Prediction of Ultimate Bearing Capacity of Eccentrically Loaded Rectangular Foundations using ANN

B P Sethy[1], C R Patra[2], N Sivakugan[3], B M Das[4]

[1]Research Scholar, National Institute of Technology, Rourkela, India, email: barada.jeetu@gmail.com
[2]Professor, National Institute of Technology, Rourkela, India, email: crpatra19@yahoo.co.in
[3]Associate Professor, James Cook University, Townsville, Australia, email: siva.sivakugan@jcu.edu.in
[4]Dean Emeritus, California State University, Sacramento, USA, email: brajamdas@gmail.com

ABSTRACT: Extensive laboratory model tests were conducted on a rectangular embedded foundation resting over homogeneous sand bed and subjected to an eccentric load to determine the ultimate bearing capacity. The depth of embedment varies from 0 to 1B with an increment of 0.5B; where *B* is the width of foundation and the eccentricity ratio (*e*/*B*) varies from 0 to 0.15 with an increment of 0.05. Based on the laboratory model test results, a neural network model is developed to estimate the reduction factor (*RF*). The reduction factor can be used to estimate the ultimate bearing capacity of an eccentrically loaded foundation from the ultimate bearing capacity of a centrally loaded foundation. A thorough sensitivity analysis was carried out to determine the important parameters affecting the reduction factor. Importance was given on the construction of neural interpretation diagram, and based on this diagram, whether direct or inverse relationships exist between the input and output parameters was determined. The results from artificial neural network (ANN) were compared with the laboratory model test results and these results are well matched.

Keywords: *Eccentric load, rectangular foundation, depth of embedment, sand, neural network, reduction factor*.

INTRODUCTION

During the last thirty years, a number of laboratory model test results and few field test results have been published that are related to the ultimate bearing capacity of shallow foundation resting over homogeneous sand bed and clay. Most of the experimental studies were related to centric loading and embedded foundation condition. However, none of the published studies address the effect of load eccentricity on the ultimate bearing capacity of rectangular foundation using ANN. The purpose of this study is to develop a neural network model from the results of laboratory model tests to estimate the reduction factor. Artificial neural network (ANN) is an artificial intelligence system inspired by the behavior of human brain and nervous system. In the present study a feed forward back propagation neural network model has been used to predict the reduction factor of eccentrically loaded rectangular foundation. Backpropagation neural network is most suitable for prediction problems and Levenberg-Marquadrt algorithm is adopted as it is efficient in comparison to gradient descent backpropagation algorithm (Goh *et al*. 2005; Hornik *et al*. 1989). By drawing a neural interpretation diagram relationship in between input and output are found out. A prediction model is developed based on the weights of the ANN model. The developed reduction factor is compared with the experimental reduction factor.

ANALYSIS AND DATA

All the laboratory model tests were conducted using a poorly graded sand with effective grain size D_{10} = 0.325 mm, uniformity coefficient C_u = 1.45, and coefficient of gradation C_c = 1.15. Model foundations used for the tests had dimensions of 100 mm × 100 mm (B/L = 1), 100 mm × 200 mm (B/L = 0.5), 100 mm × 300 mm (B/L = 0.33) and 100 mm × 500 mm ($B/L \approx 0$). Mild steel plates 30-mm thick were used to make the model foundations. The bottom of the foundation was made rough by applying glue and rolling the steel plate over sand.

Forty eight laboratory model tests were conducted. Three parameters e/B, B/L and D_f/B are used as inputs in the ANN model, and the output is the reduction factor RF given by

$$RF = \frac{q_{u(B/L,\,D_f/B,\,e/B)}}{q_{u(B/L,\,D_f/B,\,e/B=0)}} \tag{1}$$

where $q_{u(B/L,D_f/B,e/B)}$ is the ultimate bearing capacity with eccentricity ratio e/B and B/L ratio and at an embedment ratio D_f/B and $q_{u(B/L,D_f/B,e/B=0)}$ is the ultimate bearing capacity with centric vertical loading (e/B = 0) with B/L ratio and at an embedment ratio D_f/B.

Out of 48 tests, 36 tests are considered for training and the remaining 12 are considered for testing. All the inputs and output are normalized in the range of [-1, 1] before training. A feed-forward back-propagation neural network is used with hyperbolic tangent sigmoid function and linear function as the transfer function. The network is trained with Levenberg-Marquardt (LM) algorithm as it is efficient in comparison to gradient descent back-propagation algorithm. The ANN has been implemented using MATLAB V 7.11.0(R2015b).

RESULTS AND DISCUSSION

Three inputs and one output parameters were considered in the ANN model. The schematic diagram of the ANN architecture is shown in FIG. 1. which was computed from the database. The number of neurons in hidden layer is varied and the optimum number was taken based on mean square error (mse) value which was maintained at 0.001. In this ANN model there were six neurons evaluated in hidden layer as shown in

FIG. 2. Therefore the final ANN architecture as 3-6-1[i.e. 3 (input) – 6 (hidden layer neuron) – 1 (output)].

Mean square error (MSE) is defined as

$$MSE = \frac{\sum_{i=1}^{n} \left(RF_i - RF_p\right)^2}{n} \tag{2}$$

Coefficient of efficiency, R^2 is defined as

$$R^2 = \frac{E_1 - E_2}{E_1} \tag{3}$$

where,

$$E_1 = \sum_{i=1}^{n} \left(RF_i - \overline{RF}\right)^2 \tag{4}$$

and

$$E_2 = \sum_{i=1}^{n} \left(RF_p - RF_i\right)^2 \tag{5}$$

where, RF_i, $\overline{RF}$ and RF_p are the experimental, average experimental, predicted *RF* values respectively; and n = number of training data.

The coefficient of efficiency (R^2) is found to be 0.995 for training and 0.902 for testing as shown in FIGS. 3. and 4. The weights and biases of the network are presented in Table 3. These weights and biases can be utilized for interpretation of relationship in between the inputs and output, sensitivity analysis and framing an ANN model in the form of an equation. The residual analysis was carried out by calculating the residuals in between experimental reduction factor and predicted reduction factor for training data. Residuals can be defined as the difference between the experimental and predicted *RF* value and is given by

$$e_r = RF_i - RF_p \tag{6}$$

The residuals are plotted with the experimental number as shown in FIG. 5. It is observed that the residuals are evenly distributed along the horizontal axis of the plot. Therefore it can be said that the network is well trained and can be used for prediction with reasonable accuracy.

SENSITIVITY ANALYSIS

Sensitivity analysis was carried out for selection of important input variables. Different approaches have been suggested to select the important input variables. The Pearson correlation coefficient is one of them in selecting proper inputs for the ANN model. It was approached by Guyon and Elisseff (2003) and Wilby *et al*. (2003). Goh (1994) and Sahin *et al*. (2002) Behera, *et.al*. (2013) have used Garson's algorithm (Garson 1991) in which the input-hidden and hidden-output weights of trained ANN model are partitioned and the absolute values of weights are taken to select the important input variables. It does not provide information on the effect of input variables in terms of direct or inverse relation to the output. Olden *et al*. (2004) proposed a connection weights approach based on the neural interpretation diagram (NID), in which the actual values of input-hidden and hidden-output weights are taken. Table 4 shows the cross-correlation of the three input parameters with the reduction factor (*RF*) value. From the table it can be seen that *RF* is highly correlated to e/B with a values of 0.975 followed by D_f/B and B/L. The relative importance, *quantified through the parameter* S_i of three input parameters as per Garson's algorithm is presented in Table 5. The e/B is found to be the most important input parameters with relative importance value being 45.08% followed by 36.41% for B/L and 18.51% for D_f/B. As per the connection weight approach (Olden *et al*. 2004) the relative importance of the present input variables is also presented in Table 5. B/L is the most important input parameter (S_i = 8.6) followed by D_f/B (S_i = 1.38) and e/B (S_i = -1.06). The S_i values being positive imply that both B/L and D_f/B are directly related and e/B is indirectly related to *RF*. In other words increase in B/L or D_f/B leads to increase in *RF* and leads to increase in ultimate bearing capacity. Increasing e/B decreases the *RF*, and hence decreases the ultimate bearing capacity.

NEURAL INTERPRETATION DIAGRAM (NID)

Ozesmi and Ozesmi (1999) proposed neural interpretation diagram for visual interpretation of the connection weight among the neurons. For the present study with the weights as obtained and shown in Table 3, an NID is presented in FIG. 7. The lines joining the input-hidden and hidden output neurons represent the weights. The positive weights are represented by solid lines and negative weights by dashed lines and the thickness of the line is proportional to its magnitude.

It is seen from Table 5 that S_i values for parameters B/L and D_f/B are positive indicating that both the parameters are directly related to *RF* values, whereas S_i values for parameter e/B being negative is indirectly related to *RF* values. This is shown in FIG. 7. Therefore, the developed ANN model is not a black box and could explain the physical effect of input parameters on the output.

ANN MODEL EQUATION FOR REDUCTION FACTOR BASED ON TRAINED NEURAL NETWORK

A model equation is developed using the weights obtained from trained neural network model (Goh *et al*. 2005). The mathematical equation relating input parameters (B/L, e/B and D_f/B) to output given by

$$RF_n = f_n\left\{b_0 + \sum_{k=1}^{h}\left[w_k\, f_n\left(b_{hk} + \sum_{i=1}^{m} w_{ik} X_i\right)\right]\right\} \tag{7}$$

where RF_n is the normalized value of RF in the range [-1, 1], f_n is the transfer function, h is the number of neurons in the hidden layer, X_i is the normalized value of inputs in the range [-1, 1], m is the number of input variables, w_{ik} is the connection weight between the i^{th} layer of input and k^{th} neuron of hidden layer, w_k is the connection weight between the k^{th} neuron of hidden layer and single output neuron, b_{hk} is the bias at the k^{th} neuron of hidden layer and b_0 is the bias at the output layer.

The model equation of RF of shallow rectangular foundations subjected to eccentrically inclined load was formulated using the values of the weights and biases shown in Table 3 as per the following steps.

Step 1

The input parameters were normalized in the range [-1, 1] by the following expressions

$$X_n = 2\left(\frac{X_n - X_{\min}}{X_{\max} - X_{\min}}\right) \tag{8}$$

Step 2

Calculate the normalized value of reduction factor (RF_n) using the following expressions

$$A_1 = -0.0679\left(\frac{B}{L}\right)_n + 0.9077\left(\frac{e}{B}\right)_n + 0.0742\left(\frac{D_f}{B}\right)_n + 2.1 \tag{9}$$

$$A_2 = 11.43\left(\frac{B}{L}\right)_n - 18.11\left(\frac{e}{B}\right)_n - 0.95\left(\frac{D_f}{B}\right)_n + 20.89 \tag{10}$$

$$A_3 = 24.94\left(\frac{B}{L}\right)_n + 15.28\left(\frac{e}{B}\right)_n + 13.52\left(\frac{D_f}{B}\right)_n + 3.88 \tag{11}$$

$$A_1 = 26.69\left(\frac{B}{L}\right)_n + 1.16\left(\frac{e}{B}\right)_n - 14.61\left(\frac{D_f}{B}\right)_n + 10.28 \tag{12}$$

$$A_1 = 0.56\left(\frac{B}{L}\right)_n + 2.18\left(\frac{e}{B}\right)_n 0.83\left(\frac{D_f}{B}\right)_n - 1.86 \tag{13}$$

$$A_1 = 1.13\left(\frac{B}{L}\right)_n + 0.74\left(\frac{e}{B}\right)_n - 0.41\left(\frac{D_f}{B}\right)_n + 0.94 \tag{14}$$

$$B_1 = -4.36\left(\frac{e^{A_1} - e^{-A_1}}{e^{A_1} + e^{-A_1}}\right) \tag{15}$$

$$B_2 = -0.11\left(\frac{e^{A_1} - e^{-A_1}}{e^{A_1} + e^{-A_1}}\right) \tag{16}$$

$$B_3 = 0.14\left(\frac{e^{A_1} - e^{-A_1}}{e^{A_1} + e^{-A_1}}\right) \tag{17}$$

$$B_4 = -0.26\left(\frac{e^{A_1} - e^{-A_1}}{e^{A_1} + e^{-A_1}}\right) \tag{18}$$

$$B_5 = -0.52\left(\frac{e^{A_1} - e^{-A_1}}{e^{A_1} + e^{-A_1}}\right) \tag{19}$$

$$B_6 = -0.63\left(\frac{e^{A_1} - e^{-A_1}}{e^{A_1} + e^{-A_1}}\right) \tag{20}$$

$$C_1 = B_1 + B_2 + B_3 + B_4 + B_5 + B_6 + 4.27 \tag{21}$$

$$RF_n = C_1 \tag{22}$$

Step 3

Denormalize the RF_n value obtained from Eq. 22 to actual *RF* as

$$RF = 0.5(RF_n + 1)\left(RF_{\max} - RF_{\min}\right) + RF_{\min} \tag{23}$$

$$RF = 0.5(RF_n + 1)\left(1 - 0.52\right) + 0.52 \tag{24}$$

FIG. 6. Shows the comparison of reduction factor obtained from Eq. 23 and Eq. 1. It can be seen that the ANN results are closer to the experimental value. The deviation between

the experimental and predicted *RF* is within ± 10% except two values as shown in Table 1. The proposed ANN model can be used as an effective tool in predicting the *RF* and hence, the ultimate bearing capacity of an eccentrically loaded rectangular footing.

CONCLUSION

Based on developed neural network model, the following conclusions may be drawn.

1. The errors are distributed evenly along the centerline as per residual analysis. It can be concluded that the network was well trained and can predict the reduction factor *RF* with reasonable accuracy.
2. Based on Pearson correlation coefficient, it was observed that e/B is the most important input parameter followed by B/L and D_f/B and as per the Garson's algorithm e/B is the most important input parameter followed by B/L and D_f/B.
3. The developed ANN model could explain the physical effect of inputs on the output, as described in NID. It has been observed that e/B is inversely related to *RF*, whereas B/L and D_f/B are directly related to *RF*.
4. A model equation is developed based on the trained weights of ANN

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Table 1. Database used for ANN model and compared with experimental results

Data Type	Expt. No.	***B*/*L***	***e*/*B***	D_f /***B***	Experimental q_u (kN/m^2)	$RF_{expt.}$	RF_{ANN}	Deviation (%)
Training	1	0	0	0	166.67	1.00	1.00	0.00
	2	0	0.1	0	109.87	0.66	0.66	-0.04
	3	0	0.15	0	86.33	0.52	0.52	-0.49
	4	0	0.05	0.5	226.61	0.86	0.85	0.66
	5	0	0.1	0.5	195.22	0.74	0.74	-0.04
	6	0	0.15	0.5	164.81	0.62	0.63	-0.53
	7	0	0	1	353.16	1.00	1.01	-0.68
	8	0	0.05	1	313.92	0.89	0.89	0.31
	9	0	0.1	1	278.6	0.79	0.80	-1.78
	10	0.33	0	0	131	1.00	1.02	-2.30
	11	0.33	0.05	0	109	0.83	0.83	0.58
	12	0.33	0.15	0	71	0.54	0.54	0.16
	13	0.33	0	0.5	224	1.00	1.00	-0.02
	14	0.33	0.1	0.5	181	0.81	0.81	-0.09
	15	0.33	0.15	0.5	161	0.72	0.71	0.63
	16	0.33	0.05	1	289	0.86	0.87	-1.11
	17	0.33	0.1	1	265	0.79	0.76	3.31
	18	0.33	0.15	1	239	0.71	0.71	0.42
	19	0.5	0	0	128	1.00	0.98	1.78
	20	0.5	0.05	0	102	0.80	0.80	-0.37
	21	0.5	0.1	0	86	0.67	0.66	2.32
	22	0.5	0	0.5	212	1.00	1.01	-0.98
	23	0.5	0.05	0.5	175	0.83	0.83	-0.94
	24	0.5	0.15	0.5	134	0.63	0.63	-0.42
	25	0.5	0	1	327	1.00	0.99	0.79
	26	0.5	0.1	1	230	0.70	0.72	-2.32
	27	0.5	0.15	1	200	0.61	0.62	-1.30
	28	1	0.05	0	102	0.84	0.84	0.59
	29	1	0.1	0	78	0.64	0.65	-0.82
	30	1	0.15	0	67	0.55	0.55	1.09
	31	1	0	0.5	238	1.00	1.00	-0.04
	32	1	0.05	0.5	198	0.83	0.85	-2.06
	33	1	0.1	0.5	176	0.74	0.74	-0.69
	34	1	0	1	339	1.00	1.00	0.29

	35	1	0.05	1	294	0.87	0.85	1.62
	36	1	0.15	1	227	0.67	0.66	1.50
Testing	37	0	0.05	0	133.42	0.80	0.80	0.24
	38	0	0	0.5	264.87	1.00	1.01	-0.52
	39	0	0.15	1	245.25	0.69	0.82	-17.82
	40	0.33	0.1	0	94	0.72	0.69	4.06
	41	0.33	0.05	0.5	195	0.87	0.85	2.29
	42	0.33	0	1	336	1.00	0.97	3.31
	43	0.5	0.15	0	68	0.53	0.57	-6.50
	44	0.5	0.1	0.5	152	0.72	0.80	-11.80
	45	0.5	0.05	1	265	0.81	0.81	-0.51
	46	1	0	0	121	1.00	0.98	1.70
	47	1	0.15	0.5	143	0.60	0.58	4.07
	48	1	0.1	1	258	0.76	0.79	-3.99

Table 2. Statistical values of the parameters

Parameters	**Maximum value**	**Minimum value**	**Average Value**	**Standard deviation**
e/B	0.15	0	0.075	0.056
B/L	1	0	0.46	0.36
D_f/B	1	0	0.5	0.41
RF	1	0.52	0.8	0.15

Table 3. Values of connection weights and biases

Neuron		**Weight**				
		w_{ik}		w_k	**Bias**	
	B/L	e/B	D_f/B	RF	b_{hk}	b_0
Hidden neuron 1(k=1)	-0.0679	0.9077	0.0742	-4.3646	2.1037	4.2743
Hidden neuron 2(k=2)	11.4264	-18.1075	-0.9497	-0.1099	20.8869	
Hidden neuron 3(k=3)	24.9425	15.2804	13.5236	0.1446	38838	
Hidden neuron 4(k=4)	26.6906	1.1618	-14.609	0.2608	10.2778	
Hidden neuron 5(k=5)	0.5598	2.1791	-0.8329	-0.5202	-1.8638	
Hidden neuron 6(k=6)	1.131	0.7402	-0.4105	-0.6329	0.9429	

Table 4. Cross-correlation of input and output for reduction factor

	B/L	***e/B***	***D_f/B***	***RF_{expt}***
(B/L)	1	-0.1	0	0.012
(e/B)		1	0	0.975
(D_f/B)			1	0.167
RF_{expt}				1

Table 5. Relative importance of different inputs as per Garson's algorithm and connection weight approach

Parameters	Garson's algorithm		Connection weight approach	
	Relative importance	Ranking of input as per relative importance	S_i values as per connection weight approach	Ranking of input as per relative importance
B/L	36.41	2	8.6	1
e/B	45.08	1	-1.06	3
D_f/B	18.51	3	1.38	2

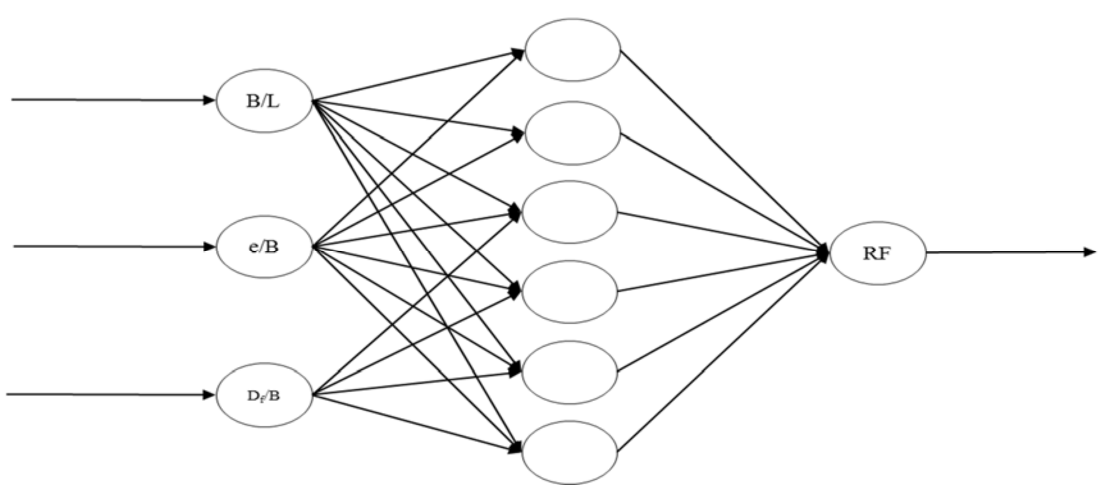

FIG.1. ANN architecture

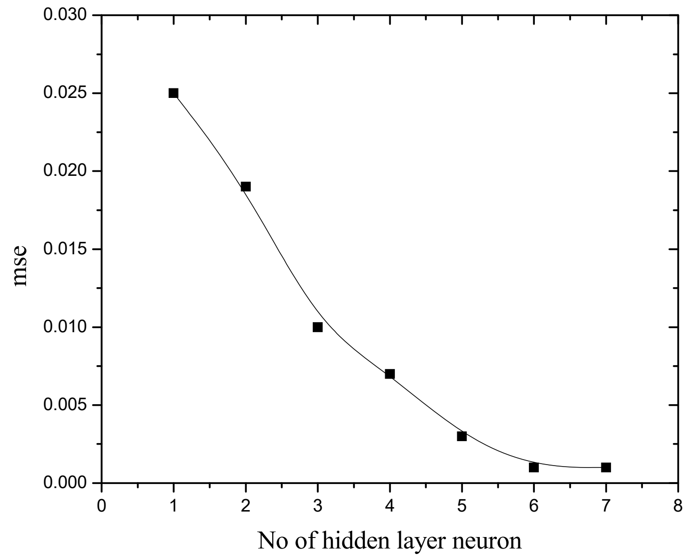

FIG. 2. Variation of hidden layer neuron with mean square error (mse)

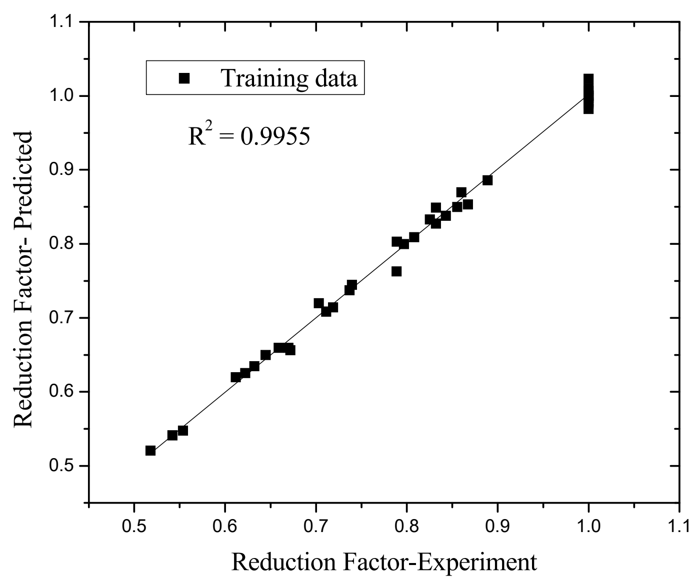

FIG. 3. Correlation between prediction reduction factors with experimental reduction factor for training data

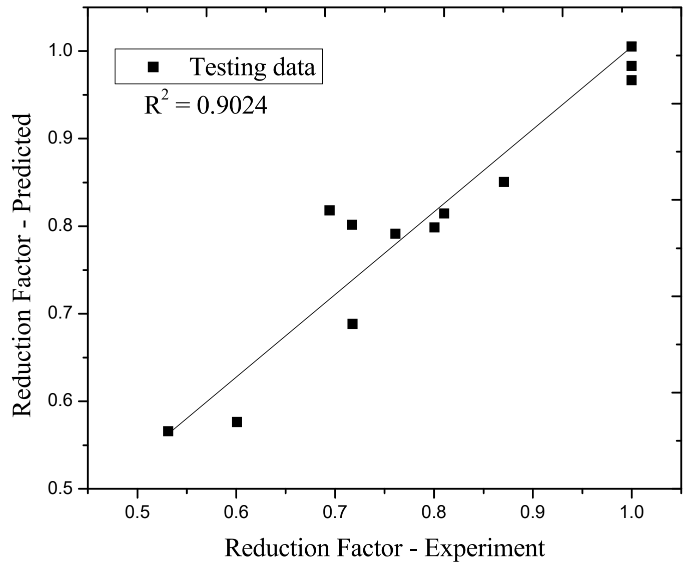

FIG. 4. Correlation between prediction reduction factors with experimental reduction factor for testing data

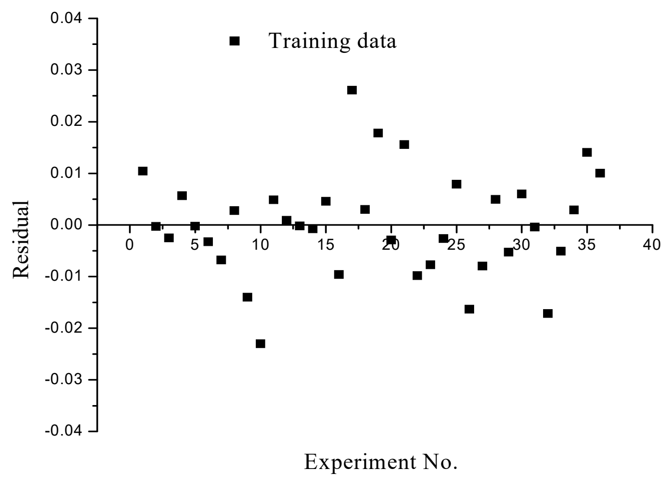

FIG. 5. Residual distribution of training data

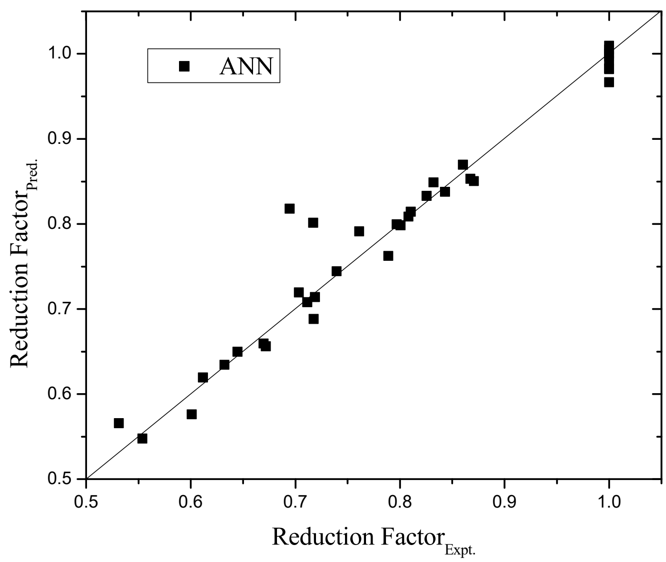

FIG. 6. Comparison of ANN results with experimental RF

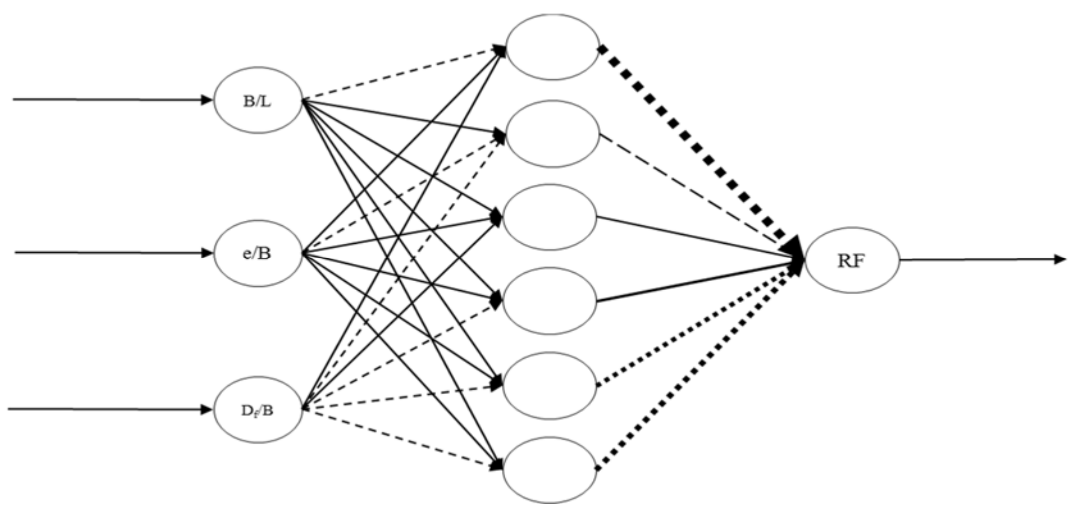

FIG. 7. Neural interpretation diagram (NID) showing lines representing connection weights and effects of inputs on reduction factor (RF)