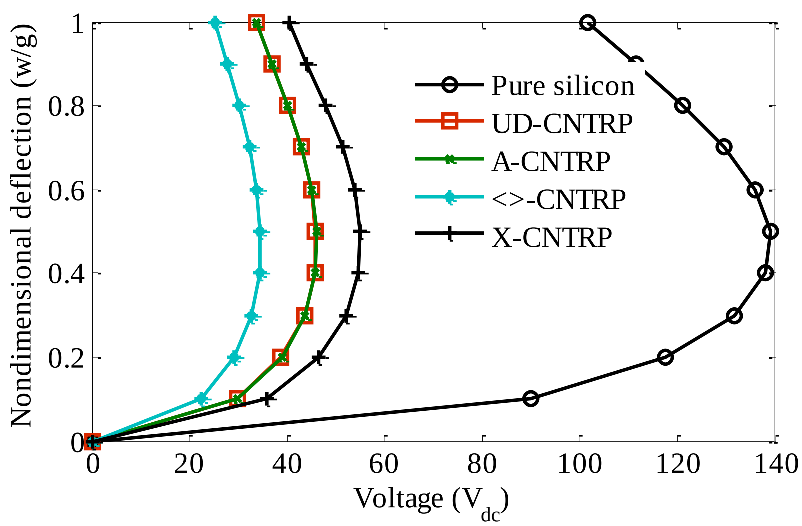
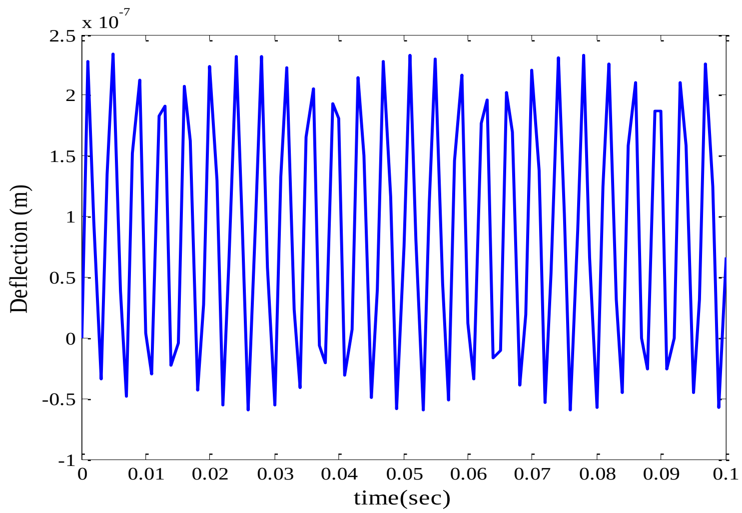
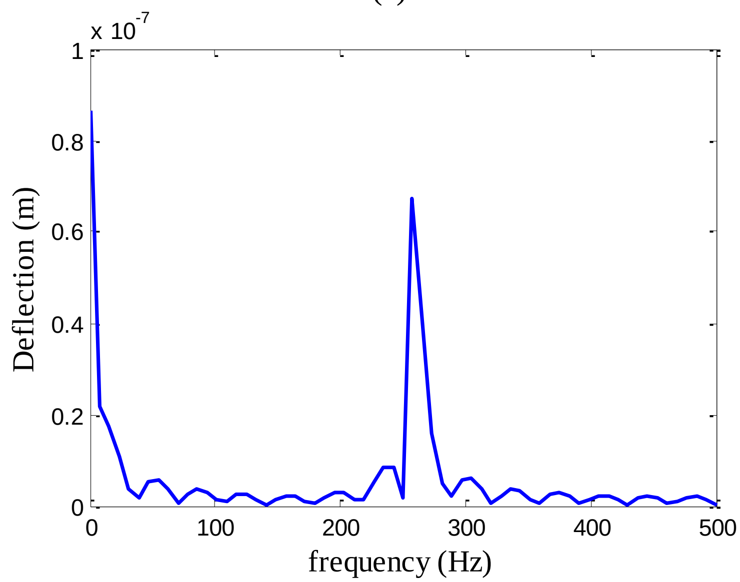


Fig. 3 Static pull-in analysis of CNT reinforced polymer composite microbeam

The dynamic behavior analysis of CNT reinforced polymer composite micro beam resonator is predicted by considering squeeze film damping and time dependent electrostatic force. The time and frequency response of CNT reinforced polymer composite micro beam are shown in Figure 4 at $V_{ac}=1$ volt, $V_{dc}=20$ volt. The pull-in instability occurs when deflection of free end of beam attains the value equals to gap. In figure it can be seen that defection of beam is lower than gap at this particular voltage and it behaves like a stable system.



(a)

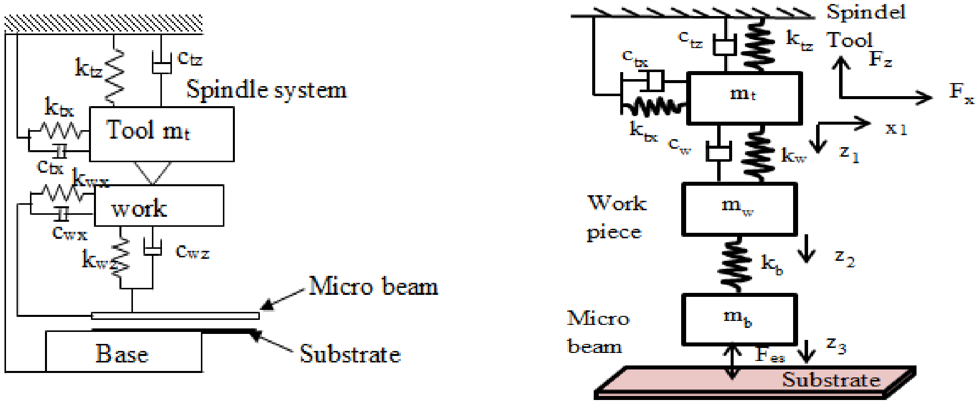


(b)

Fig. 4 Dynamic deflection response at V_{ac}=1 volt, V_{dc}=20 volt

3. Application of the resonant sensor in Machining

There are several applications of micromechanical sensors in machining domain. MEMS accelerometer is one such kind. Here, in this work, microbeam resonator application is illustrated to measure the displacements of workpiece during milling operation. Dynamic behavior of machine tool determines the surface quality during machining operations. Dominant frequencies are always related to intrinsic properties of machining systems, like components of machine structures, tools, fixtures and workpiece. Fig.5 (a) shows the dynamic model of the microbeam sensor mounted below the work piece during machining. A simplified model with proper notation of external and restoring forces is shown in Fig. 5 (b).



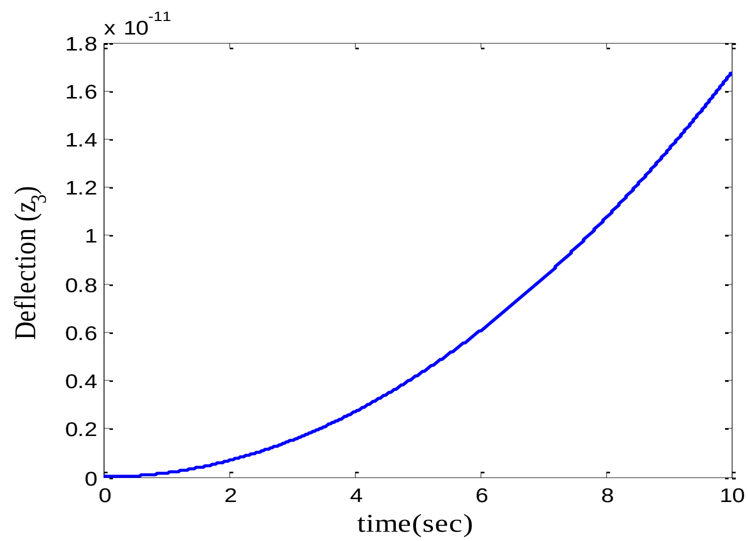
(a) Dynamic model (b) Simplified model

Fig.5

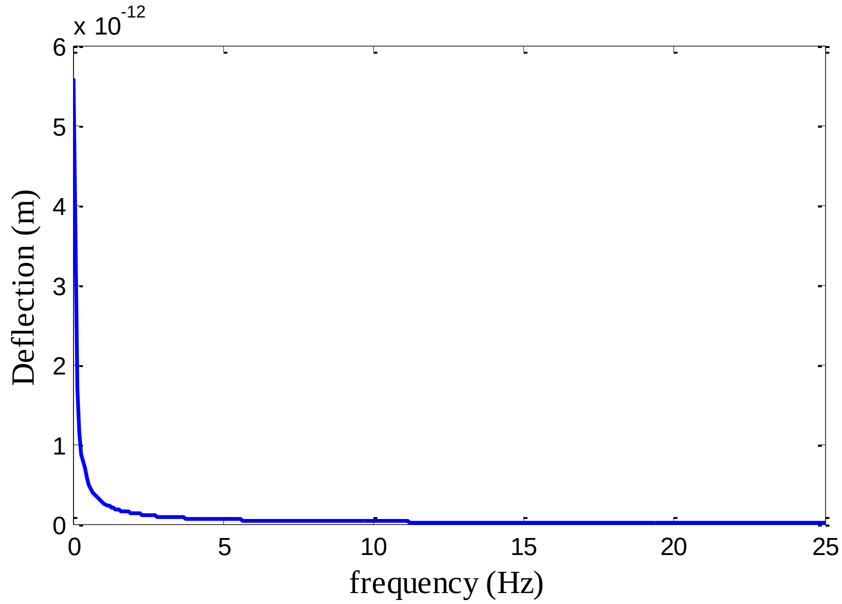
The equations of system can be written as

$$\begin{bmatrix} m_t & 0 & 0 & 0 \\ 0 & m_t & 0 & 0 \\ 0 & 0 & m_w & 0 \\ 0 & 0 & 0 & m_b \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{z}_1 \\ \ddot{z}_2 \\ \ddot{z}_3 \end{Bmatrix} + \begin{bmatrix} c_{tx} & 0 & 0 & 0 \\ 0 & c_{tz}+c_w & -c_w & 0 \\ 0 & -c_w & c_w & 0 \\ 0 & 0 & 0 & c_s \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{Bmatrix} + \begin{bmatrix} k_{tx} & 0 & 0 & 0 \\ 0 & (k_{tz}+k_w) & -k_w & 0 \\ 0 & -k_w & (k_w+k_b) & -k_b \\ 0 & 0 & -k_b & k_b \end{bmatrix} \begin{Bmatrix} x_1 \\ z_1 \\ z_2 \\ z_3 \end{Bmatrix} = \begin{Bmatrix} F_x \\ F_z \\ 0 \\ F_{es} \end{Bmatrix} \quad (11)$$

where m_t, m_w c_t, c_w, k_t, k_w are the mass, damping and stiffness of spindle tool and workpiece respectively. Also, m_b, k_b and c_s are the mass, stiffness and squeeze film damping of micro beam sensor. In this analysis, F_x and F_z are the time dependent tool forces exerted on workpiece, whereas F_{es} electrostatic forced between substrate and micro beam. Figure 6 shows the dynamic response of the sensor mass due to deflection of the workpiece with the following input data: m_t=1 kg, m_w=2 kg, $k_{tx}=k_{tz}=1\times10^5$ N/m, $k_w=0.5\times10^5$ N/m, $c_{tx}=c_{tz}=k_{tx}/100$ N-s/m, $c_w=k_w/100$ N-s/m, $F_x=0.1\times e^{-t}$ N, $F_z=0.2\times e^{-2t}$ N.



(a)



(b)

Fig. 6 Time and frequency responses of deflection observed by micro beam sensor.

The micro beam frequencies are of order 1 kHz to several mega Hz, so the machining force components of higher order frequencies are only tuned and the sensor can predict the vibrations of flexible workpiece system. The fabrication procedure of FG-CNT polymer composite is also simple and process is economical.

4. Conclusions

In present work, functional graded CNT reinforced polymer composite microbeam is implemented as a sensing element in milling operation towards prediction of workpiece deflection. A micro beam resonator was considered under an electrostatic actuation and squeeze film damping effect. Static and dynamic pull-in studies were conducted by approximate method and results were compared with theoretical one. Further, a four degree of freedom model is used for milling machining process with a micobeam sensor mounted on workpiece. Time and frequency responses at the resonator were obtained for a test case of machining forces. The results signify that micro beam sensor is capable to measure the very low frequency vibration in machining operation. The resonator needs to be fabricated and testing is required at CNC milling center in workshop. As a capacitive sensor the voltage output from the resonator dictates its sensitivity. Further, the scope for using the resonator as an energy harvesting device during machining operation has to be perused in future.

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