

EVALUATION OF EMPIRICAL EQUATIONS FOR DAM BREACH PARAMETERS AND ITS APPLICATION IN INDIAN DAM FAILURE CASES

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ABSTRACT

Our society gets huge benefits from the water storage dams, but the consequences are devastating if a dam fails. It causes extensive damage to the life and properties mostly due to short warning time. Important factors governing failure studies are the breaching parameters that help to quantify the risk associated with dam break floods. Many empirical equations have been developed for predicting the breaching parameters associated with peak outflow, and these equations are generally developed by regression analysis from the record of dam failure data. The present paper is focused to develop the empirical formulas for earthen embankments that can predict breach parameters and peak outflow based on past failures. The database of 157 past dam failure cases has been compiled with the inclusion of new 15 dam failure data from India. A multivariable linear and nonlinear regression method for both dimensional and non-dimensional form of breaching parameters and predictors is used to develop the relationship. The results obtained from the regression equations for breach parameters and peak outflow is compared with observed data and the uncertainties in the results are evaluated. The recently developed regression equations for breach parameter predictions along with the older equation for earthen dams are studied for Indian dam failures.

Keywords: Breach parameters, Dam safety, Earthen dam, India dam failure cases, Regression model

1 INTRODUCTION

Consequences of natural disasters are devastating causing immense loss to the human life and property. In recent times flood events comes under the top natural disasters. Around the world dam safety has gained maximum awareness as the dam failure produces flash floods at the entire downstream area in a short time. Therefore, in recent years the dam break analysis draws attention from the researchers. Dam break analysis includes data collection, estimation of breach parameters, derive dam breach outflow to the downstream, preparation of inundation map and then preparation of emergency action plan. Hence, the first step in dam break analysis is to predict accurate breach parameters and for that it is important to understand the dam breaching process. The breach parameters includes breach depth, average breach width, breach side slope, breach time and peak outflow.

In general three types of approaches are used for breach parameter prediction: (1) Comparative approach which is based on comparisons with one or more very similar dams that have failed. (2) Empirical formulas based on case study data, and (3) Physical based dam breach models. Many investigators such as Kirkpatrick (1977), SCS (1981), Singh and Snorrason (1984), MacDonald and Langridge-Monopolis (1984), Bureau of Reclamation (1988), Froehlich (1995a,b and 2008), Zhang (2009) and Pierce (2010) uses analytical approach to predict peak outflow as a function of various dam and reservoir parameters, with the empirical relation developed from case study data. On the other hand most of the physical based models depends on hydraulic principles and sediment transport formulas (e.g., Fread 1977, 1988), Singh and Scarlatos (1985)). Whal et al. (2008) summarized the progress on physical based dam breach model.

All three approaches have their shortcomings. First two models depend largely on case studies and that have high uncertainty. Physical based models lack understanding of the mechanisms of breach development and its inability to model that mechanism. Still, the most acceptable and widely used approach is parametric that make use of breach parameters calculated from regression based

formulas. The reliability of breach parameters are mostly depends on the number of case study involved in developing the regression equations. Froelich (1995a) uses 63 cases for the prediction of average breach width and 21 cases for the prediction of breach formation time. Wahl (1998) compiled 108 case study data and named it super compilation dam failure data. Xu and Zhang (2009) mention 75 case study data in his study with inclusion of some data from super compilation. He proposed nonlinear regression model for developing empirical equations. The present study pays great efforts in collecting the dam failures data from the available reports from India.

In India 37 dam failures are recorded out of which 15 dam failure case reports are available. The data of 156 dam failures are used for the study which includes Wahl (1998) super compilation, Zhang (2009) and 15 new cases. The data comprises with mostly homogeneous dams of about 61%. The variation of dam height is uniform from 1m to 60m with unknowns of 26.9%. Reservoir capacity largely varies between 1.0×10^6 to 1.0×10^8 m³. Most of the embankments failed due to overtopping (51.3%, seepage or piping failure consists of 37.8% and sliding failure consists of 5.8% of all the data, while the rest have failed differently. Table 1 represents dam failure data statistics. Further the details of 15 new cases from India are documented in Table 2. We note that the mentioning of dam erodibility as low, medium or high is based on construction era, soil type near the construction place and compaction methods used for making of dam.

Breach shape geometry is either taken as triangular or trapezoidal. Many case histories generally show trapezoidal breach shape. For defining geometric breach parameters as trapezoidal shape one needs to know any three combinations of breach depth (H_b), breach top width (B_t), breach average width (B_{avg}), breach bottom width (B) or breach side slope. Fig 1 shows the breach parameter for trapezoidal shape geometry.

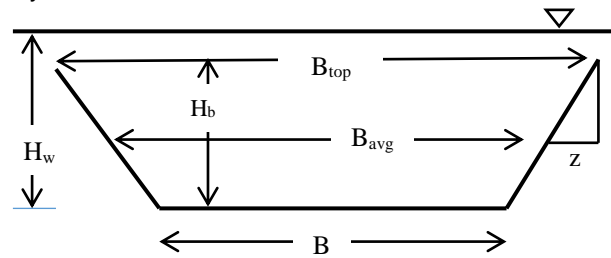


Figure 1 Geometric representation of breach parameters

Two more parameters are set for dam breach analysis. These are the peak discharge or peak outflow rate (Q_p) and failure time (t_f). Following Wahl (1988) failure time is divided into two phenomena that is breach initiation time and breach formation time. The breach initiation time begins with the first flow over or through a dam that will initiate warning, evacuation, or heightened awareness of the potential for dam failure. Breach formation time is the duration of time between the first breaching of the upstream face of the dam until the breach is fully formed. Practically breach initiation time is not possible to evaluate and continuous research is going on physical models of dam for evaluation of parameters. Therefore, the developed failure time equations are breach formation time. In dam break analysis the peak discharge and time of occurrence of peak discharge is important factor to be evaluated. This time of occurrence of peak discharge is dependent on the exact prediction of breach parameters and most importantly the breach formation time.

Table 1. Dam failure data statistics

Total Numbers of Dam Failure = 156			
Dam Type	Dam Height (m)	Reservoir Capacity (Mm ³)	Failure Mode
Homogeneous dams = 61%	<10 = 18.6%	<1.0 = 21.8%	Overtopping = 51.3%
Dam with corewalls = 10.9%	10-15 = 20%	1.0-10 = 17.3%	Seepage erosion/Piping = 37.8%
Zoned filled dams = 5.8%	15-30 = 18.6%	10-100 = 21.8%	Sliding = 5.8 %
Concrete faced dam = 3.2 %	30-60 = 18%	100-1000 = 6.4%	Unknown = 5.1 %
Rockfill & composite dams = 3.2%	60-100 = 3.8%	Unknown = 32.7%	
Unknown = 12.2%	Unknown = 6.9%		

Table 2. Indian dam failure list

SI.No.	Dam name	Year Built	Year Failed	Dam type	Dam height Hd (m)	Dam crest width	U/P slope (H:V)	D/S slope (H:V)	length (m)	Reservoir capacity V_d (10^6)	Design Flood Discharge (m^3/s)	Depth of water above breach invert H_w (m)	Failure mode	Dam erodibility	Observed peak Inflow (m^3/s)
1	Kodaganar	1974	1977	HD	12.75	6	2:1	2:1	2425	123.00	1274.26	13.2	O	ME	7079
2	Ghurlijor	1956		HD	12.19	3			593.3	2.179	204.46	13.99	O	HE	228
3	Panshet		1961	CD	63.56	6			765	310.61	1162	64.76	S & O	ME	10763
4	Khadakwasla	1879		CD	36.09	4			1539	86.00	2974.82	38	O	HE	16000
5	Palem Vagu		2008	HD	46	6	2.5:1	2.5:1	810	35.60	1416		P	LE	2425
6	Palem Vagu	unfinished	2006	HD	36	56	2.5:1	2.5:1	810			36.5	O	LE	
7	Kaddem	1958	1959	HD	30.78	3.28				215.30	4955.44	31.24	O	HE	14158
8	Kaila	1952	1959	HD	23.08	3.5			213.3	13.98			S	HE	
9	Dantiwada	1965	sept, 1973	HD	61	6			4881	464.00	6654	61.6	O	ME	11950
10	Machhu-II	1972	Aug,1979	CD	22.56	6.1	3:1	2:1	5210	100.55	5663	28.66	O	ME	16307
11	Mitti	1982	1988	HD	16.02					17.40		16.1	O	ME	
12	Pratappura	1930	2005	HD	10.67	3			2500	4.12			P	HE	
13	Jamuniya	1915	2002	HD	15.4	3.3			2772	9.209	108		P	HE	1800
14	Nandgavan		2005	CD	18.64				735	2.06	855.16	19.64	O	HE	4100
15	Nanaksagar	1962	aug,1967	HD	16.5				19200	209.80	1600	16.6	O	HE	9711

HD = Homogeneous Dam, CD= Composite Dam, O = Overtopping Failure, P= Piping, S= Seepage, HE= High erodibility, ME=Medium Erodibility, & LE=Low erodibility

3 METHODOLOGY USED FOR PRESENT BREACH EQUATIONS

This paper evaluates five breaching parameters (breach depth, breach top width, breach average width, peak outflow rate and failure time) individually as outcomes of multivariate regression analysis. We consider three predictors [dam height (H_d), reservoir capacity (V), dam average thickness (W)] and three discrete variables [dam type (X_4), failure mode (X_5), dam erodibility (X_6)] that are expressed as dummy variables in multivariate regression analysis. Xu and Zhang (2009) expressed outcome and predictors non-dimensionally and evaluated it as additive form (linear) and multiplicative form (non-linear). We follow the same approach of additive and multiplicative multivariable regression analysis for dimensionally and non-dimensionally stability. Dimensional and non-dimensional breach parameters are shown in Table 5.

Table 5 Breaching parameters and predictors for multivariate regression

Breaching Parameters	Predictors
Dimensional (m, m³/s, hr)	Dimensional (m, hr)
$Y_1=H_b$ (Breach Depth)	$X_1=H_d$ (Dam height)
$Y_2=B_t$ (Breach Top Width)	$X_2 = V^{1/3}$ (Reservoir capacity)
$Y_3=B_{avg}$ (Average Breach Width)	$X_3=W$ (Average thickness of dam)
$Y_4=Q_p$ (Peak Discharge)	
$Y_5=T_f$ (Failure time)	
Non-dimensional	Non-dimensional
$Y_1 = H_b/H_d$ (Breach Depth)	$X_1 = H_d/ H_r$ (Dam height)
$Y_2 = B_t/H_b$ (Breach Top Width)	$X_2 = V_w^{1/3}/H_w$ (Reservoir Shape Coefficient)
$Y_3 = B_{avg}/H_b$ (Average Breach Width)	
$Y_4 = Q_p/\sqrt{gV_w^{5/3}}$ (Peak Discharge)	Dam Type
$Y_5 = T_f/T_r$ (failure time)	With Corewalls
	Concrete Faced
	Failure Mode
	Overtopping
	Piping/seepage erosion
	Dam Erodibility
	High Erodibility (HE)
	Medium Erodibility (ME)
	Low Erodibility (LE)
	Homogeneous/zoned-fill

Multivariable regression is a linear transformation of the X variables such that the sum of squared deviations of the observed and predicted Y is minimalized. In multivariable linear regression the value of the dependent variable depends on several independent variables instead of one. This paper is following the multiplicative regression procedure proposed in Xu and Zhang (2009). Here we add one more predictor to the regression equation and carryout the analysis for both dimensional and non-dimensional forms. Additive form (linear) and multiplicative form (nonlinear) as shown below are used to establish empirical relationships.

Additive (linear) multivariate regression equation is written as

$$Y_i = b_0 + b_1X_1 + b_2X_2 + b_3X_3 + (b_{41}X_{41} + b_{42}X_{42}) + (b_{51}X_{51}) + (b_{61}X_{61} + b_{62}X_{62}) \quad [1]$$

Multiplicative (non-linear) multivariate regression equation is expressed as

$$Y_i = b_0X_1^{b_1}X_2^{b_2}X_3^{b_3}(X_{41}^{b_{41}}X_{42}^{b_{42}})(X_{51}^{b_{51}})(X_{61}^{b_{61}}X_{62}^{b_{62}}) \quad [2]$$

where, Y_i ($i=1, 2...5$) are the five breaching parameters (dependent variable) X_{is} = predictors and b_{is} = regression coefficient. Equation (2) can be rearranged to additive form by taking logarithm to both side of equation as

$$\ln Y_i = \ln b_0 + b_1 \ln X_1 + b_2 \ln X_2 + b_3 \ln X_3 + b_{41} \ln X_{41} + \dots + b_{62} \ln X_{62} \quad [3]$$

The process of establishing the empirical formula for dam breach parameters includes the following steps:

1. While selecting predictors ($X_1, X_2, X_3, X_4, X_5,$ and X_6) from the case study for finding breach parameter (Y_i), it may be possible that some predictors are not known. Proper selection of predictors is the first step of finding Y_i .
2. Conduct multivariate regression analysis (additive and multiplicative) for all the six predictors and then carryout the regression by considering different combinations of predictors. The model with higher value of regression coefficient R^2 is preferred.
3. The selected model is used to establish the empirical formula for breach parameters.

4 PROPOSED EMPIRICAL EQUATION

4.1 Breach depth

We propose two equations for breach depth. For the formulation of breach depth, the additive and multiplicative regression analysis result for different cases are summarized and a high regression coefficient model is chosen for establishing the empirical formula for beach depth. Eq. [4] and Eq. [5] are developed from the multiplicative nonlinear regression model. Fig. 2 and Fig. 3 show the result for observed versus predicted breach depth from Eq. (4) and Eq. (5) respectively.

$$H_b = 0.618 H_d^{0.61} V^{0.05} W^{0.18} \quad [4]$$

$$H_b = 0.805 H_d^{0.855} (V)^{0.029} e^{C_1} \quad [5]$$

where, $C_1 = b_1 + b_2$, $b_1 = -0.064$ for overtopping, -0.111 for Piping Failure, $b_2 = 0.246$ for High erodibility, 0.196 for Low erodibility

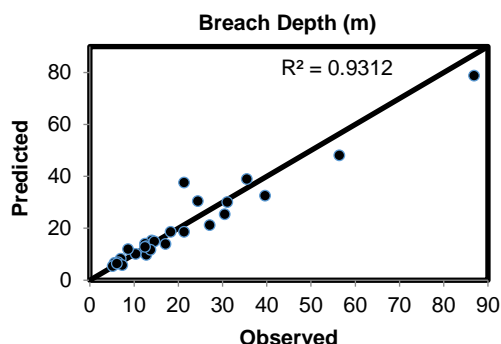


Figure 2 Observed Vs Predicted H_b for Eq. [4]

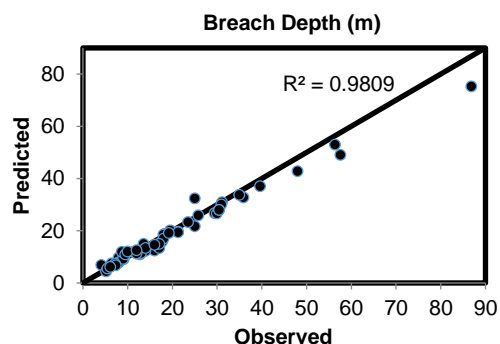


Figure 3 Observed Vs Predicted H_b for Eq. [5]

4.2 Breach top width

Using the same analysis procedure, the regression results for all the six breaching parameters and their combination are obtained. The best prediction model has been selected for developing the empirical formula for breach top width. For breach top width the non-dimensional multiplicative model with regression coefficient of 0.628 has been selected. The developed Eq. [6] gives correlation coefficient of 0.83 (Fig.4) for observed versus predicted breach top width.

$$B_t = 0.59(H_b)(H_d)^{0.089}(V_w^{1/3}/H_w)^{0.528} e^{C_2} \quad [6]$$

where, $C_2 = b_1 + b_2 + b_3$, $b_1 = -0.095$ for DC, -0.206 for HD and ZD, $b_2 = 0.447$ for Overtopping, $b_3 = 0.7$ for HE, 0.216 for ME

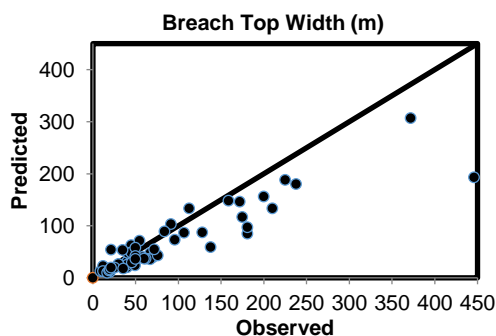


Figure 4 Observed Vs Predicted B_t

4.3 Average breach width

Multiplicative regression model having 0.667 coefficient of determination can be used for the formulation of average breach width (Eq. [7]).

$$B_{avg} = 0.425H_bH_d^{0.125}\left(V_w^{0.33}/H_w\right)^{0.66}e^{C_3} \quad [7]$$

where, $C_3 = b_1 + b_2 + b_3$, $b_1 = -0.166$ for dam with DC, -0.302 for HD and ZD, $b_2 = 0.429$ for overtopping, $b_3 = 0.628$ for HE, 0.238 for ME

4.4 Peak outflow rate

Two new empirical Eq. [8] along with Eq. [9] are proposed for the calculation of peak discharge from the breached dam. Exact prediction of peak discharge is an important task because this can be routed to the downstream. Many factors such as reservoir capacity, height of dam etc. are involved in affecting the peak discharge. With these factors many researchers develop there empirical formulas for peak discharge. Here, multivariable regression analysis are carried out for peak discharge prediction with new control variables such as breach parameters (breach depth, average breach width etc.) and try to bring out the best model by inserting each non-dimensional breach parameter as a predictor with the combination of five predictors (dam height, reservoir shape coefficient, dam type, failure mode, and dam erodibility). We found that average breach width is the most important factor on peak discharge estimate as compared to the other breach parameters. The linear correlation coefficient for observed peak discharge versus non dimensional average breach width is 0.46 and it shows an increasing trend i.e. with the increase in breach width, the peak discharge increases. Formulation of peak discharge follows the same procedure as followed in breach depth formulation with only difference being the inclusion of non-dimensional average breach width as a new predictor.

$$Q_p = \left(\frac{B_{avg}}{H_b}\right)^{0.641}(V_w)^{0.25}(H_w)^{1.738}e^{C_4} \quad [8]$$

where, $C_4 = b_1 + b_2 + b_3$, $b_1 = 0.416$ for DC, 0.113 for HD and ZD, $b_2 = 0.218$ for Overtopping, $b_3 = 0.9$ for HE, 0.848 for ME and

$$Q_p = 0.0105(H_d)^{0.272}(H_w)^{1.358}(V_w)^{0.38}e^{C_4} \quad [9]$$

where, $C_4 = b_1 + b_2 + b_3$, $b_1 = 0.176$ for DC, 0.033 for HD and ZD, $b_2 = 0.75$ for Overtopping, $b_3 = 1.52$ for HE, 1.06 for ME

4.5 Failure time

The reliability of prediction of failure time is too insignificant from the documented data of Xu-Zhang (2009) that does not clearly justify weather it is a breach initiation time or breach formation time. In case study data by Whal (1988) the breach formation time is clearly mentioned but the number of data sets are only a few. Multivariate regression analysis (additive and multiplicative) is carried out. The regression model selected for the formulation of breach failure time is additive form as shown in Eq. [10]

$$T_f = 0.311 + 0.029(H_d) + 0.23(V_w^{1/3}/H_w) + C_5 \quad [10]$$

where, $C_5 = b_1 + b_2 + b_3$, $b_1 = 0.164$ for DC, 0.725 for HD/ZD, $b_2 = 0.145$ for Overtopping, $b_3 = -2.49$ for HE, -2.11 for LE

5 RESULT COMPARISSON WITH EXISTING BREACH EQUATION

The observed versus predicted average breach width and peak outflow for present study are compared with that of Xu-Zhang (2009), Froehlich (2008) and Bureau of Reclamation (1988) in Figure 5 and 6. The developed equation in the present study gives higher degree of accuracy in terms of correlation coefficient as can be seen in the two Figures. The linear equation developed for failure time is better than that proposed by Xu-Zhang (2009) as can be seen in terms of correlation coefficient given in the Figure 7.

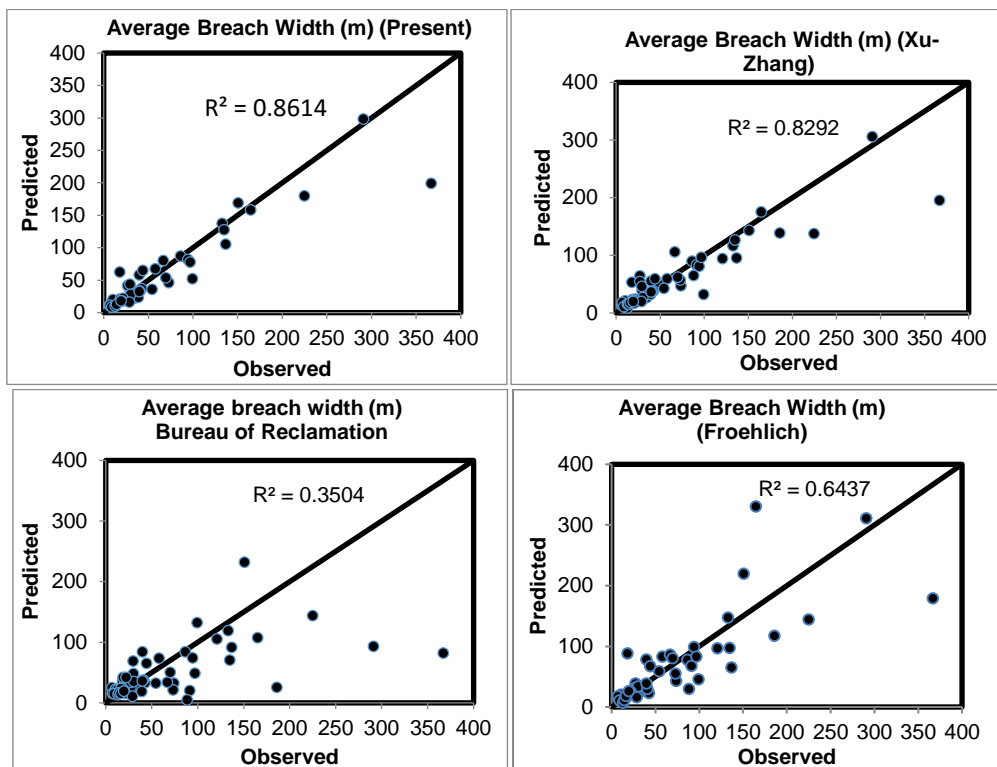


Figure 5 Comparison between average breach width formula developed by different researchers

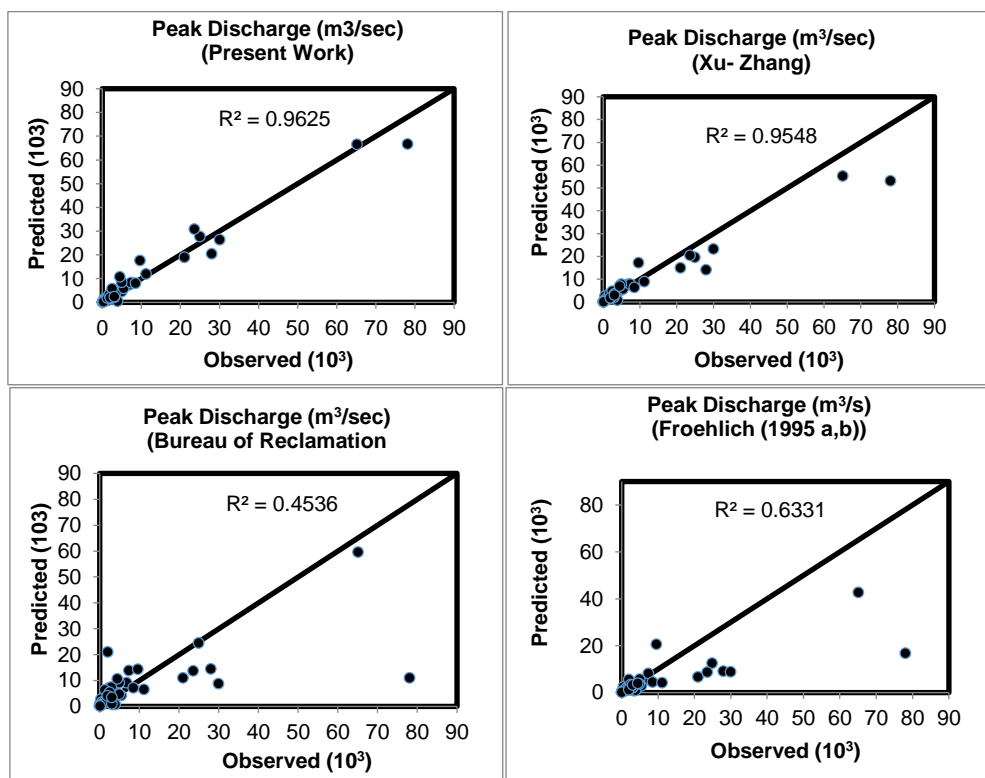


Figure 6 Comparison between peak outflow rate (m³/s) formulas developed by different researcher

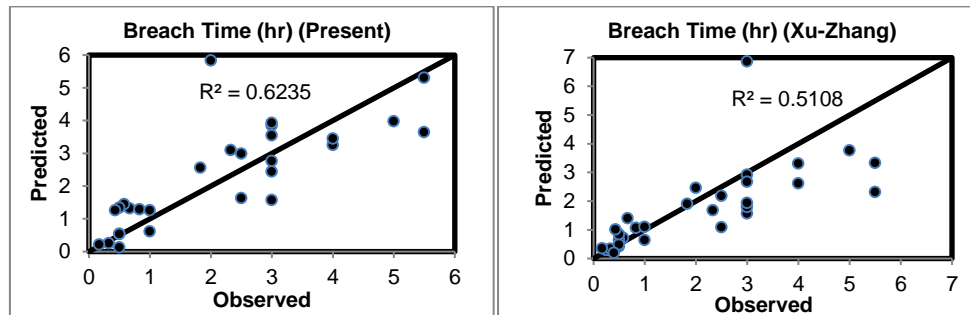


Figure 7 Comparison of failure time (hour) with Xu-Zhang (2009) work

6 BREACH PARAMETER PREDICTION FOR INDIAN CASE STUDIES

Case studies carried out using data from Indian dams are presented in Table 6 to demonstrate the application of breach predictor equations and the uncertainties. Whal (1988) study has found large uncertainty in the breach parameters and as such their application for dam break analysis is under question mark. Standard error of estimate has been calculated for breach parameters and it is found that all the breach parameter equations resemble huge difference between observed vs predicted values. If we consider the Standard error of estimate for average breach width calculation the Bureau of Reclamation (BR) (1982, 1988), Froehlich (1995) and present work gives less uncertainty. In most of the cases the observed data for peak out flow is not available because the discharge measurement stations are not placed at downstream region. It is quite relevant that peak outflow from breached dam is more as compared to the peak inflow to the reservoir. Prediction of breach time using the proposed equation is found quite comparable with that using Froehlich (2008) approach for Indian dams. The equation proposed by Xu-Zhang, over predict the breach time. Over prediction of breach time affects the emergency action plan as it under predicts the peak outflow from the dam breach. On the other hand the approach by Bureau of Reclamation under predict the breach time. Usually for breach depth calculations, the dam height has been used for the analysis purposes which are quit relevant. The present model for the calculation of breach depth is found to be quite comparable with that proposed by Xu-Zhang which can be adequate for Indian dam failure conditions.

7 CONCLUSION

Several techniques are available for estimating the breach parameters resulting from dam overtopping and subsequent failure. These techniques are predominately empirical based on fitting relationships between the key parameters. This paper has presented new equations based on 156 dam failure cases using both linear and nonlinear multivariable regression methods for predicting embankment breach parameters and peak outflow rate. For the first time dam failure data of Indian dams are collected and presented in this paper. Comparisons are made with the other developed empirical equations.

It is found that the present developed equations are better suited to calculate the breach parameters with higher degree of accuracy in terms of correlation coefficient than the other approaches proposed by Bureau of Reclamation (BR), Froehlich and Xu-Zhang. Further, the predictor parameters such as the dam height, reservoir volume and erodibility are more influencing factors in order for predicting the breach parameters than the other predictors. Dam average thickness also plays important role for the calculation of breach depth. Developed equations need more analysis and improvement by including more dam failure data. No sensitivity analysis has been carried out in this paper to rank the influence of each parameter affecting the dam failures.

VARIABLES

B, B_{avg} = breach width (average), m
 B_t = breach top width, m
 g = acceleration due to gravity, 9.8 m/s²
 H_b , h_b = height of breach, m
 H_d , h_d = height of dam, m
 H_r = dam reference height = 15 m
 H_w , h_w = height of water above breach bottom, m

Q_p = peak outflow, m³/s
 t_f , T_f = failure time, hr
 T_r = time reference = 1 hr
 V_w = volume of water above breach bottom, m³
 V = Storage capacity of reservoir m³
 W = average dam thickness
 t_f^* = Dimensionless breach formation time, $t_f / \sqrt{gh_b}$

Table 6 Comparisons of breach parameters for dam failure cases in India

DAM	Average Breach Width (m)					Peak Outflow (m ³ /sec)				Breach time (Hr)					Breach Depth (m)		
	Observed	BR (1982,1988)	Froehlich (2008)	Xu-Zhang	Present	BR (1982,1988)	Froehlich (2008)	Xu-Zhang	Present	Observed	BR (1982,1988)	Froehlich (2008)	Xu-Zhang	Present	Observed	Xu-Zhang	Present
Kodaganar	415	40	158	101	82	2260	3624	6190	4199		1.7	5.2	6.9	8.3	12.2	12.1	13.9
Ghurlijor	60	42	43	56	50	2516	1185	1936	1516		0.5	0.6	0.7	1.3	12	11.6	12.5
Panshet		194	289	321	194	42850	34230	103481	94539		3.2	1.9	1.8	2.7	63	55.8	56.4
Khadakwasla	70	114	173	198	257	15982	12100	31038	19641	4	1.9	1.6	1.5	1.9	36	33.3	35.2
Palem Vagu	90	0	86		32				3992	2	0.9	1.5	5.9	4.0	24	45.9	31.5
Palem Vagu (Unfinished)	215	110				14835								2.1	36	32.8	
Kaddem	137.2	94	224	239	211	11124	12442	35151	39431	4	2.5	3.1	2.7	3.9	30	28.3	31.5
Kaila		0	63		67				4114		0.7	0.9	1.0	1.6	23.08	23.9	21.7
Dantiwada	110	185	322	326	219	39062	36215	114355	89281		3.5	2.7	2.5	3.6	56	53.0	55.1
Machhu-II		86	166	100	145	9484	8931	15333	11374	2	1.8	2.7	4.0	3.8	22.56	21.4	22.5
Mitti dam		48				3263	2604	3591	2894							15.1	16.0
Pratappura									735				1.0	2.3		11.3	10.8
Jamuniya	50		51		51				1815	6	0.6	1.1	1.1	2.1	15.4	16.2	15.2
Nandgavan	90	59	46	67	83	4714	1775	2917	3422	0.33	0.5	0.4	0.5	0.1	18.64	17.8	18.0
Nanaksagar	150	50	199	212	164	3453	5637	15523	14131		2.2	5.2	3.9	7.3	16.8	15.6	18.5
Standard Error of Estimate		135.4	121.9	158.9	136.6	13754	11181	46892	35300		2.50	2.29	2.92	2.12	12.2	12.1	13.9

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